Solving Multi-Criteria Shortest Path by Optimization with Morphological Filters

Amar Kateb Hachemi Amar
Department of Computer Science, Faculty of Mathematics and Computer Science, University of Science and Technology of Oran - Mohamed Boudiaf, Algeria
amar.katebhachemiamar@univ-usto.dz (corresponding author)

Mohammed Amin Tahraoui
LIA Laboratory, Department of Informatics, Faculty of Exact Sciences and Informatics, University of Hassiba Benbouali of Chlef, Algeria
m.tahraoui@univ-chlef.dz

Abderrahim Belmadani
Department of Computer Science, Faculty of Mathematics and Computer Science, University of Science and Technology of Oran - Mohamed Boudiaf, Algeria
abderrahim.belmadani@univ-usto.dz

Received: 1 November 2024 | Revised: 5 December 2024 | Accepted: 9 December 2024

ABSTRACT

The challenge of determining the shortest path within a multimodal transportation network involves identifying the most efficient travel route while considering various interconnected modes of transportation, such as roads, railways, and public transit. This problem becomes increasingly complex when numerous criteria and modes are involved, complicating the decision-making process. This study proposes a novel approach to computing the shortest path in multimodal networks, focusing on four modes of transportation: metro, trams, buses, and taxis. The optimization criteria include distance, travel time, and monetary cost. The proposed method utilizes a new metaheuristic called Optimization by Morphological Filters (OMF), inspired by image processing techniques. This approach was compared with the Genetic Algorithms (GA) and the Non-Dominated Sorting Genetic Algorithm II (NSGA-II). Experiments were carried out using graph models of multimodal transport networks that closely resemble real-world scenarios varying in size. Furthermore, the proposed method was evaluated using a real network from the city of Lyon, France. The results demonstrate that the OMF approach performs well in terms of convergence to optimal solutions and computation time.

Keywords-multicriteria optimization; shortest path; multimodal transport networks; optimization by morphological filters; graph modeling

I. INTRODUCTION

A multimodal transport network can be defined as an integrated system that combines various modes of transport. Its primary goal is to facilitate the movement of passengers and goods from one location to another. The network is characterized by a series of routes and pathways connecting different nodes. These nodes, which include terminals and interchange points, serve as critical junctures where travelers can switch from one mode of transportation to another. In a multimodal system, passengers enjoy the flexibility to use a combination of transportation modes based on their individual needs and preferences. By leveraging the strengths of different modes, the system aims to mitigate the drawbacks associated with each, thus enhancing overall efficiency and convenience.

Transport planning is a widely researched topic due to its relevance to real-world applications. Most research efforts in this field focus on two key aspects: modeling transport networks and solving routing issues. The first involves defining how to represent a transport system, while the second pertains to developing algorithms to address the routing challenges faced by travelers and transport operators. The modeling of multimodal transport systems is crucial and has been the focus of numerous studies. In [1], a mathematical framework of multilayer networks was presented and applied to survey models of multimodal infrastructures, focusing on integrated real-world mobility patterns. In [2], the multimodal discrete network design problem was investigated, a super network topology was presented, and a bi-objective programming model was proposed to optimize network operations.

With the evolution of navigation systems, the transport planning problem has received considerable attention. Many studies aim to identify optimal routes in multimodal transport networks to optimize various criteria, such as distance and time cost. However, several challenges arise, including difficulties in managing live multimodal transport networks. The situation becomes particularly complex when the criteria to be optimized conflict with each other, such as balancing time and cost or prioritizing distance while ensuring safety.

The most exact methods for route planning problems in multimodal transportation networks are based on classical SP algorithms such as Dijkstra, Bellman-Ford, and A*. Various extensions of these algorithms have been proposed to find exact solutions for SP in multimodal transportation networks. These extensions consider the complexities introduced by transfer stations and aim to optimize various criteria such as travel time, cost, and environmental impact. For example, in [3], an extension of Dijkstra's algorithm was proposed to solve multiobjective path planning problems in urban multimodal transportation systems. This approach was based on transforming the problem into a classical SP with a single objective function by aggregating a weighted sum of modal characteristics along a path. In [4], an A*-label setting algorithm was proposed to compute constrained SPs in a multimodal network. In this approach, each edge was characterized with a vector of resource consumption in addition to travel time. In [5], two improved algorithms were proposed to solve the MCSP problem in a multimodal transport network, using transfer delay alone and in conjunction with the arrival time window. In [6], two approaches were presented to solve the SP problem in multimodal transportation networks using the k-shortest path algorithm and Geographic Information Systems (GIS).

Most problems in multimodal transportation belong to multiobjective optimization problems. Therefore, heuristics and metaheuristics have emerged as invaluable tools for solving several routing issues. In [7], a new heuristic method known as the Path Composition Approach was presented, which identifies the optimal path in a bimodal network based on user preferences. This method works by partitioning the entire path into distinct subpaths and then recombining them to construct the final paths. In [8], a new time-dependent SP algorithm was introduced, which used the A* algorithm within a heuristically restricted search space in a multimodal transportation network. In [9], an advanced hybrid heuristic approach was introduced, combining a GA with Variable Neighborhood Search (VNS). In [10], a memetic algorithm was proposed, in which a GA was combined with the Hill Climbing (HC) local search algorithm to solve the MCSP problem in a multimodal network. In [11], a new approach was proposed for the time-dependent multimodal transport problem, based on Dijkstra and ant colony optimization. In [12], a memetic approach was proposed to solve the problem of the Hub-and-Spoke-based Road-Rail Intermodal Transportation (HS-RRIT) network. This approach integrated a GA to effectively traverse the search space, complemented by two local search methods, namely shift and exchange, to exploit information in the search space.

