Evaluating the Impact of Weighting Methods on the Stability of Scores for Alternatives in Multi-Criteria Decision-Making Problems

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ABSTRACT

Criteria weights play a crucial role in Multi-Criteria Decision Making (MCDM) problems when selecting the best alternative from a set of options. This study aims to compare three objective weighting methods: MEthod based on the Removal Effects of Criteria (MEREC), Entropy, and Symmetry Point of Criterion (SPC). These methods were applied to a case study involving the ranking of eight sustainable energy development alternatives, each characterized by seventeen criteria. Four representative MCDM methods, the Simple Additive Weighting (SAW), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Proximity Indexed Value (PIV), and Root Assessment Method (RAM), were also deployed. The results revealed that the Entropy method provided the most stable and consistent performance, followed by the MEREC method, with the SPC method showing the least stability.

Keywords-weight method; MEREC method; entropy method; SPC method; MCDM

I. INTRODUCTION

The MCDM methods have gained widespread recognition for their effectiveness in identifying optimal alternatives across various fields, including economics, engineering, medicine, and education [1, 2]. Two critical outcomes influencing the reliability of the MCDM are the method used to determine criteria weights and the selection of the MCDM approach itself. Both decisions substantially affect the ranking of alternatives [3-5]. Among these, the choice of a method to calculate criteria weights is particularly significant, as it directly shapes the final ranking [6, 7].

The weighting methods in MCDM can be classified into three categories: subjective, objective and hybrid methods. The subjective methods rely on the evaluator's knowledge and experience to assign weights to criteria. However, these methods are prone to inaccuracies when evaluators lack sufficient experience, potentially leading to suboptimal decisions. The objective methods, on the other hand, determine the criteria weights solely from numerical data, eliminating subjective biases. The hybrid methods combine elements of both the subjective and objective approaches, seeking to balance the evaluator input with data-driven insights [8]. Among these, the objective weighting methods are the most widely adopted due to their impartial nature, as they are unaffected by the decision-maker's personal judgments [9, 10].

The objective weighting methods encompass a variety of techniques, including MEREC [11], Entropy [12], SPC [13], Logarithmic Percentage Change-driven Objective Weighting (LOPCOW) [14], Criteria Importance Through Intercriteria Correlation (CRITIC) [15], FUCA [16], Criterion Impact Loss (CILOS) [17], and Integrated Determination of Objective Criteria Weights (IDOCRIW) [18]. Among these, the MEREC and Entropy are the most widely used. Numerous studies have evaluated the high accuracy of these two methods and recommended their application in decision-making scenarios [19].

In contrast, the SPC is a recently proposed method for determining criteria weights, introduced in January 2023 [13]. Research applying the SPC remains limited, with only a few studies exploring its use in specific contexts, such as evaluating woodworking machines [20], delivery drones [21], batteries for new energy vehicles [22], biomass sources for biofuel production [23], and technology change alternatives [24]. However, no comprehensive evaluation exists that assesses the performance of objective weighting methods in general, or the MEREC, Entropy, and SPC methods, in particular. A comparative analysis of these three methods is essential to guide users in selecting the most appropriate one. This necessity serves as the primary motivation for this study.

To compare the objective weighting methods, the latter must be integrated with the MCDM approaches to solve practical decision-making problems. Given the vast number of the MCDM methods available -ranging into the hundreds-evaluating all of them in a single study is infeasible [25, 26]. This research focuses on four representative MCDM methods: SAW, TOPSIS, PIV, and RAM.

- SAW method: It was elected as it is the foundational MCDM method and forms the basis for many subsequent methods [27]. Despite its simplicity, SAW continues to be widely applied across diverse fields, including the selection of industrial robots and flexible production systems [28], suppliers [29], and educational institutions [30].
- TOPSIS method: It was chosen for its popularity and extensive use [31]. This method has been applied in various fields, such as analyzing emerging big data [32] and selecting airplanes [33].
- PIV method: It was highlighted for its ability to minimize the rank reversal [34]. This advantage makes it a preferred option in contents, such as selecting metal cutting options [35], gearbox manufacturing materials [36], and forklifts [37].
- RAM method: A relatively recent method [38], RAM has also been utilized in studies that involve the ranking of the financial health of banks [39] and the evaluation of the digital transformation index of geographical areas [40].

