Multi-Objective Optimization of a Two-Stage Helical Gearbox with Two Gear Sets in First Stage to Reduce Volume and Enhance Efficiency using the EAMR Technique

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ABSTRACT

This study presents the results of a work employing the Evaluation by an Area-based Method of Ranking (EAMR) methodology to address the Multi-Objective Optimization Problem (MOOP) of a two-stage helical gearbox comprising two gear sets in the initial stage. The objective of this study is to identify the most critical design parameters for minimizing the volume of the gearbox while optimizing its efficiency. In this study, three key design parameters were selected for analysis: the wheel face width coefficients X_{ba} for the first and second stages, as well as the gear ratio of the first stage u_1. Furthermore, the EAMR technique was employed to address the Multi-Criteria Decision-Making (MCDM) challenge, with the entropy method used to ascertain the weight criterion for resolving the MOOP. The study's findings offer valuable insights into the optimal values for three primary design parameters, which are essential for the development of a two-stage helical gearbox with two gear sets in the initial stage.

Keywords-EAMR method; entropy method; helical gearbox; gear ratio

I. INTRODUCTION

A gearbox is an essential component of a mechanical power transmission system. Its function is to amplify torque and reduce the speed of the motor shaft relative to the working shaft. Such devices are employed in a multitude of applications, including those pertaining to automotive systems, hoisting apparatuses, agricultural machinery, and more. Consequently, numerous academics are actively engaged in the pursuit of the optimal design of the gearbox. Authors in [1] evaluated four objective functions: the minimum size, weight, tooth deflection, and maximum life of a spur gear pair. They employed the modified iterative weighted Tchebycheff method to achieve this. One disadvantage of this approach is that achieving convergence is dependent on the initial sample vector. Furthermore, the time required for convergence of the solution is often considerable. Authors in [2] put forth a methodology for the construction and optimization of multi-spindle gear trains. The optimization algorithm employed a two-stage procedure. The initial phase entailed the direct search approach, which was employed to exclude impracticable alternatives. The subsequent phase involved the implementation of a heuristic strategy. Authors in [3] evaluated the most efficient gear ratios for mechanical driven systems by employing a three-stage bevel helical gearbox and a chain drive. The objective of this study is to reduce the cross-sectional area of the system. In addition, six input parameters were taken into account, comprising the overall system ratio, the permissible contact stress, the face width coefficients for both the bevel and helical gear sets, and the output torque. The study yielded results that enabled the estimation of the impact of input characteristics on the optimal ratios. Furthermore, equations were proposed to facilitate the calculation of optimal gear ratios. Authors in [4] examined the most efficient gear ratios for mechanical drive systems comprising a two-stage helical gearbox with double gear sets in the initial stage and a chain drive. The objective of the study was to minimize the system length, which has been selected as the objective function for the optimization problem. Furthermore, the study investigated the input parameters, including the overall system ratio, the wheel face width coefficients for the first and second stages, the permissible contact stress, and the output torque. The derived equations demonstrated that the optimal gear ratios may be accurately and readily computed. Authors in [5] employed a customized adaptive random search technique to optimize the weight of the helical gear pair. The design variables considered were the gear module, helix angle, pinion teeth, and face width. The restrictions were contact stress and tooth-bending strength. The limitation of this approach is that it is a non-deterministic random search technique, which is only efficient for a limited number of design variables. Authors in [6] examined the failure mode characteristics of spur gear pairs made from 20 regularly used gear materials, using both full depth 20° and 25° pressure angles. The objective function was the center distance, while the bending, pitting, interference, and scoring failures were regarded as constraints. Authors in [7] investigated the optimization of the prediction of optimal partial ratios for three-step helical gearboxes with second-step double gear sets. The study had several objectives, including minimizing the length of the gearbox, minimizing the cross-sectional dimension of the gearbox, and minimizing the mass of the gears. The study concentrated on the equilibrium of a mechanical system comprising three gear units and the conditions of their resistance. Three optimization problems were conducted with the objective of determining the minimum length of the gearbox, the minimum cross-sectional dimension of the gearbox, and the minimum mass of the gears. Moreover, regression analysis was used to develop explicit models for computing the partial ratios of the gearboxes. Authors in [8] sought to identify the most efficient method for calculating the gear ratios of a two-stage helical reducer. The objective of the study was to reduce the surface area of the cross-section of the reducer. In light of the findings of the study, two methods were put forth for calculating the most efficient gear ratios of a two-stage reducer.