In [13], a hybrid approach was proposed, combining population-based simulated annealing with an exact method to solve the intermodal freight transport problem. GA has been widely used to solve multicriteria optimization problems. In [14], a new GA was introduced for the SP problem in a multimodal transportation network. This approach was based on two novel genetic operators, namely hypercrossover and hypermutation, designed to optimize the solution space more effectively. In [15], the generalized traveling salesman problem was examined, which aims to find the minimum cost tour in a clustered set of cities. A hybridization between GA and Nearest Neighbor Search (NNS) was proposed, and a heuristic mutation operator was used. In [16], an Elitism Multi-Objective Evolutionary (EMOE) algorithm was proposed to address a multiobjective routing problem within a multimodal public transport network. The routing objectives were to minimize travel expense, time, and discomfort with three modes of transport, including tram, bus, and taxi. The proposed algorithm, based on the Non-Dominated Genetic Algorithm II (NSGAII), was implemented and evaluated using simulated data from a large network consisting of 150 vertices and 2600 edges.

This study proposes a novel method using Optimization with Morphological Filters (OMF) to tackle the MCSP problem in multimodal transportation networks. OMF is a recent neighborhood-based stochastic optimization algorithm inspired by morphological transformations that searches for the global optimum in a multidimensional space using morphological filters. The OMF method was initially introduced in [17]. Since then, it has been applied in various fields, demonstrating its versatility and effectiveness. It was employed in [18] to address engineering optimization problems without relying on penalty functions. Additionally, in [19], its application was extended to tackle combined economic and emission dispatch problems in power systems. To date, OMF has not been used to solve MCSP in multimodal transportation networks, which constitutes the primary objective of this research. This study considers various modes of transport, including metro, trams, buses, and taxis, reflecting the diverse choices available to travelers. Several criteria are analyzed, including distance, total travel time, and monetary cost, to ensure a comprehensive assessment of transportation options.

The proposed approach was rigorously evaluated using network models of varying sizes that closely resemble real-world conditions. Additionally, it was tested on an actual transportation network using data from the city of Lyon, France. This practical application demonstrates the effectiveness of OMF in optimizing multimodal transportation planning and highlights its potential to improve decision-making in complex urban environments. Furthermore, the results indicate that OMF outperforms traditional methods in several key aspects, such as solution quality and computational efficiency. By integrating various transportation modes and optimizing for multiple criteria, this approach paves the way for more sustainable and user-friendly transportation systems.

II. MULTIMODAL TRANSPORTATION NETWORK MODELING

The multimodal transportation network is modeled by a multigraph model, where each transportation mode is presented by a subgraph. The union between all graphs forms the global graph. Three transportation modes are considered: Tram, Bus, and Taxi. The SP is calculated by considering three criteria: distance, travel time, and cost.

A. Multigraph Representation

Let $G = (V, E)$ be a multigraph representing a multimodal transport network where V is the set of vertices (stations, stops) and E is the set of edges that represent the connections between nodes. Let $G_i = (V_i, E_i)$ be the subgraph that represents the transportation network for mode i. $V_i \subset V$ and $E_i \subset E$ are the mode-specific vertices and links, respectively. Unlike a partitioned graph, it is not mandatory for two distinct subgraphs G_x and G_y to have disjoint vertex sets, i.e., $V_x \cap V_y = \emptyset$ is not required. However, it is essential that the edge sets are disjoint, meaning $E_x \cap E_y = \emptyset$ must be always held. Any node $i \in V_x \cap V_y$ is a transfer node. In other words, a transfer point is a node that belongs to more than one subgraph, facilitating transfers between different transport modes within the network. This transfer process may involve various considerations, such as waiting times, accessibility features, and the availability of information on connecting services. For example, in [20], acceptable wait times beyond the scheduled bus arrival time at bus stops were determined, and predictive models were developed to provide additional tools to decision-makers to improve transportation.

Understanding these transfers is crucial for optimizing travel efficiency and improving the overall user experience in multimodal transportation systems. This framework enables the analysis of various transportation modalities within a single integrated network, enhancing operational efficiency and user experience. Figure 1 illustrates an example of a multigraph representation of a multimodal transportation network with three modes and six transfer nodes, highlighting the interactions and connections between different transportation systems.

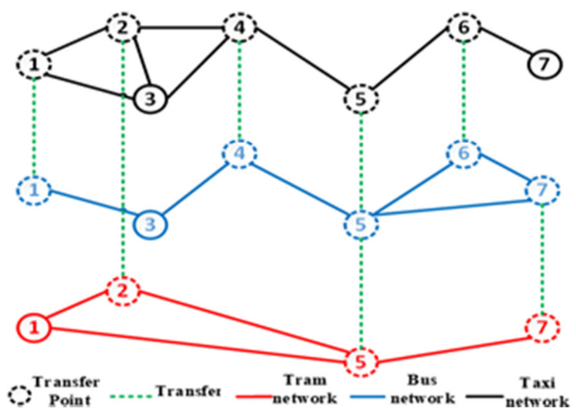

Fig. 1. The proposed multimodal transportation network model.