II. METHODOLOGY

A. Weight Methods

1) MEREC Method

The MEREC method determines the criteria weights through six systematic steps [11]:

- **Step 1:** Create a decision matrix *X* with *m* rows and *n* columns, where *m* is the number of alternatives and *n* is the number of criteria for each alternative. Each element x_{ij} represents the value of criterion *j* for the alternative *i*.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & x_{ij} & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

- **Step 2:** Normalize the matrix whether the criterion is a benefit (B) or a cost (C) criterion.

$$n_{ij} = \frac{min\,(x_{ij})}{x_{ij}} \quad if \quad j \in B \quad (2)$$

$$n_{ij} = \frac{x_{ij}}{max\,(x_{ij})} \quad if \quad j \in C \quad (3)$$

where $min\,(x_{ij})$is the minimum value of the criterion j for all alternatives, and $max\,(x_{ij})$ is the maximum value of the criterion j for all alternatives. The benefit criteria can be machining productivity, product quality, economic development, etc., while the cost criteria can be price, pollution level, maintenance cost, and unemployment rate.

- **Step 3:** Calculate the overall performance S_i of each alternative using:

$$S_i = ln\left[1 + \left(\frac{1}{n}\sum_{j=1}^{n}\left|ln\left(n_{ij}\right)\right|\right)\right] \quad (4)$$

- **Step 4:** Calculate the modified performance S'_{ij} of each alternative:

$$S'_{ij} = ln\left[1 + \left(\frac{1}{n}\sum_{k,k\neq j}^{n}\left|ln(n_{ik})\right|\right)\right] \quad (5)$$

- **Step 5:** Calculate the absolute deviation E_j for each criterion:

$$E_j = \sum_{i}^{m}\left|S'_{ij} - S_i\right| \quad (6)$$

- **Step 6:** Determine the weight w_j for each criterion using:

$$w_j = \frac{E_j}{\sum_k E_k} \quad (7)$$

2) Entropy Method

To calculate the weights of the criteria utilizing the Entropy method, the following steps are performed [12]:

- **Step 1:** This step is identical to Step 1 of the MEREC method.
- **Step 2:** Normalize the decision matrix to ensure that all values are dimensionless, using:

$$n_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2} \quad (8)$$

- **Step 3:** Calculate the value of the Entropy measurement degree e_j for each criterion:

$$e_j = \sum_{i=1}^{m}\left[n_{ij} \times ln(n_{ij})\right] - \left(1 - \sum_{i=1}^{m} n_{ij}\right) \times ln\left(1 - \sum_{i=1}^{m} n_{ij}\right) \quad (9)$$

- **Step 4:** Calculate the weight w_j for each criterion:

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{n}(1 - e_j)} \quad (10)$$

3) SPC Method

The following steps are applied sequentially to calculate the weights for the criteria using the SPC method [13]:

- **Step 1:** Similar to step 1 of the MEREC method.
- **Step 2:** Calculate the *SPC* value for each criterion according to:

$$SPC_j = \frac{max(x_{ij}) + min(x_{ij})}{2} \quad (11)$$

where $i = 1,2,\dots,m; \forall j \in [1,$

- **Step 3:** Create the absolute distance matrix *D*:

$$D = |d_{ij}|_{mxn} = \begin{bmatrix} |x_{11} - SPC_1| & |x_{12} - SPC_2| & \cdots & |x_{1n} - SPC_n| \\ |x_{21} - SPC_1| & |x_{22} - SPC_2| & \cdots & |x_{2n} - SPC_n| \\ \cdots & \cdots & \cdots & \cdots \\ |x_{m1} - SPC_1| & |x_{m2} - SPC_2| & \cdots & |x_{mn} - SPC_n| \end{bmatrix} \quad (12)$$

- **Step 4:** Create the matrix of symmetric modules *R*:

$$R = |r_{ij}|_{m\times n} = \begin{bmatrix} \left|\frac{\sum_{i=1}^m d_{i1}}{m\times x_{11}}\right| & \left|\frac{\sum_{i=1}^m d_{i2}}{m\times x_{12}}\right| & \cdots & \left|\frac{\sum_{i=1}^m d_{in}}{m\times x_{1n}}\right| \\ \left|\frac{\sum_{i=1}^m d_{i1}}{m\times x_{21}}\right| & \left|\frac{\sum_{i=1}^m d_{i2}}{m\times x_{22}}\right| & \cdots & \left|\frac{\sum_{i=1}^m d_{in}}{m\times x_{2n}}\right| \\ \cdots & \cdots & \cdots & \cdots \\ \left|\frac{\sum_{i=1}^m d_{i1}}{m\times x_{m1}}\right| & \left|\frac{\sum_{i=1}^m d_{i2}}{m\times x_{m2}}\right| & \cdots & \left|\frac{\sum_{i=1}^m d_{in}}{m\times x_{mn}}\right| \end{bmatrix} \quad (13)$$

