New Non-Integer Indirect Adaptive Control for a Class of Non-Integer Order System with Non Prior Nowledge

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*Abstract*— In this study a new non integer indirect adaptive control problem with reference model is suggested for class of non-integer order systems. The objective of model reference control is to include the output of the controlled plant to track the output of a given reference fractional model by using the concept of on-line goal adaptation. The stability of the closed-loop system is analyzed via Lyapunov method. Finally, a simulation results with Matlab are presented to illustrate the effectiveness of the proposed method of indirect fractional model reference adaptive control.

Keywords- ***Non Integer Order System; Fractional Adaptive Control; MRAC; Control System; Lyapunov Stability***

Introduction

Model reference adaptive control (MRAC) is one of the main approaches of adaptive control, the fundamental goal of adaptive control strategy is to treat with system uncertainty and/or time varying system parameters. The essential idea in the in the adaptive control is to design the controller which adapts itself to plant uncertainty or time variance in the plant dynamics. Adaptive control is one of the important research in the literature on control when the model of the system is uncertain, see e.g. the papers [1], [2], [3], [4] and the books [5] [6], [7], furthermore, references in that. Fundamental research efforts have been focused on the model reference adaptive control (MRAC) designs for linear plants with parametric uncertainty. In general case, in model reference adaptive control, the goal is to have a system that follows a certain system as a reference model. In reality, generally adaptive controller is designed to achieve this goal, despite the uncertainties that may occur in the parameters of the system [8].

Fractional-order calculus is another area of mathematics that deals with integrals and derivatives from non-integer orders. In other words, it is a generalization of the traditional calculus that leads to similar concepts and tools, but with a much wider applicability. In the last three decades, fractional calculus has been rediscovered by engineers and scientists. He is applied in an increasing number of fields, namely in the area of control theory, as an example, modeling of viscoelastic materials has been done in many old and recent works using fractional order derivatives [10-11].

The success of non-integer order controllers is unquestionable with a lot of success due to emerging of effective methods in differentiation and integration of non-integer order equations.

Based on the results already reported in the literature, we propose to in this paper, novel method to control a class

of linear fractional-order systems based in MRAC configuration. The controller is developed to ensure the

tracking the reference model Numerical simulations show the efficiency of the proposed scheme.

This paper is organized as follows: Mathematical preliminaries definitions are presented in Section 2. Model reference adaptive fractional control, stability proof Section 3. Simulation results are illustrated in Section 4 Finally, concluding remarks are drawn in Section 5.

Fractional-Order Definition and Preliminaries

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* 1. *Fractional calculus*

Fractional-order calculus or non-integer-order calculus is the generalization of the classical integer order calculus. There are several definitions in literature for fractional order derivative and integral. Fractional calculus is considered as an extension of integer-order calculus to non-integer order calculus. The theory of derivatives and integral of non-integer order was firstly mentioned by Leibnitz in 1965. After that, another definitions were generated by Liouville, Grünwald, Letnikov and Riemann. The integro-differential operator is given bywhere ;

 (1)

where *a* denote lower limits and *t* are the upper limits respectevely of the operator, and ∝∈R is the order of integration or differentiation[12-14]

The Riemann–Liouville derivative definition of the order *α* is described as

 (2)

Where and is the Gamma function, *f(t)*  *is*  a continuous time function.

The following definition is given by caputo[13,14]

 (3)

The Grünwald–Letnikov’s derivative definition can be written as[13,14];

 (4)



Where *h* denote the time increment ,  is a flooring operator and the binomial coefficients.

In this study the Grünwald–Letnikov’s derivative definition is used due to wide applications in engineering and well-understood physical interpretation.

* 1. *Some properties of non-integer derivatives*

Two general properties of the fractional-order derivative will be utilized, which are reviewed underneath:

**Property 1** The additive order law

 (5)

**Property 2** Caputo fractional derivative operator is a linear operator

 (6)

*a* and *b*: are real constants.

**Property 3** The fractional order derivative of  where ;

 (7)

In general, any fractional-order single input single output system can be described by a fractional differential equation of the form [11,12]

 (8)

Where indicates the fractional-orders and denote the system parameters.

If , the system (8) is called a commensurate order system, otherwise system (8) indicates an incommensurate order system

Model reference adaptive fractional control

Adaptive control is a well-researched topic in control theory that spans several. Decades The model reference adaptive control system is one of an important scheme in adaptive control. It may be treat as an adaptive system in which the desired performance is expressed in terms of a reference model. The configuration of the model reference adaptive control block is given in figure 1. In the configuration MRAC the desired behavior is specified by a reference model, and the parameters of the controller are adjusted based on the error, which is the difference between the outputs of the closed-loop system and the outputs of the reference model. the mechanism for adjusting the parameters of the controller in a model reference adaptive control can be obtained by applying stability theory of Lyapunov.

The MRAC structure is divided in four main parts; the plant, the controller, the reference model and the adjustment mechanism.

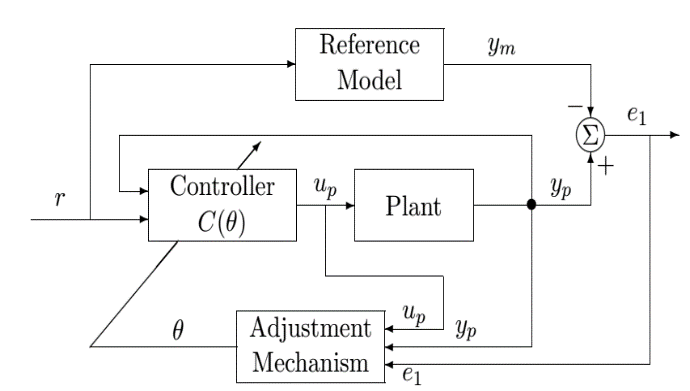


Fig.1. Block diagram scheme of adaptive control with reference model.

