Hybrid Multi-Criteria Decision Making Methods: Combination of Preference Selection Index Method with Faire Un Choix Adèquat, Root Assessment Method, and Proximity Indexed Value

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ABSTRACT

This study presents an investigation into the hybridization of Multiple Criteria Decision Making (MCDM) methods. The Preference Selection Index (PSI) method is used in two distinct ways: first, for its traditional purpose of ranking alternatives, and second, to calculate criteria weights. These criteria weights are utilized to rank the alternatives provided by other MCDM methods, including the Faire Un Choix Adéquat (FUCA), Root Assessment Method (RAM), and Proximity Indexed Value (PIV), resulting in the creation of three hybrid models: FUCA-PSI, RAM-PSI, and PIV-PSI. The effectiveness of these hybrid approaches is tested by ranking 20 Vietnamese cities based on their digital transformation efforts. The results demonstrate that the hybrid approaches produce a highly correlated ranking, as evidenced by the Spearman rank correlation coefficient found among these methods, with the lowest being 0.8571. Both the PSI method and the three hybrid models identified the same top alternative, confirming the reliability and accuracy of the rankings.

Keywords-hybrid models; MCDM methods; PSI; FUCA; RAM; PIV; digital transformation

I. INTRODUCTION

The MCDM methods play a crucial role in assisting managers and experts in making optimal decisions when faced with multiple alternatives [1]. Over the years, advancements in technology and data have stimulated the development of more than 200 MCDM methods, enhancing accuracy and efficiency [2]. These methods have been applied in fields like education [3, 4], health [5], and engineering [6], contributing to sustainable development by optimizing decision processes.

A critical step in MCDM is determining the weights of criteria, as they reflect the priority and influence of each factor on the final decision, ensuring fairness and reasonability [7]. Methods for calculating criteria weights are categorized into three main groups: objective, subjective, and combined weight approaches [8, 9]. Subjective weights rely on the decision-maker's experience and option, while the objective ones are based on data and calculations, but may lack flexibility. The complexity increases when combining both weight types to achieve balance, ensuring that the decision is comprehensive and aligned with the set goals [10, 11].

This study introduces a novel approach to criteria weighting deploying the PSI method. PSI has been employed as an MCDM technique to rank alternatives, offering the ability to automatically calculate criteria weights without user input [12]. This advantage has led to its application in various fields, including the selection of 3D printers [13], scholarship recipients [14], machining methods [15], evaluation of air quality in offices [16], and the ranking of transportation companies [17].

Until now, no former studies have explored the utilization of the weights derived during the PSI method with the purpose of integrating them with other MCDM methods. In the current study, three methods were considered, i.e. FUCA, RAM, and PIV, resulting in hybrids named FUCA-PSI, RAM-PSI, and PIV-PSI. Each method was chosen for its unique strengths. FUCA does not require data normalization [18], RAM, a recently developed method, balances beneficial and non-beneficial criteria [19], and PIV minimizes the phenomenon of rank reversal [20].

II. METHODOLOGY

A. *Hybrid Model of PSI with MCDM Methods*

To hybridize PSI with MCDM methods, the sequence of applying these methods needs to be clarified. Assume there are *m* alternatives to be ranked, and *n* is the number of criteria for each alternative. A criterion where the expectancy is the-larger-the-better is denoted as type *B*, while a criterion where the expectancy is the-smaller-the-better is denoted as type *C*. The value of the criterion *j* for alternative *i* is denoted as x_{ij}. A decision matrix is then formed as:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

where *i = 1, 2, …, m*, and *j = 1, 2, …, n*.

1) PSI

It is important to perform the following processes in a systematic order:

- Normalize the data:

$$n_{ij} = \frac{x_{ij}}{\max(x_{ij})} \quad (2)$$

where $j \in B$.

$$n_{ij} = \frac{\min(x_{ij})}{\mathrm{x_{ij}}} \quad (3)$$

where $j \in C$.

- Calculate the average normalized values:

$$n = \frac{\sum_{i=1}^{m} n_{ij}}{m} \quad (4)$$

- Calculate the preference value for each criterion:

$$\varphi_j = \sum_{i=1}^{m} \left(n_{ij} - n\right)^2 \quad (5)$$

- Calculate the weight for each criterion:

$$w_j = \frac{1-\varphi_j}{\sum_{j=1}^{n}(1-\varphi_j)} \quad (6)$$

- Calculate the score for each alternative:

$$\theta_i = \sum_{j=1}^{n} n_{ij} \cdot w_j \quad (7)$$

- Rank the alternatives based on the principle that the best alternative is the one with the highest score.

2) FUCA

- Rank the alternatives for each criterion. Let r_{ij} be the rank of alternative *i* for criterion *j*. r_{ij} will be ranked 1 if the value x_{ij} is the smallest and if *j* belongs to type C, or the largest if *j* belongs to type B, and vice versa.
- Calculate the scores of the alternatives:

$$S_i = \sum_{j=1}^{n} r_{ij} \cdot w_j \quad (8)$$

3) RAM

- Normalize the data:

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad (9)$$

- Calculate the normalized values considering the criteria weights:

$$y_{ij} = r_{ij} \cdot w_j \quad (10)$$

- Calculate the total normalized scores considering the criteria weights:

$$S_{+i} = \sum_{j=1}^{n} y_{+ij} \quad (11)$$

where $j \in B$.

$$S_{-i} = \sum_{j=1}^{n} y_{-ij} \quad (12)$$

where $j \in C$.