- **Step 5:** Calculate the modulus of symmetry of criterion Q:

$$Q = |q_{1j}|_{1\times n} = \left|\frac{\sum_{i=1}^m r_{i1}}{m} \quad \frac{\sum_{i=1}^m r_{i2}}{m} \quad \dots \quad \frac{\sum_{i=1}^m r_{in}}{m}\right| \quad (14)$$

- **Step 6:** Calculate the weights of the criteria W:

$$W = |w_{1j}|_{1\times n} = \left|\frac{q_1}{\sum_{j=1}^n q_j} \quad \frac{q_2}{\sum_{j=1}^n q_j} \quad \cdots \quad \frac{q_j}{\sum_{j=1}^n q_j}\right| \quad (15)$$

B. MCDM Methods

1) SAW Method

The procedure for ranking the alternatives through the *SAW* method consists of four steps [27]:

- **Step 1:** Similar to step 1 of the MEREC method.
- **Step 2:** Normalize the values for each criterion:

$$n_{ij} = \frac{x_{ij}}{max(x_{ij})} \quad if \quad j \in B \quad (16)$$

$$n_{ij} = \frac{min(x_{ij})}{x_{ij}} \quad if \quad j \in C \quad (17)$$

- **Step 3:** Calculate the score V_i for each alternative:

$$V_i = \sum_{j=1}^n w_j \cdot n_{ij} \quad (18)$$

- **Step 4:** Rank the alternatives according to the principle that the best alternative is the one with the highest V_i score.

2) TOPSIS Method

The procedure to rank the alternatives using the TOPSIS method consists of the following seven steps [31]:

- **Step 1:** Similar to step 1 of the MEREC method.
- **Step 2:** Normalize the decision matrix values x_{ij} using:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (19)$$

- **Step 3:** Calculate the weighted normalized values Y_{ij}:

$$Y_{ij} = w_j . n_{ij} \quad (20)$$

- **Step 4:** Determine the positive ideal solution A^+ and the negative ideal solution A^- for each criterion:

$$A^+ = \{y_1^+, y_2^+, \dots, y_j^+, \dots, y_n^+\} \quad (21)$$

$$A^- = \{y_1^-, y_2^-, \dots, y_j^-, \dots, y_n^-\} \quad (22)$$

where: y_j^+ and y_j^- are the best and worst values of criterion *j*, respectively.

- **Step 5:** Determine the values of S_i^+ and S_i^- in accordance with:

$$S_i^+ = \sqrt{\sum_{j=1}^n (y_{ij} - y_j^+)^2} \quad (23)$$

$$S_i^- = \sqrt{\sum_{j=1}^n (y_{ij} - y_j^-)^2} \quad (24)$$

where $i = 1,2,\dots,m$.

- **Step 6:** Calculate the values of C_i^*:

$$C_i^* = \frac{S_i^-}{S_i^+ + S_i^-} \quad (25)$$

where $i = 1,2,\dots,m; 0 \le C_i^* \le 1$.

- **Step 7:** Rank the alternatives according to the principle that the option with the highest C_i^* is the best one.

3) PIV Method

The following steps are applied sequentially to rank the alternatives using the PIV method [34]:

- **Step 1:** Similar to step 1 of the MEREC method.
- **Step 2:** Similar to step 2 of the TOPSIS method.
- **Step 3:** Similar to step 3 of the TOPSIS method.
- **Step 4:** Evaluate the weighted proximity index:

$$u_{ij} = Y_{ij\,max} - Y_{ij} \quad if \quad j \in B \quad (26)$$

$$u_{ij} = Y_{ij} - Y_{ij\,min} \quad if \quad j \in C \quad (27)$$

- **Step 5:** Determine the overall proximity value d_i in accordance with:

$$d_i = \sum_{j=1}^n u_i \quad (28)$$

- **Step 6:** Rank the alternatives according to the principle that the best alternative is the one with the smallest deviation score d_i.