Authors in [9] employed a Genetic Algorithm (GA) to optimize the volume of a two-stage helical gear train. The objective function was augmented with static and dynamic penalty functions to address design constraints, including contact stress, bending stress, the number of teeth on the gear and pinion, module, and face width of the gear. The results obtained through the use of the GA were compared to those obtained using a deterministic design technique. It was found that the GA outperformed the deterministic approach. Authors in [10] employed two sophisticated optimization techniques, namely Particle Swarm Optimization (PSO) and Simulated Annealing (SA), with the objective of identifying the most optimal combination of design parameters that would result in the smallest weight of a spur gear train. The constraints were formulated in accordance with American Gear Manufacturers Association (AGMA) standards with the objective of minimizing the weight of a basic spur gear pair that involves mixed integers, using PSO and SA. Athors in [11], determined the most efficient gear ratios for mechanical driven systems. The study focused on the implementation of a chain drive and a two-step helical gearbox, wherein the initial step involved the use of double gear sets with the objective of minimizing the system's cross-sectional area. The study examined a number of input parameters, including the total system ratio, the wheel face width coefficients of both helical gear sets, the permissible contact stress, and the output torque. The results demonstrated that the optimal ratios can be achieved with a high degree of precision by employing the recommended models. Authors in [12] determined the most efficient partial transmission ratios for mechanical drive systems. The study concentrated on the usage of a chain drive and a three-step helical reducer with the objective of reducing the height of the system's cross-section. The optimization problem yielded equations that provide the optimal partial ratios of the chain drive and three phases of the reducer. Authors in [13] proposed a multi-objective optimization approach using GA to identify the optimal module, shaft diameter, and rolling bearing for a single-stage spur gearbox. The problem was defined by considering gear volume, shaft diameter, and rolling bearing dimensions as the objective functions, while tooth root fracture and surface fatigue failure were treated as constraints. Authors in [14] employed the EAMR approach to conduct a multi-objective optimization of a two-stage helical gearbox. This approach involves measuring and rating alternatives based on a compromise solution. The objective of this study was to identify the most favorable primary design parameters that will enhance gearbox efficiency while reducing gearbox volume.

A substantial body of research has been conducted on a variety of topics related to MCDM. Authors in [15] employed the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method to develop a model for enhancing the university accreditation process. Authors in [16] employed the technique of TOPSIS to identify the optimal main design elements for a two-stage helical gearbox, while authors in [17] employed the FGDEMATEL approach to prioritize the potential causes of project risks. In order to resolve the MOOP of a two-stage helical gearbox comprising two gear sets in the initial stage, this study employs the EAMR strategy to address the MCDM issue and the entropy technique for determining the criteria weights. Moreover, the objective of the study is to reduce the volume of the gearbox while simultaneously enhancing its efficiency. The results of the analysis facilitated the identification of several critical design factors for the gearbox.

II. OPTIMIZATION PROBLEM

A. Determining Gearbox Volume

The gearbox volume V_{gb}, shown in Figure 1, can be determined by:

$$V_{gb} = L \times B \times H \quad (1)$$

where *L*, *B* and *H* can be found by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \quad (2)$$

$$B = 2 \times b_{w1} + b_{w2} + 4 \times \delta \quad (3)$$

$$H = max(d_{w21}, d_{w22}) + 8.5 \times \delta \quad (4)$$

where δ=7÷10 (mm) [18], b_{wi}, d_{w1i}, d_{w2i} are the width of the gear, the pitch diameter of the pinion and the gear of i_{th} stage (i=1÷2) which can be calculated by:

$$b_{wi} = X_{bai} \times a_{wi} \quad (5)$$

$$d_{w1i} = 2 \times a_{wi}/(u_i + 1) \quad (6)$$

$$d_{w2i} = 2 \times a_{wi} \times u_i/(u_i + 1) \quad (7)$$

where X_{bai} and a_{wi} (i=1÷2) are the wheel face width coefficient and the gearbox center distance of stage I, a_{wi} is found by [18]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (8)$$

where T_{1i} (i=1÷2) is the torque on the pinion of i^{th} stage which can be found by:

$$T_{11} = T_{out}/\left(2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \quad (9)$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (10)$$

B. Determining Gearbox Efficiency

The gearbox efficiency (%) is calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (11)$$

where P_l is the total power loss in the gearbox which is determined by [19]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (12)$$

where P_{lg}, P_{lb}, P_{ls}, and P_{Z0} are the power losses in the gearings, in bearings, in seals, and in the idle motion, respectively [20].