- Rank the alternatives in descending order according to their scores:

$$RI_i = \sqrt[2+S_{-i}]{2 + S_{+i}} \quad (13)$$

4) PIV

- Normalize the data:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad (14)$$

- Calculate the normalized values considering criteria weights:

$$v_{ij} = w_j \times n_{ij} \quad (15)$$

- Calculate the weighted proximity index:

$$u_i = max\left(v_{ij}\right) - v_i \quad (16)$$

where $j \in B$.

$$u_i = v_i - min\left(v_{ij}\right) \quad (17)$$

where $j \in C$.

- Calculate the scores of the alternatives:

$$d_i = \sum_{j=1}^{n} u_i \quad (18)$$

The alternative with the smallest score is ranked first, and so on.

III. CASE STUDY

The effectiveness of the hybrid methods FUCA-PSI, RAM-PSI, and PIV-PSI was tested by ranking Vietnamese cities based on their digital transformation efforts. Comparing cities within a country regarding digital transformation is essential for fostering economic growth and encouraging healthy competition. Such comparisons provide a basis for assessing each city's level of development, helping to identify areas that require investment and support. These efforts enhance the efficiency of management and public services while contributing to the creation of a modern and comfortable living environment for residents [21].

Table I presents data on the digital transformation of twenty Vietnamese cities, coded as CT1 to CT20 [22]. The evaluation was based on eight parameters, including digital awareness index, digital institution index, digital infrastructure index, digital human resources index, cyber information security index, digital government activity index, digital business activity index, and digital social activity index, all classified as type B criteria. A brief explanation of the significance of each parameter is provided below:

- **Digital Awareness Index (C1):** Evaluates the level of understanding and awareness within the community and government regarding the importance of digital transformation.
- **Digital Institution Index (C2):** Assesses the capability to implement policies, regulations, and legal frameworks that support digital transformation.
- **Digital Infrastructure Index (C3):** Measures the development and quality of information technology and telecommunications infrastructure.
- **Digital Human Resources Index (C4):** Evaluates the skills and capabilities of the workforce in using and developing digital technologies.
- **Cyber Information Security Index (C5):** Assesses the security and safety of information and data in the digital environment.
- **Digital Government Activity Index (C6):** Measures the level of digital technology application in local government activities and services.
- **Digital Business Activity Index (C7):** Evaluates the level of digital technology utilization in business activities and economic development.
- **Digital Social Activity Index (C8):** Measures the level of digital technology adoption and interaction in social life.

TABLE I. PARAMETERS FOR DIGITAL TRANSFORMATION

City	C1	C2	C3	C4	C5	C6	C7	C8
CT1	1.0000	0.9000	0.8253	0.8487	0.7578	0.8575	0.7836	0.5197
CT2	0.8917	0.9000	0.8258	0.6267	0.5580	0.8406	0.7611	0.3590
CT3	0.9581	0.9000	0.7684	0.6250	0.5382	0.8385	0.5904	0.4505
CT4	0.9500	0.9000	0.6823	0.6794	0.6423	0.8262	0.6787	0.3001
CT5	0.9000	0.8000	0.7233	0.7376	0.7226	0.7325	0.7556	0.2947
CT6	1.0000	0.8000	0.7522	0.7276	0.5624	0.7342	0.6499	0.3678
CT7	0.8417	0.9000	0.7002	0.6458	0.6384	0.7193	0.7286	0.3190
CT8	0.9500	0.8000	0.7453	0.6277	0.6216	0.7779	0.4829	0.2961
CT9	0.8643	0.9000	0.5586	0.6411	0.5134	0.6478	0.7764	0.3348
CT10	0.9500	0.8000	0.5684	0.6593	0.4961	0.6911	0.6581	0.3648
CT11	0.9500	0.9000	0.6105	0.7561	0.4956	0.7347	0.4756	0.3270
CT12	0.9417	0.8000	0.7264	0.5921	0.5527	0.7312	0.5222	0.3505
CT13	0.9417	0.7000	0.7257	0.6190	0.2614	0.7271	0.7499	0.3711
CT14	0.9077	0.9000	0.5756	0.6465	0.3620	0.6481	0.7469	0.3623
CT15	1.0000	0.8000	0.5177	0.6584	0.6079	0.6604	0.5924	0.3712
CT16	1.0000	0.8000	0.6412	0.7072	0.5949	0.5953	0.6591	0.2853
CT17	0.9333	0.7000	0.7273	0.7051	0.5227	0.7379	0.5537	0.2984
CT18	0.9917	0.9000	0.4861	0.5752	0.5816	0.6418	0.7594	0.2555
CT19	1.0000	0.8000	0.7284	0.7193	0.4641	0.7707	0.4193	0.2924
CT20	0.8750	0.8000	0.6443	0.7538	0.6060	0.6388	0.5691	0.3093