4) RAM Method

The procedure for ranking the alternatives using the RAM method is [38]:

- **Step 1:** Similar to step 1 of the MEREC method.
- **Step 2:** Similar to step 2 of the TOPSIS method.
- **Step 3:** Similar to step 3 of the TOPSIS method.
- **Step 4:** Calculate the total weighted normalized score of the criteria:

$$S_{+i} = \sum_{j=1}^n Y_{+ij} \quad if \quad j \in B \quad (29)$$

$$S_{-i} = \sum_{j=1}^n Y_{-ij} \quad if \quad j \in C \quad (30)$$

- **Step 5:** Calculate the relative importance index of each alternative:

$$RI_i = \sqrt[2+S_{-i}]{2 + S_{+i}} \quad (31)$$

- **Step 6:** Rank the alternatives in descending order of their RI_i scores.

III. RESULTS AND DISCSSION

In this section, the impact of different weighting methods on the decision-making outcomes is evaluated by ranking eight sustainable energy development alternatives. The alternatives considered include A_1: Hydropower, A_2: Geothermal energy, A_3: Biomass energy, A_4: Wind energy, A_5: Solar energy, A_6: Concentrated solar power, A_7: Coal technology, A_8: Oil-fired power plants. These alternatives are assessed based on seventeen criteria, which characterize each energy option in terms of various environmental, economic, and technical factors. The criteria, denoted as C_1 to C_{17} are as follows:

- Capital cost (C_1): The total initial investment required to construct and install the energy system.
- Fixed operation and maintenance cost (C_2): Recurring costs that occur regardless of the electricity production, such as employee salaries, insurance, and taxes.
- Variable operation and maintenance cost (C_3): Costs that vary with the system's operating level, such as fuel (if applicable), consumables, etc.
- Reliability (C_4): The system's ability to operate continuously and stably with minimal interruptions or failures.
- Capacity (C_5): The maximum amount of electricity that the system can produce.
- Technology maturity (C_6): The level of development and popularity of the technology, including efficiency and cost.
- Resource availability (C_7): The abundance and accessibility of natural energy sources, such as sun, wind, and water.
- Load-following capability (C_8): The system's ability to supply sufficient electricity to meet demand.
- Land area (C_9): The land area required for constructing and operating the energy system.
- CO_2 emissions (C_{10}): The amount of carbon dioxide released into the environment during electricity generation, directly impacting the climate change.
- NOx emissions (C_{11}): The amount of nitrogen oxides released, causing air pollution and acid rain.
- SO_2 emissions (C_{12}): The amount of sulfur dioxide released, also causing air pollution and acid rain.
- CH_4 emissions (C_{13}): The amount of methane released, a potent greenhouse gas.
- Water consumption (C_{14}): The amount of water used in electricity generation.
- Job creation (C_{15}): The number of direct and indirect jobs created by the project.
- Safety risk (C_{16}): The potential risks that could cause accidents or harm to people and the environment.
- Social acceptance (C_{17}): The level of acceptance of the project by the local community.

C-type criteria are C_1, C_2, C_3, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, *and* C_{16}, whereas the remaining are B-type criteria. The specific values for each criterion across the alternatives are provided in Table I.

TABLE I. SONE SUSTAINABLE DEVELOPMENT TECHNOLOGIES

Criteria	Alternatives							
	A_1	**A_2**	**A_{31}**	**A_4**	**A_5**	**A_6**	**A_7**	**A_8**
C_1	2000	880	8000	1250	5200	2800	800	600
C_2	40	12	77	34	13.5	50	16	10
C_3	2	1	1	1	17	10	3.5	10
C_4	4	2	2	4	5	4	4	4
C_5	50	27	52	45	85	82.5	70	55
C_6	5	4	3	5	4	4	3	5
C_7	25	23000	15400	1800	87.6	3.61	162.82	65.09
C_8	4	1	3	1	3	2	2	4
C_9	750	35	40	100	18	5000	2.5	2.5
C_{10}	12	49.174	16	25	18.913	70	800	700
C_{11}	0.03	0.178	0.065	0.06	0.28	0.9	2	1
C_{12}	0.015	0.257	0.04	0.05	0.02	0.5	3.5	4.5
C_{13}	1	2	1	4	2	40	5.5	8
C_{14}	68	1	3.02	1	150	135	78	78
C_{15}	0.27	0.87	0.23	0.17	0.25	0.21	0.11	0.11
C_{16}	0.945	2.45e-4	2.45e-4	1.89e-3	1.74e-3	1.49e-2	1.08	1.69
C_{17}	68	94	94	69	56	56	32	30