C. Objectives and Constraints

In this work, the MOOP is designed with two distinct objectives. The objective of minimizing gearbox volume:

$$\min f_1(X) = V_{gb} \quad (13)$$

and maximizing gearbox efficiency:

$$\max f_2(X) = \eta_{gb} \quad (14)$$

where *X* represents the vector in the design that replicates the variables. A two-stage helical gearbox is comprised of five primary design elements: u_1, X_{ba1}, X_{ba2}, AS_1, and AS_2 [18]. Additionally, findings indicate a correlation between the maximum and optimal values of AS_1 and AS_2 [21]. Therefore, the three main design features, u_1, X_{ba1}, and X_{ba2}, were used as variables in the optimization problem of this work. In consequence of the aforementioned circumstances, we are now proceeding with the distribution of:

$$X = \{u_1, Xba_1, Xba_2\} \quad (15)$$

With this gearbox, u_i=1÷9; $Xba_i = 0.25 \div 0.4$ (i=1÷2) [18]. Therefore, there are two constraints in the MOOP:

$$1 \le u_i \le 9 \quad (13)$$

$$0.25 \le Xba_i \le 0.4 \quad (14)$$

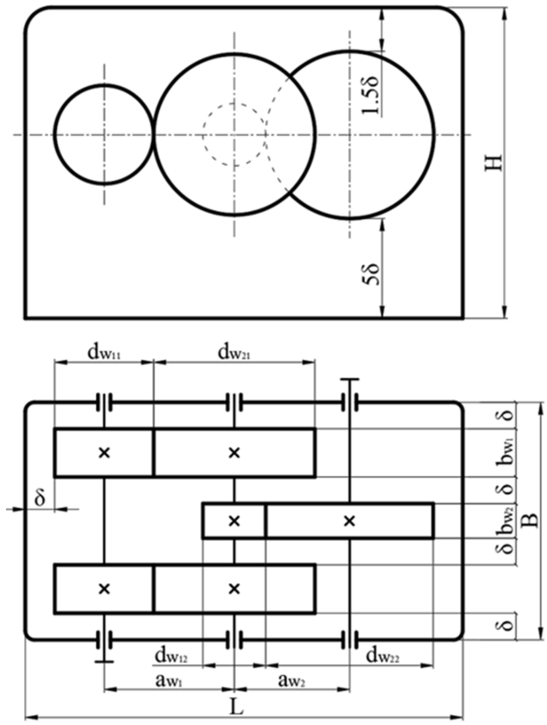

Fig. 1. Determining gearbox volume.

III. METHODOLOGY

A. Method to Solve MOOP

The objective of this research is twofold: to enhance the efficiency of the gearbox and to reduce its volume. Table I presents the three essential design factors that serve as the basis for the investigation. Moreover, the methodology from [20] was used to address the MOOP, as presented in Figure 2. The process is comprised of two distinct steps. The initial step in solving the single-objective optimization problem, as indicated in Table I, is to reduce the gaps between the input variables. The subsequent phase of the process is aimed at solving the multi-objective optimization problem by selecting the optimal primary design variables. In the event that the difference between the levels of the variables in the initial stage is less than 0.02, the lesser of the two differences between the input components will be selected for re-evaluation of the EAMR issue.