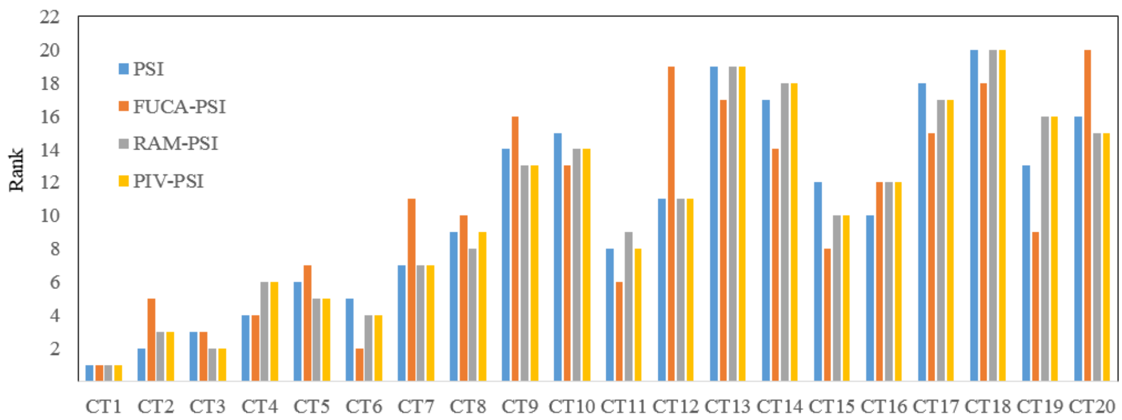

Fig. 1. Ranking of alternatives of the 20 Vietnamese cities.

IV. RESULTS AND DISCUSSION

Table II presents the values of the criteria weights C1 to C8, calculated from (1)-(6). Figure 1 illustrates the ranking of alternatives as determined by the PSI method and the three hybrids, FUCA-PSI, RAM-PSI, and PIV-PSI. While the rankings of the cities' digital transformation efforts vary across the methods, such differences are expected when using MCDM approaches [23-25]. However, both PSI and the FUCA-PSI, RAM-PSI, and PIV-PSI hybrid models are consistently identifying City 1 as the city with the best digital transformation efforts. This confirms that the established hybrid models have high accuracy and are completely reliable for use.

TABLE II. CRITERIA WEIGHT VALUES

Criteria Weights	Values
C1	0.1508
C2	0.1420
C3	0.1183
C4	0.1403
C5	0.0959
C6	0.1359
C7	0.0980
C8	0.1189

To evaluate the similarity of the ranking results of the cities using different methods, Spearman's rank correlation coefficient S was used:

$$S = 1 - \frac{6\sum_{i=1}^{m} D_i^2}{m(m^2-1)} \quad (19)$$

where D_i is the difference in the ranking of the alternative i when ranked by different methods [26, 27].

The calculated values of Spearman's coefficient are summarized in Table III.

TABLE III. SPEARMAN'S RANK CORRELATION COEFFICIENT

	PSI	FUCA-PSI	RAM-PSI	PIV-PSI
PSI	1	0.8571	0.9759	0.9774
FUCA-PSI		1	0.8301	0.8361
RAM-PSI			1	0.9985
PIV-PSI				1

The S values are all close to 1, indicating minimal differences in the rankings of the cities across the different methods [28]. Even the smallest S value of 0.8571 between PSI and FUCA-PSI suggests a very high level of agreement, with only small variations in the rankings [29]. Notably, the S coefficient of 0.9985 between RAM-PSI and PIV-PSI indicates that the rankings of the two hybrid models are almost identical. As confirmed in Figure 1, the rankings of 18 cities are consistent between these two hybrids, with only a swap in cities 8 and 11 having taken place. This consistency demonstrates the high accuracy of the proposed methods, confirming that FUCA-PSI, RAM-PSI, and PIV-PSI are highly effective tools in the field of MCDM.

V. CONCUSIONS

In this study, three hybrid models of Multi-Criteria Decision Making (MCDM) methods have been proposed combining the Preference Selection Index (PSI) with Faire Un Choix Adéquat (FUCA), Root Assessment Method (RAM), and Proximity Indexed Value (PIV) approaches. These hybrids -FUCA-PSI, RAM-PSI, and PIV-PSI- are utilized to rank the alternatives by deploying the FUCA, RAM, and PIV methods, while the criteria weights are determined by the PSI method. The ranking of twenty cities in Vietnam based on their progress in digital transformation has been carried out to evaluate the proposed hybrids. The following conclusions can be drawn:

- All three proposed hybrids demonstrate high accuracy, with RAM-PSI and PIV-PSI showing strong performance.
- Among the 20 Vietnamese cities, CT1 is ranked as the highest in digital transformation.
- The study also suggests that the proposed hybrid models be applied to other problems, while future work should explore the development of new hybrids combining PSI with other MCDM methods.

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