The next step in th evaluation process is to calculate the weights of the criteria using three weighting methods: MEREC, Entropy, and SPC, and then rank the alternatives employing the SAW, TOPSIS, PIV, and RAM. The objective is to assess how different weighting methods influence the ranking of the alternatives. Using the formulas outlined in Section II, the weights of the criteria were calculated, as summarized in Table II.

TABLE II. WEIGHT OF THE CRITERIA

Criteria	Weigh Method		
	MEREC	Entropy	SPC
C_1	0.0421	0.0464	0.0013
C_2	0.0314	0.0509	0.0007
C_3	0.0403	0.0589	0.0018
C_4	0.0161	0.0751	0.0002
C_5	0.0208	0.0501	0.0002
C_6	0.0081	0.0739	0.0001
C_7	0.1071	0.0463	0.2811
C_8	0.0227	0.0785	0.0003
C_9	0.1287	0.0464	0.1710
C_{10}	0.1981	0.0468	0.0079
C_{11}	0.0513	0.073	0.0042
C_{12}	0.0804	0.0634	0.0264
C_{13}	0.0598	0.0525	0.0049
C_{14}	0.0449	0.0485	0.0086
C_{15}	0.0167	0.0731	0.001
C_{16}	0.1158	0.0664	0.4901
C_{17}	0.0157	0.0498	0.0002

The results demonstrated that the weights of the criteria changed significantly depending on the method utilized. This variation can be attributed to the different data normalization approaches employed by each method (Section II), which is in accordance with previous studies [41-42]. It is also noticeable that when using the SPC method, the weights for C_7 is 0.2811 and that of C_{16} is 0.4901. These weights are significantly larger than those of the other criteria. This can be explained by the large variation in the values of C_7 and C_{16} across the alternatives. Specifically, the maximum value of C_7 is 2300 at A_2 and the minimum is 3.61 at A_6, resulting in a change factor of 6371.19. Similarly, C_{16} has a maximum value of 1.69 at A_8 and a minimum of 0.000245 at A_2 and A_3, resulting in a change factor of 6897.96. The SAW method was employed to calculate the V_i values of the alternatives. Table III summarizes the V_i scores and rankings of the alternatives with the three different cases of criteria weights.

TABLE III. RANKING OF ALTERNATIVES USING THE SAW METHOD

Alternatives	MEREC		Entropy		SPC	
	Vi	Rank	*Vi*	Rank	*Vi*	Rank
A_1	0.5038	3	0.5933	2	0.0471	7
A_2	0.5214	2	0.5988	1	0.8034	1
A_3	0.5813	1	0.5634	3	0.7175	2
A_4	0.3535	5	0.4801	4	0.1168	6
A_5	0.3612	4	0.4680	5	0.1235	5
A_6	0.1254	8	0.2955	8	0.0122	8
A_7	0.2614	7	0.3535	7	0.1770	3
A_8	0.2841	6	0.4302	6	0.1760	4
max/min	4.6339		2.0262		65.9240	

Similarly, the TOPSIS method was applied to calculate the C_i^* values of the alternatives. Table IV summarizes the C_i^* scores and rankings of the alternatives with the three different cases of criteria weights. For the PIV method, the d_i values of the alternatives were calculated. Table V summarizes the d_i scores and rankings of the alternatives with the three different cases of criteria weights. Finally, the RAM method was deployed to calculate the RI_i values of the alternatives. Table VI summarizes the RI_i scores and rankings of the alternatives with the three different cases of criteria weights.

It is evident that the choice of the weighting method significantly influences the scores and rankings of the alternatives, irrespective of whether the SAW, TOPSIS, PIV, or RAM method is used for ranking. A summary of the scores and the corresponding changes in the rankings across these four methods is provided in Table VII. The analysis reveals that the ratio of scores between the alternatives varies notably depending on the weighting method employed. Among all the cases examined, the most stable results were consistently observed when the Entropy method was used to calculate the weights of the criteria. In contrast, the SPC method consistently produced the least stable results across all scenarios.