B. Considered Criteria

Let P be a multimodal path between a starting node $SN = a_1$ and a destination node $DN = a_n$ in graph G. This path is represented as an ordered sequence of triplets, each consisting of edges connecting the nodes along the journey. The segments between these two nodes can be expressed as follows:

$$P = [(a_1, a_2, m_1), (a_2, a_3, m_1), (a_3, m_1 \rightarrow m_2), \ldots (a_{n-1}, a_n, m_k)]. \quad (1)$$

where:

- Each segment (a_i, a_{i+1}, m_k) represents a connection from node a_i to node a_{i+1} using mode m_k.
- Each segment $(a_i, m_k \rightarrow m_l)$ represents a transfer from mode m_k to mode m_l at the transfer node a_i.

The criteria considered for calculating the SP include distance, travel time, and cost.

1) Distance

The global distance of a given path $Dist(P)$ is defined as:

$$Dist(P) = \sum_{(a_i,a_j,m_k)\in P} dist(a_i, a_j, m_k) \quad (2)$$

where (a_i, a_j, m_k) is the distance between nodes a_i and a_j when using mode m_k. This distance reflects the physical separation between the two nodes, measured along the specific route taken by mode m_k.

2) Travel Time

This factor accounts for the duration of the journey, including potential delays and transfer times, and is crucial for passengers prioritizing efficiency and timely arrivals. The travel time of a given path P is calculated according to:

$$Time(P) = \sum_{(a_i,a_j,m_k)\in P} time(a_i, a_j, m_k) + \sum_{(a_i,m_k,m_l)\in P} tranf\_time(a_i, m_k, m_l) \quad (3)$$

where $time(a_i, a_j, m_k)$ is the travel time between nodes a_i and a_j when using mode m_k. This travel time accounts for the duration of the journey along the designated route, influenced by factors such as speed, traffic conditions, and any operational delays specific to that transport mode. Additionally, $Transf\_time(a_i, m_k \rightarrow m_l)$ represents the waiting time at the transfer node a_i when switching from mode m_k to mode m_l. This waiting time may include the duration it takes for passengers to disembark, locate their next mode of transport, and wait for its arrival.

3) Cost

This criterion evaluates the monetary expenses associated with the journey, encompassing ticket prices, transfer fees, and any additional charges that may apply.

$$Cost(P) = \sum_{(a_i,a_j,m_k)\in P} cost(a_i, a_j, m_k) \quad (4)$$

where (a_i, a_j, m_k) is the travel cost between nodes a_i and a_j when using mode m_k. This cost encompasses all associated expenses for that segment of the journey, including ticket prices, fuel costs, surcharges, and any applicable fees specific to mode m_k.

III. PROPOSED APPROACH

The proposed approach is based on OMF. This approach is based on mathematical morphology transformations, mostly employed in the field of image processing. The basic element called erosion transformation is employed to explore the search space and locate the local optimum close to the structuring element, also called the morphological filter. Many parallel filters are used, which ensures that the system searches more intensely. To diversify the search, a method has also been included that prevents blockage at the level of local optimums and allows the examination of previously unexplored places.

A. The General OMF

The general OMF algorithm is given as follows.

```
initialize (NF, NN, FS, and ε)
for i = 0 to NF do
  Place randomly each filter Fi in the
  space search then assign it a filter
  size FSi
end for
repeat
  for i = 0 to NF do
    Calculate neighbors of Fi
    if there is a local optimum then
      move Fi to the local optimum
    else
      Reduce FSi
    end If
  end For
until (∑_{i=1}^{NF}(FS_i) < ε)
return the global optimum
```

A set of *NF*-size filters is placed within a search space, where each filter's neighboring filters (*NN*) are then identified. If a neighboring filter shows an improvement in the objective function, it replaces the current filter. If no improvement is found, the current filter's size (*FS*) is decreased, and neighboring filters are recalculated. This cycle continues until the combined filter sizes reach a small threshold value ε, approaching zero.

B. Optimization with Morphological Filters for Multimodal Transportation (OMF-MT)

The main contribution of this study is the application of the OMF method to the multimodal shortest path problem, called OMF-MT. Figure 2 shows the detailed steps of the proposed algorithm. An additional parameter is introduced, the stopping criterion for the optimization process, which replaces the Filter Size (*FS*) and the threshold value ε in the general OMF. Additionally, two neighborhood strategies are proposed: one focused on depth and the other on width. The primary goal is to enable the OMF to identify a collection of nondominated solutions that effectively approximate the Pareto front. Filters and neighbors in the OMF-MT represent a potential path within the multimodal transportation network from the source *SN* to the destination *DN*.