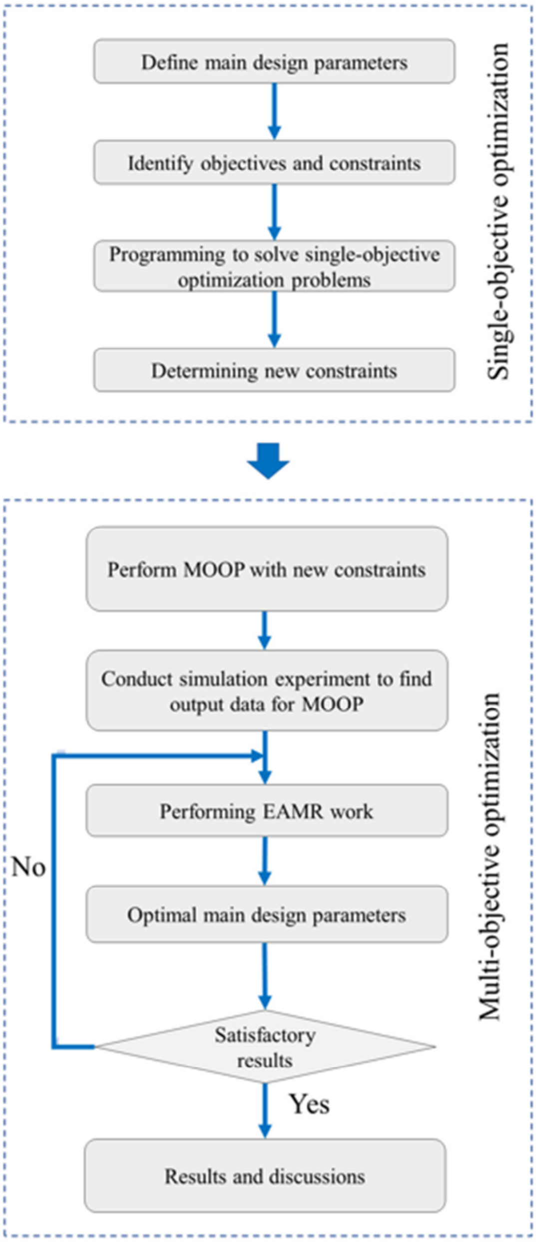

Fig. 2. The procedure to solve MOOP.

TABLE I. INPUT FACTORS

Factor	Minimal value	Maximal value
u_1	1	9
X_{ba1}	0.25	0.4
X_{ba2}	0.25	0.4

B. Methodology to Solve MCDM

The MCDM problem was solved using the EAMR method.

- To ensure the optimal application of this methodology, it is essential to observe the following stages with close attention:
- Build a decision-making matrix [22]:

$$X_d = \begin{bmatrix} x_{11}^d & \cdots & x_{1n}^d \\ x_{21}^d & \cdots & x_{21}^d \\ \vdots & \cdots & \vdots \\ x_{m1}^d & \cdots & x_{mn}^d \end{bmatrix} \quad (15)$$

where $1 \leq d \leq k$, k is the decision maker's number, d is the decision maker's indication.

- Find the mean value of each option by:

$$\bar{x}_{ij} = \frac{1}{k}\left(x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k\right) \quad (16)$$

- Calculate the creation weights.
- Compute each criterion's weighted average:

$$\bar{w}_j = \frac{1}{k}\left(w_j^1 + w_j^2 + \cdots + w_j^k\right) \quad (17)$$

- Find n_{ij} by:

$$n_{ij} = \frac{\bar{x}_{ij}}{e_j} \quad (18)$$

where e_j can be determined by:

$$e_j = max_{i\in\{1,\ldots,m\}}\left(\bar{x}_{ij}\right) \quad (19)$$

- Determine the normalized weight by:

$$v_{ij} = n_{ij} \cdot \bar{w}_j \quad (20)$$

- Find the criteria's normalized score: for the gearbox efficiency objective:

$$G_i^+ = v_{i1}^+ + v_{i2}^+ + \cdots + v_{im}^+ \quad (21)$$

and for the gearbox volume objective:

$$G_i^- = v_{i1}^- + v_{i2}^- + \cdots + v_{im}^{-+} \quad (22)$$

- Determine the ranking's (RV) values from *Gi+* and *Gi-*:
- Calculate the options' evaluation score by:

$$S_i = \frac{RV\left(G_i^+\right)}{RV\left(G_i^-\right)} \quad (23)$$

The best option is the one with the highest *Si* value.