TABLE IV. RANKING OF ALTERNATIVES USING THE TOPSIS METHOD

Alternatives	MEREC		Entropy		SPC	
	Ci^*	Rank	Ci^*	Rank	Ci^*	Rank
A_1	0.6691	5	0.6462	4	0.4119	6
A_2	0.9438	1	0.7834	1	0.9966	1
A_3	0.8346	2	0.6524	3	0.8507	2
A_4	0.7365	3	0.6539	2	0.6558	3
A_5	0.6993	4	0.6019	5	0.6384	4
A_6	0.5060	6	0.4620	6	0.5629	5
A_7	0.4348	7	0.4446	8	0.3938	7
A_8	0.4284	8	0.4540	7	0.2774	8
max/min	2.2029		1.7620		3.5928	

TABLE V. RANKING OF ALTERNATIVES USING THE PIV METHOD

Alternatives	MEREC		Entropy		SPC	
	d_i	Rank	d_i	Rank	d_i	Rank
A_1	0.2035	5	0.1771	3	0.4704	6
A_2	0.0384	1	0.0871	1	0.0032	1
A_3	0.1053	2	0.1788	4	0.0807	2
A_4	0.1305	3	0.1620	2	0.2203	3
A_5	0.1903	4	0.2127	5	0.2416	4
A_6	0.3930	6	0.3389	7	0.4209	5
A_7	0.4373	7	0.3491	8	0.4999	7
A_8	0.4514	8	0.3329	6	0.6386	8
max/min	11.765		4.0080		202.4785	

TABLE VI. RANKING OF ALTERNATIVES USING THE RAM METHOD

Alternatives	MEREC		Entropy		SPC	
	RI_i	Rank	RI_i	Rank	RI_i	Rank
A_1	1.4162	4	1.4231	4	1.4016	6
A_2	1.4301	1	1.4404	1	1.4118	1
A_3	1.4171	3	1.4271	2	1.4056	4
A_4	1.4180	2	1.4257	3	1.4104	2
A_5	1.4119	5	1.4198	5	1.4065	3
A_6	1.3819	7	1.3994	8	1.3526	8
A_7	1.3851	6	1.3996	7	1.3933	7
A_8	1.3805	8	1.4012	6	1.4016	5
max/min	1.0360		1.0294		1.0438	

TABLE VII. COMPARISON OF OBTAINED RESULTS USING MCDM TECHNIQUES AND THE WEIGHTING METHODS: MEREC, ENTROPY, AND SPC

MCDM Methods		Weight Methods		
		MEREC	Entropy	SPC
SAW	V_{max}/V_{min}	4.6339	2.0262	65.9240
TOPSIS	C^*_{max}/C^*_{min}	2.2029	1.7620	3.5928
PIV	d_{max}/d_{min}	11.7650	4.0080	202.4785
RAM	RI_{max}/RI_{min}	1.0360	1.0294	1.0438

IV. CONCLUSION

The determination of criteria weights is a critical element in Multi-Criteria Decision Making (MCDM), as it directly influences the evaluation of alternatives. Objective weighting methods, which rely on the data structure rather than subjective judgment, are essential for eliminating the decision-maker bias. This study compared the stability of alternative rankings when using four different MCDM techniques— Simple Additive Weighting (SAW), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Proximity Indexed Value (PIV), and Root Assessment Method (RAM)—while employing three objective weighting methods: MEthod based on the Removal Effects of Criteria (MEREC), Entropy, and Symmetry Point of Criterion (SPC).

The results consistently demonstrated that the alternative rankings achieved the highest stability when using the Entropy method to calculate the criteria weights. In contrast, the rankings were least stable when the SPC method was utilized. Specifically, when deploying the SAW method, the ratio of the highest score to the lowest score was 2.0262 with the Entropy method, 4.6339 with the MEREC method, and 65.9240 with the SPC method. Similarly, when the TOPSIS method was applied, the ratios were 1.7620, 2.2029, and 3.5928, respectively. In the case of the PIV method, the corresponding ratios were 4.0080, 11.7650, and 202.4785, respectively, while when using the RAM method, the ratios were 1.0294, 1.0360, and 1.0438. This conclusion aligns with previous research [19] and is consistent with the widespread use of the Entropy method in published studies, such as [43].

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