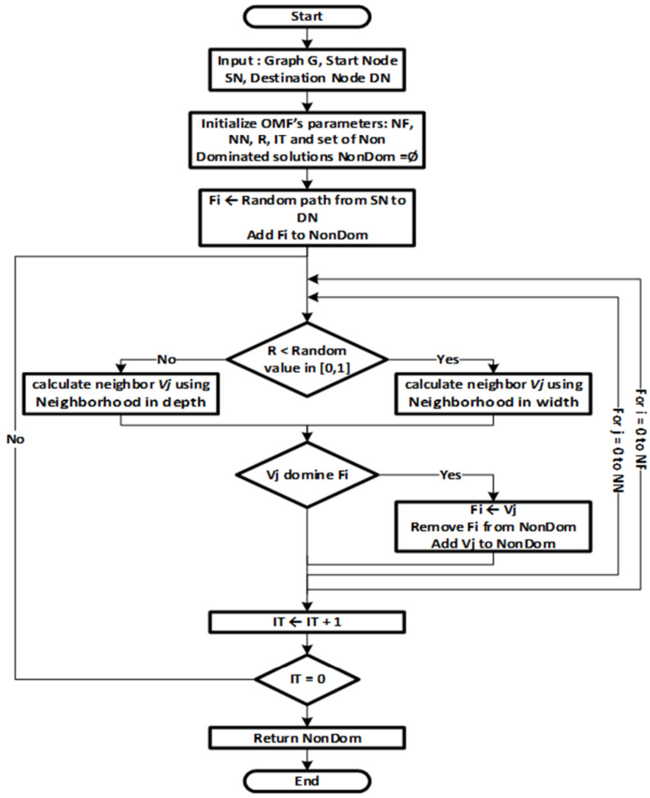

Fig. 2. The OMF-MT steps.

The algorithm begins with an initial graph G, a start node SN, and a destination node DN. The objective is to find a set of non-dominated solutions *NonDom* between *SN* and *DN*. After initializing parameters, including *NF*, *NN*, *IT*, and *R*, the algorithm creates *NF* filters *Fi*. Then, it enters a loop that continues until the iteration *IT* count reaches zero. Within each iteration, the algorithm explores neighboring paths by iterating over *NN*. For each neighbor, a random value determines whether a width-based or depth-based neighboring strategy is used. The algorithm then evaluates whether the newly calculated neighbor *Vj* dominates the current best path *Fi*. If so, *Fi* is updated to *Vj* and it is added to *NonDom*. This process iteratively refines the path by balancing exploration and exploitation through a mix of width-based and depth-based strategies. The loop terminates when the specified number of iterations is completed, and the final set of non-dominated solutions *NonDom* is returned. The algorithm leverages stochastic elements to effectively navigate the search space, aiming to identify an optimal path that dominates the initial one based on predefined criteria. This approach not only enhances the adaptability of the algorithm but also promotes the discovery of diverse solutions that meet the varying preferences of users in a multimodal transportation context. The details of the proposed parameters are presented in the following sections, providing comprehensive insight into their definitions and applications in the algorithm.

1) IT (Iterations Number)

This parameter serves as the stopping condition for the optimization process.

2) Neighborhood Strategies

Two neighborhood strategies are proposed as follows:

- Depth Neighborhood Strategy: This method preserves the filter's path while adjusting transportation modes along the edges to identify new neighbors. For instance, in Figure 3, the path from node 1 to node 6 originally utilized Bus, Bus, and Tram. After applying the depth neighborhood system, the path remains structurally the same, but the modes are modified to Taxi, Bus, and Bus, thus enhancing flexibility and potentially improving travel efficiency.

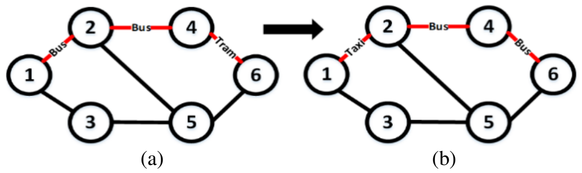

Fig. 3. Neighborhood in depth.

- Width Neighborhood Strategy: In this system, new random paths are generated while maintaining the same starting and destination nodes. Figure 4 illustrates this neighborhood system. In case (a), the path from node 1 to node 6 passes through nodes 2 and 4. After applying the width neighborhood system in (b), a new path is calculated that crosses nodes 2 and 5, showcasing the system's ability to explore alternative routing options.

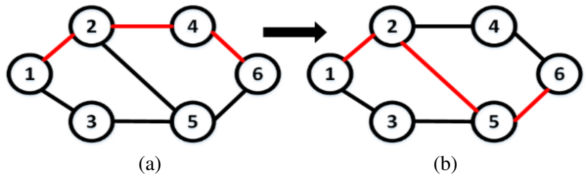

Fig. 4. Neighborhood in width.

3) Probability R

This parameter determines which neighborhood calculation method to apply at each iteration, allowing for a dynamic preference between width and depth approaches based on the current optimization context.

4) Dominance Relationship

The proposed OFM uses the dominance relationship described by the following formula.

$$\forall I \in X, I_{P2} \geq I_{P1}\ and\ \exists J \in X, J_{P2} > J_{P1} \quad (5)$$

where I_{P1} and I_{P2} are a given criterion in the set of criteria $X = \{D, T, C\}$, corresponding respectively to two paths P_1 and P_2. According to the formula P_1 dominates P_2.

IV. EXPERIMENTS AND RESULTS

The proposed method was implemented on a PC with an i7 processor at 2.7 GHz and 8 GB RAM using a routing simulator that provides a multimodal transportation network model and allowed the calculation and visualization of the optimal path between two nodes. Two experiments are presented:

- Comparison Experiment: A performance comparison was performed between the proposed OMF-MT, Genetic Algorithm (GA), and NSGA-II algorithms.
- Real-World Application: The second experiment involved applying the OMF-MT to a real-life scenario using the platform provided by Transports en Commun Lyonnais (TCL) [21].

A. Comparison Experiment

The proposed OMF-MT method was compared with GA and NSGAII, with tests on three randomly generated graph instances, as presented in Table I.