To introduce the model reference fractional adaptive control problem, in this article, we consider a class of commensurate fractional order systems formulated as

 (9)

where and  are the system state and the control variable respectively, *a* and *b* are unknown constant

parameters but sgn(*b*) is known. The fractional reference model was chosen to generate the desired trajectory which the plant output has to flow. The reference model is given by

 (10)

Subject to , the reference model is stable, i.e., *am<*0 and *r(t)* is the reference input. The parameters are known constants,  are measured at each time *t.*

The design objective is to make the tracking error converge to 0. Let us first design a Model Reference Control (MRC), that is, the control

design assuming all the parameters are known, to ensure that the output follows .

We express the closed-loop system in terms of the time derivative of the tracking error

 (11)

The basic state feedback state tracking problem of this section is to design a state feedback law *u*(*t*) to control the  
plant given by (9). Firstly, we define an ideal controller that perfectly cancels out the uncertainty and enables follows as

 (12)

Where

 and  (13)

The superscript (∗) denotes ideal constant values which are unknown.

Then, the tracking error equation is established as

 (14)

The tracking error converges to zero exponentially and the system is asymptotically stable.

When the plant parameters a, b are unknown, (12) cannot be implemented. To solve the control problem, we develop a new fractional indirect adaptive control design, which first adaptively updates the estimates of the plant parameter *a* and *b* and then calculates the controller parameters and in (12) from the plant parameter estimates. The real control input is

 (15)

Where

 (16)

 (17)

and  are the estimate of  respectively, at time *t*, and search for an adaptive law to generate and on-line.

Let  be the estimation errors. Now, the plant model (7) is expressed as

 (18)

Substituting (15), (16) and (17) into (18)



Based in the tracking error , from (19) and (9) we have:

 (20)

* **Lemma:** *assume* *be a continuous and* *derivable function. Then, for any time instant* *,*  
  *the following inequality holds* [15,16,17]*.*

 (21)

In order to analyze the closed-loop stability and to find the adaptive laws we consider the following measure for the error .

 (22)

The positive real design parameters and are often referred to as adaptive gains, as they can affect the speed of parameter adaptation. The taking fractional derivative of (22) with respect to time and using Lemma 1, one has

 (23)



 (25)

Noting that , Based on the updated estimates of and , we can calculate the parameters;

 (26)

From (24) and (26) we have

 (27)

This implies that as a function of *t* not increase, that is are bounded and so *u(t)* in (18). Then in (19), and from (27), , that is . Finaly by using the lemma of Barbalat’s given in [5,17,18,19] we conclude that the system is Lyapunov stable and the tracking error converge asymptotically to zero.

# eNumerical simulations

In this section to illustrate the application of the proposed adaptive fractional scheme, let us consider an unstable system defined by

 (28)

To build the adaptive controller we choose a model reference is given by :

 (29)

where .

The values of the parameter of adaptive gain were chosen as: and .

The control law necessitates the reference model, which has  
two design parameters *am*and *bm*. Both of them can be  
chosen without restraint, nevertheless it is worth to set *am* = 1 and *bm=1* to get a 0.78 gain of reference dynamics. The initial conditions of the system and reference model are .

The reference input signal is given by equation ;

 (30)

If the parameters of the system and reference model are known, the control law can be calculatedas

 (31)

From (10) we obtained

 (32)

and

 (33)

The results of simulations in this case are shown by Figures 2-3, where we show the time responses of the plant state and the output of the reference model and the control .

Fig.2. Comparison of the state plant signal with reference signal

 Fig.3. Control *u(t)*

when the parameters of the system are unknown,  are estimated by and  respectively, in turn used to determine the controller parameters gains , uses equations (16) and (17). The on-line estimate is generated by an adaptive law (26).

The control law, based on the certainty equivalence principle, is given by (15).

The initial conditions of the system and reference model are

.

The simulation results are shown in figure 4 to 8, The results of this simulation show the time responses of the plant state  and the state of the reference model  in Fig. 2 , the follow up error ,the control . The figures 6 and 7 shows that the estimated parameter **“*a(t)”*** and **“*b(t)”*** of converges to the true value. these values are same values of the system.

Form Fig.2 to Fig.8 indicates that the proposed method is more efficient. The follow up error converge to zero and the control is bounded. The results are shown in Fig. 4. It can be seen that  and  as *t* → ∞. The convergence rate can be increased by increasing the adaptation rates 

Fig.4. State plant signal with reference signal



Fig.5. Control u(t)

Fig.6. Estimate of the parameter ‘a(t)’



Fig.7. Estimate of the parameter ‘b(t)’



Fig.8. Tracking error results by the proposed approach

CONCLUSION

The model reference indirect adaptive controller for a class of non-integer systems has been presented in this paper, a new fractional indirect adaptive control scheme is presented based on a Lyapunov function. the proposed control strategy uses a MRAC configuration. The effectiveness of proposed scheme is confirmed by numerical simulation, these simulations are carried out on MATLAB software running in Windows 7, where the step size is set to be 0.001 s, and the other settings are kept default. Simulation results has verified correctness of the developed theoretical result.

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