C. Method to Calculate Criteria Weights

The criteria weights for the present analysis were established through the application of the entropy approach. In employing this approach [23], the indicator's normalized values were determined:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2} \quad (27)$$

along with the computation of the entropy for each indicator:

$$me_j = -\sum_{i=1}^{m}[p_{ij} \times ln(p_{ij})] - (1 - \sum_{i=1}^{m} p_{ij}) \times ln(1 - \sum_{i=1}^{m} p_{ij}) \quad (28)$$

and the calculation of the weight of each indicator:

$$w_j = \frac{1 - me_j}{\sum_{j=1}^{m}(1 - me_j)} \quad (29)$$

IV. SINGLE-OBJECTIVE OPTIMIZATION

In this study, the direct search approach was employed for the purpose of optimizing a single objective. Moreover, a Matlab computational application was employed to assess two distinct single-objective problems: the enhancement of η_{gb} and the minimization of A_b. The following observations provide a concise summary of the program's findings: Figure 3 presents a correlation between η_{gb} and u_1, in which there is a specific value of u_1 where η_{gb} reaches its maximum level, indicating an optimal value. Figure 4 provides a visual representation of the relationship between the variables u_1 and V_{gb}. The value of u_1 that yields optimal results is associated with the lowest value of V_{gb}. Moreover, Figure 5 shows the relationship between the optimal values of u_1 and u_{gb}. The constraints imposed on the variable u_1 were established based on the optimal values of u_1, as presented in Table II.

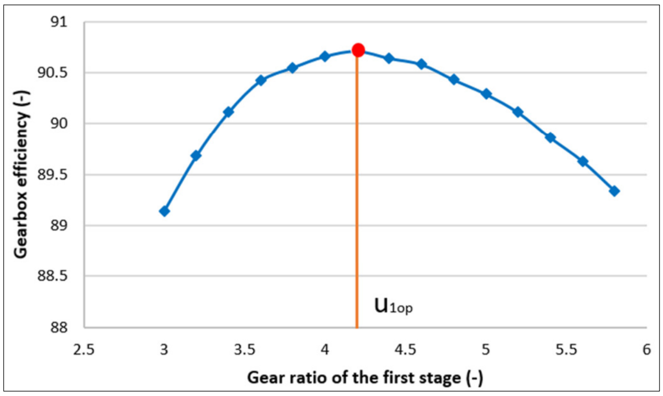

Fig. 3. Relation between u_1 and η_{gb}.

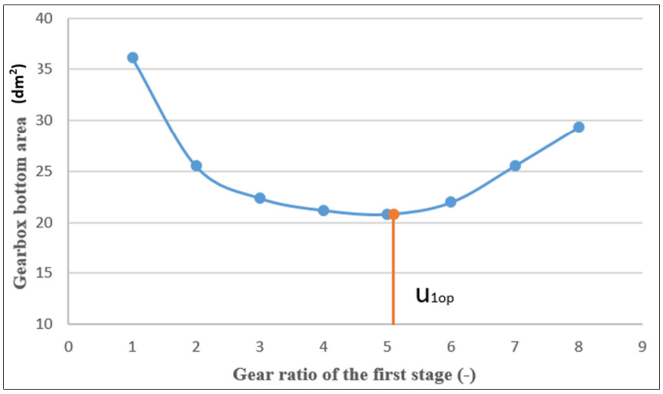

Fig. 4. Relation between u_1 and V_{gb}.

V. MULTI-OBJECTIVE OPTIMIZATION

A computer program has been developed for the purpose of conducting simulation experiments. The study examined the range of values for u_{gb}, which spanned from 5 to 25 with increments of 5. The following section presents the solutions to the problem with u_{gb}=20. The established gearbox ratio was used for the initial 125 testing cycles. The experiment will provide EAMR with the requisite output data, notably the gearbox volume efficiency, which will then be used as input parameters for solving the MOOP.