TABLE I. INSTANCES OF USED GRAPHS FOR COMPARISON

Graph	Nodes	Edges
G1	120	420
G2	960	3660
G3	2600	10100

The performance of each approach was evaluated using computation time and proximity (*Prox*) based on the Euclidean distance in the Pareto front to the ideal point in the search space. Figure 5 shows the proposed proximity concept.

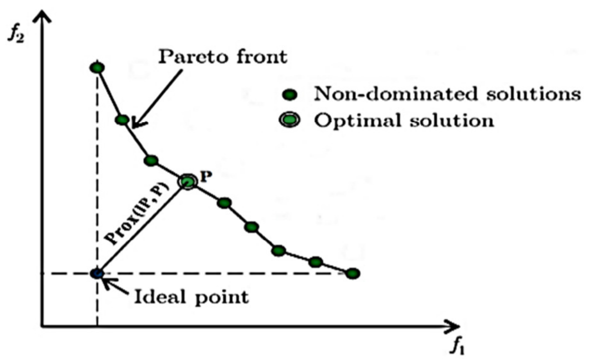

Fig. 5. Proximity demonstration.

Equations (6) and (7) indicate how to obtain the criteria of the ideal point and the calculation of proximity $Prox(IP, P)$ between the ideal point and a given solution, respectively. The solution with the smallest proximity to the ideal point is the best.

$$I_{IP} = min(I_P) P \in ND \quad (6)$$

$$Prox(IP, P) = \sqrt{\sum_{I \in X}(I_{IP} - I_P)^2} \quad (7)$$

where I_{IP} and I_P are the given criteria in the set of criteria $X = \{D, T, C\}$, corresponding respectively to the paths IP and P in the set of non-dominated solutions ND.

The GA and NSGAII parameters were as follows: population size = 100, generations number = 50, crossover rate = 0.9, and mutation rate = 0.1. The parameters of the OMF-MT algorithm included $FN = 3$, $NN = 5$, $IT = 50$, and probability $R = 0.7$. For each test, a random path calculation is requested. After that, the ideal point is calculated and the execution of each algorithm is applied. Table II and Figures 6-10 show the simulation results.

TABLE II. COMPARISON RESULTS

Network	Approach	*D*	*T*	*C*	*Prox*	Ex (ms)
G1	**IP**	**8.7**	**14.69**	**40**	**-**	**-**
	GA	9	14.8	40	0.87	136
	NSGA-II	9	14.75	40	**0.67**	225
	OFM-MT	9	14.86	40	1.07	**96**
G2	**IP**	**29.34**	**45.27**	**60**	**-**	**-**
	GA	32	48.16	80	21.48	739
	NSGA-II	29,6	45.85	60	**3.11**	1019
	OFM-MT	32	48.16	80	21.48	**329**
G3	**IP**	**51.22**	**76.94**	**160**	**-**	**-**
	GA	58	86.09	200	44.33	1358
	NSGA-II	53	78.48	160	**6.72**	2735
	OFM-MT	56	85.73	200	43.89	**640**

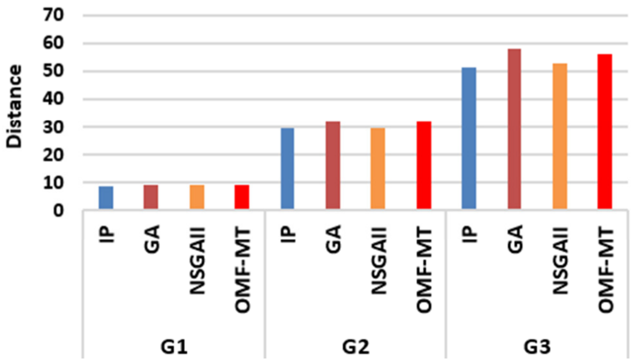

Fig. 6. Results obtained for the distance criterion.

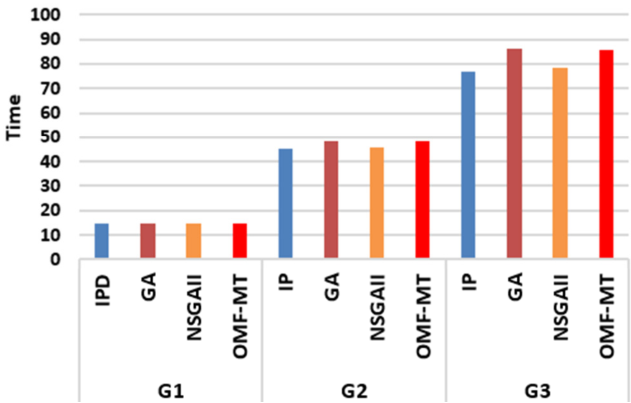

Fig. 7. Results obtained for the time criterion.

The findings reveal that NSGAII demonstrated a modest advantage over both the proposed OMF-MT and GA in terms of solution quality. Specifically, NSGAII achieved results that were nearly aligned with the ideal point, indicating greater precision in meeting optimization objectives. In contrast, the OMF-MT delivered solutions that closely matched those obtained with the GA and NSGAII, suggesting a comparable level of performance in terms of accuracy. However, a noteworthy benefit of OMF-MT is its significant reduction in computation time compared to both NSGAII and GA. This time efficiency makes OMF-MT a valuable alternative when computational resources or processing time are constrained. Although NSGAII may offer slightly better solution quality, fast execution with OMF-MT can be advantageous in some decision-making scenarios. Therefore, OMF-MT stands out as a viable compromise between solution quality and computational efficiency, achieving nearly comparable results to NSGAII while offering substantial time savings.

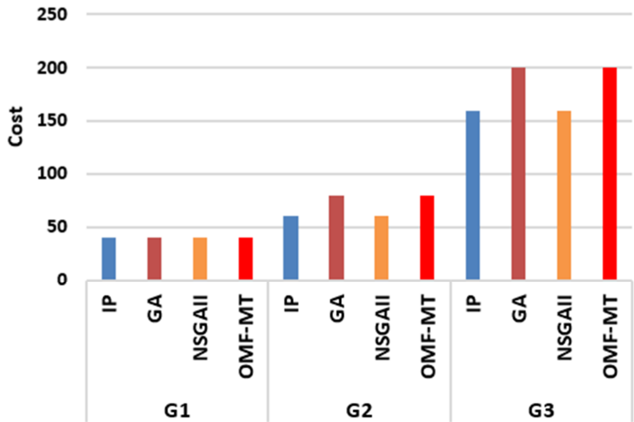

Fig. 8. Results obtained for the cost criterion.

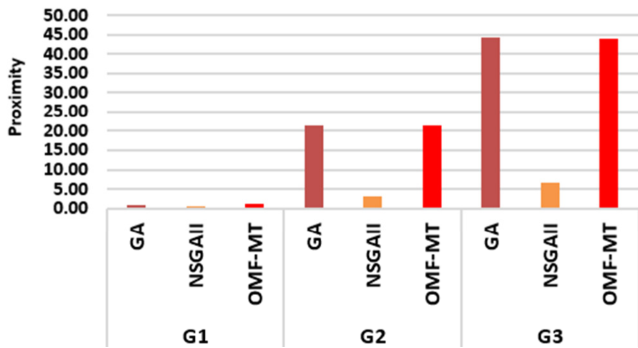

Fig. 9. Proximity metric performance results.

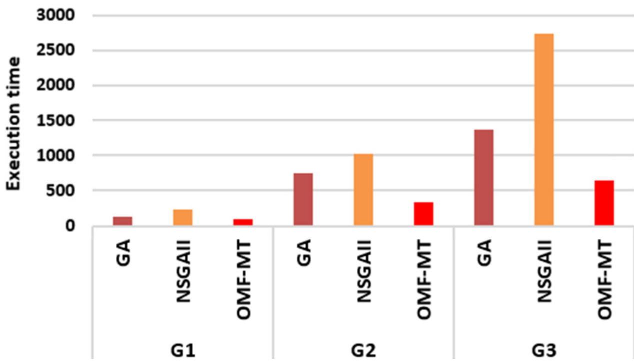

Fig. 10. Execution time performance results.

An additional experiment was carried out to evaluate the convergence behavior of the algorithms, focusing on how closely they approach the ideal point as the number of iterations increases. Experiments were performed using the G3 network. Figure 11 shows the results obtained. The results indicate that the proposed OMF-MT approach achieved faster convergence compared to NSGAII and GA. Although population-based algorithms such as GA and NSGAII generate new sets of solutions at each iteration and utilize crossover and mutation operators to enhance these solutions, this process often leads to a divergence of solutions throughout successive iterations. The continuous search for better solutions by exploring a wide solution space tends to slow the convergence rate of these algorithms. In contrast, OMF-MT leverages a local search mechanism combined with the proposed width and depth neighborhood systems, allowing it to rapidly refine and improve solutions. Integration of neighborhood systems enables OMF-MT to systematically explore solution variations while maintaining a tighter convergence path, which minimizes the divergence often seen with population-based methods. Consequently, OMF-MT achieves faster convergence.

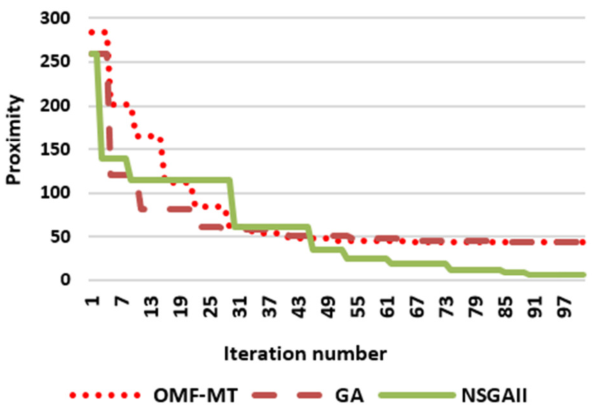

Fig. 11. Convergence analysis of approaches across iterations.

B. *Real-World Application*

This experiment involved the Lyon City network in France. The TCL platform [21] serves as the official source for multimodal transport routes in the Lyon metropolitan area, offering comprehensive travel information and route planning across various transportation modes, consisting of four metro lines, seven tram lines, more than 100 bus and trolleybus routes, and two funicular lines. It serves 72 municipalities, including 58 in the Lyon metropolitan area, covering a geographical area of 746 km² and serving a population of more than 1.3 million inhabitants. Table III presents the data for each mode of transport in the network, providing detailed information specific to the number of stops and edges of each transportation mode.

TABLE III. TCL NETWORK

Transportation mode	Stops	Edges
Tram	70	162
Metro	40	72
Bus	1460	2340
Funicular	4	4

This real-life implementation scenario considered the large lines served by metros, trams, and buses. The map is provided by TCL [22]. The test involves a travel request from the Cuire station to the Meyzieu Les Panettes station. OMF-MT was applied multiple times to generate the best optimal path. Figure 12 presents the visualization of the result path and the criteria values. It is important to note that the paths obtained in this study are consistent with the routes provided by the TCL transport service platform. Therefore, the alignment between the obtained paths and those of the TCL platform validates the accuracy and reliability of the multimodal transport network representation and the proposed OMF-MT.

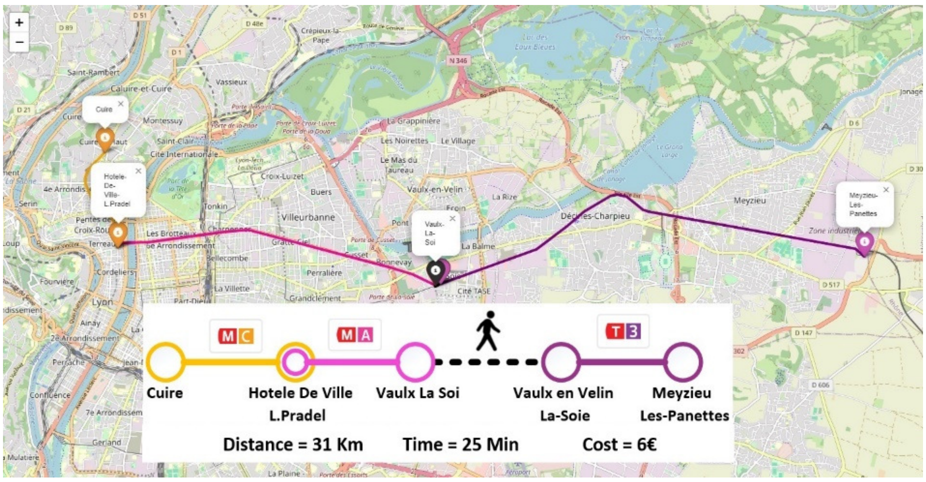

Fig. 12. Best optimal path.

V. DISCUSSION

The OMF method introduces an innovative approach, inspired by image processing techniques, to address the complex challenges of multimodal transport network optimization. Unlike traditional metaheuristic algorithms, which are computationally intensive and face scalability issues when applied to large real-world networks, OMF leverages a local search mechanism and an innovative neighborhood system to significantly enhance computational efficiency. Experiments conducted on synthetic graph models and a real-world multimodal transport network in Lyon, France, showed that OMF achieved competitive solution quality comparable to both GA and NSGA-II. In particular, OMF outperformed these algorithms in terms of computation time, offering substantial time savings without compromising the accuracy of the solutions. This makes OMF an attractive alternative when time efficiency is a critical factor, such as in real-time transportation planning or large-scale urban mobility optimization. The study also highlights several promising avenues for extending OMF to other domains. For example, the principles behind OMF's local search and neighborhood system could be adapted to optimize complex network problems in fields such as logistics, supply chain management, and even telecommunications. By providing a flexible framework that balances exploration and exploitation within solution spaces, OMF has the potential to advance both theoretical research in optimization and its practical applications across a range of industries. The OMF method not only provides a fresh perspective on solving multimodal transport optimization problems but also presents a promising tool to address large-scale optimization tasks in real-world settings. Its ability to achieve competitive solution quality while offering significant computational savings positions OMF as a robust alternative to traditional methods, especially in cases where fast and reliable decision-making is critical.

VI. CONCLUSION

This study addressed the challenges inherent in optimizing multimodal transportation networks by introducing OMF as a novel approach to solving the multicriteria shortest path problem. This study highlighted the need to integrate various transportation modes to improve efficiency, convenience, and user satisfaction in complex urban environments. Through rigorous evaluation using both simulated network models and actual data from the city of Lyon, it was shown that OMF offers significant advantages over traditional optimization methods, particularly in terms of solution quality and computational efficiency. By accommodating multiple criteria such as distance, total travel time, and monetary cost, the proposed approach provides a comprehensive framework for optimizing itineraries tailored to the diverse needs of travelers. Future research should focus on further refining the OMF algorithm and exploring its applicability to dynamic networks, where real-time data can significantly influence transportation choices. Additionally, integrating machine learning techniques to predict traveler preferences and behaviors could enhance the adaptability of the proposed approach, providing even more effective solutions in the face of evolving urban challenges.

REFERENCES

[1] L. Alessandretti, L. G. Natera Orozco, M. Saberi, M. Szell, and F. Battiston, "Multimodal urban mobility and multilayer transport networks," *Environment and Planning B: Urban Analytics and City Science*, vol. 50, no. 8, pp. 2038–2070, Oct. 2023, https://doi.org/10.1177/23998083221108190.

[2] Y. Zhou, C. Cao, and Z. Feng, "Optimization of Multimodal Discrete Network Design Problems Based on Super Networks," *Applied Sciences*, vol. 11, no. 21, Jan. 2021, Art. no. 10143, https://doi.org/10.3390/app112110143.

[3] P. Modesti and A. Sciomachen, "A utility measure for finding multiobjective shortest paths in urban multimodal transportation networks1," *European Journal of Operational Research*, vol. 111, no. 3, pp. 495–508, Dec. 1998, https://doi.org/10.1016/S0377-2217(97)00376-7.

[4] T. Y. Ma, "An A* Label-setting Algorithm for Multimodal Resource Constrained Shortest Path Problem," *Procedia - Social and Behavioral Sciences*, vol. 111, pp. 330–339, Feb. 2014, https://doi.org/10.1016/j.sbspro.2014.01.066.

[5] Y. Luo, Y. Zhang, J. Huang, and H. Yang, "Multi-route planning of multimodal transportation for oversize and heavyweight cargo based on reconstruction," *Computers & Operations Research*, vol. 128, Apr. 2021, Art. no. 105172, https://doi.org/10.1016/j.cor.2020.105172.

[6] M. Bielli, A. Boulmakoul, and H. Mouncif, "Object modeling and path computation for multimodal travel systems," *European Journal of Operational Research*, vol. 175, no. 3, pp. 1705–1730, Dec. 2006, https://doi.org/10.1016/j.ejor.2005.02.036.

[7] M. G. Battista, M. Lucertini, and B. Simeone, "Path Composition and Multiple Choice in a Bimodal Transportation Network. Volume 2: Modelling Transport Systems," presented at the World Transport Research. Proceedings of the 7th World Conference on Transport ResearchWorld Conference on Transport Research Society, 1996.

[8] A. Idri, M. Oukarfi, A. Boulmakoul, K. Zeitouni, and A. Masri, "A new time-dependent shortest path algorithm for multimodal transportation network," *Procedia Computer Science*, vol. 109, pp. 692–697, Jan. 2017, https://doi.org/10.1016/j.procs.2017.05.379.

[9] O. Dib, L. Moalic, M.-A. Manier, and A. Caminada, "An advanced GA–VNS combination for multicriteria route planning in public transit networks," *Expert Systems with Applications*, vol. 72, pp. 67–82, Apr. 2017, https://doi.org/10.1016/j.eswa.2016.12.009.

[10] O. Dib, M. Dib, and A. Caminada, "Computing Multicriteria Shortest Paths in Stochastic Multimodal Networks Using a Memetic Algorithm," *International Journal on Artificial Intelligence Tools*, vol. 27, no. 07, Nov. 2018, Art. no. 1860012, https://doi.org/10.1142/S0218213018600126.

[11] H. Ayed, C. Galvez-Fernandez, Z. Habbas, and D. Khadraoui, "Solving time-dependent multimodal transport problems using a transfer graph model," *Computers & Industrial Engineering*, vol. 61, no. 2, pp. 391–401, Sep. 2011, https://doi.org/10.1016/j.cie.2010.05.018.

[12] R. Wang, K. Yang, L. Yang, and Z. Gao, "Modeling and optimization of a road–rail intermodal transport system under uncertain information," *Engineering Applications of Artificial Intelligence*, vol. 72, pp. 423–436, Jun. 2018, https://doi.org/10.1016/j.engappai.2018.04.022.

[13] A. Abbassi, A. E. hilali Alaoui, and J. Boukachour, "Robust optimisation of the intermodal freight transport problem: Modeling and solving with an efficient hybrid approach," *Journal of Computational Science*, vol. 30, pp. 127–142, Jan. 2019, https://doi.org/10.1016/j.jocs.2018.12.001.

[14] H. C. Yu and F. Lu, "A multi-modal route planning approach with an improved genetic algorithm," in *Advances in Geo-Spatial Information Science*, CRC Press, 2012.

[15] H. Jafarzadeh, N. Moradinasab, and M. Elyasi, "An Enhanced Genetic Algorithm for the Generalized Traveling Salesman Problem," *Engineering, Technology & Applied Science Research*, vol. 7, no. 6, pp. 2260–2265, Dec. 2017, https://doi.org/10.48084/etasr.1570.

[16] H. Faroqi, "Multiobjective route finding in a multimode transportation network by NSGA-II," *Journal of Engineering and Applied Science*, vol. 71, no. 1, Mar. 2024, Art. no. 81, https://doi.org/10.1186/s44147-024-00417-7.

[17] C. N. E. H. Khelifa and A. Belmadani, "New Approach for Continuous and Discrete Optimization: Optimization by Morphological Filters," in *Heuristics for Optimization and Learning*, F. Yalaoui, L. Amodeo, and E. G. Talbi, Eds. Springer International Publishing, 2021, pp. 425–440.

[18] S. Zaoui and A. Belmadani, "Solving Engineering Optimization Problems Without Penalty," *International Journal of Computational*

Methods, vol. 18, no. 04, May 2021, Art. no. 2150007, https://doi.org/10.1142/S0219876221500079.

[19] S. Zaoui and A. Belmadani, "Solution of combined economic and emission dispatch problems of power systems without penalty," *Applied Artificial Intelligence*, vol. 36, no. 1, Dec. 2022, Art. no. 1976092, https://doi.org/10.1080/08839514.2021.1976092.

[20] S. A. Arhin, A. Gatiba, M. Anderson, B. Manandhar, and M. Ribbisso, "Acceptable Wait Time Models at Transit Bus Stops," *Engineering, Technology & Applied Science Research*, vol. 9, no. 4, pp. 4574–4580, Aug. 2019, https://doi.org/10.48084/etasr.2966.

[21] "TCL - Transports en commun à Lyon : métro, tramway, funiculaire et bus." https://www.tcl.fr/.

[22] "Plans du réseau | TCL." https://www.tcl.fr/plans-du-reseau.