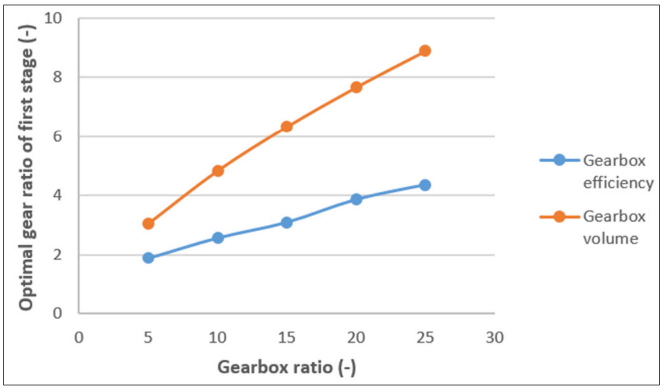

Fig. 5. Relation between u_{gb} and optimal values of u_1.

TABLE II. NEW LIMITATIONS OF u_1

ugb	u_1	
	Lower limit	Upper limit
5	1.78	3.14
10	2.47	4.93
15	3.00	6.43
20	3.77	7.77
25	4.27	9.00

This process will continue until the discrepancy between the two levels of each variable is less than 0.02. Table III presents the principal design factors and output responses for the fifth and final iteration of the EAMR work, with an u_{gb} value of 20. The criterion weights were determined using the entropy technique in the following manner: The values p_{ij} are subsequently normalized using (27). The entropy value for each indicator, M_{ej}, was calculated and the value of the criterion weight, w_j, is calculated by (29). The assigned weights for V_{gb} and η_{gb} were calculated to be 0.5431 and 0.4569, respectively. The decision matrix is initially constructed using (18). Subsequently, the mean of the selections for each criterion should be calculated using (19), while the mean weighted values can be determined by (20). Subsequently, the value of n_{ij} is estimated using (21), with due consideration of the fact that e_j, determined by (22). Subsequently, (23) should be applied in order to determine the precise value of V_{ij}. The values of G_i are determined by employing (24) for the gearbox efficiency objective and (25) for the gearbox volume target. Ultimately, the S_i value can be determined through the application of (26). The results of the option ranking and parameter computation using the EAMR approach are presented in Table IV, which displays the final iteration of

EAMR work. As evidenced in the table, option 80 represents the most optimal decision among the available alternatives. The optimal values for the crucial design components are u_1=5.90, X_{ba1}=0.25, and X_{ba2}=0.4, as specified in Table V. Figure 6 depicts the relationship between the ideal values of u_1 and u_{gb}. The given regression equation, which has an R_2 coefficient of determination of 0.9916, can be used to calculate the optimal values of u_1.

$$u_1 = 2.1768 \cdot \ln(u_{gb}) - 0.6616 \tag{24}$$

TABLE III. MAIN DESIGN FACTORS AND OUTPUT RESULTS FOR u_{gb}=20

u_{gb}	u_1	X_{ba1}	X_{ba2}	V_{gb} (dm^3)	η_{gb} (%)
1.00	5.84	0.25	0.25	23.43	90.22
2.00	5.84	0.25	0.29	22.48	90.10
3.00	5.84	0.25	0.33	21.73	90.00
4.00	5.84	0.25	0.36	21.12	89.91
5.00	5.84	0.25	0.40	20.62	89.83
6.00	5.84	0.29	0.25	24.08	88.45
…					
30.00	5.86	0.25	0.40	20.60	89.79
31.00	5.86	0.29	0.25	24.06	88.41
32.00	5.86	0.29	0.29	23.05	88.29
…					
50.00	5.86	0.40	0.40	22.37	81.27
51.00	5.88	0.25	0.25	23.40	90.14
52.00	5.88	0.25	0.29	22.46	90.02
…					
79.00	5.90	0.25	0.36	21.09	89.80
80.00	5.90	0.25	0.40	20.58	89.72
81.00	5.90	0.29	0.25	24.04	88.31
…					
123.00	5.92	0.40	0.33	23.76	81.19
124.00	5.92	0.40	0.36	22.98	81.11
125.00	5.92	0.40	0.40	22.34	81.00

TABLE IV. CALCULATED RESULTS AND RANKINGS OF OPTIONS FOR u_{gb}=20

Trial	n_{ij}		v_{ij}		G_i		S_i	Rank
	V_{gb}	η_{gb}	V_{gb}	η_{gb}	V_{gb}	η_{gb}		
1	0.9032	1.0000	0.4906	0.4569	0.4906	0.457	0.9314	49
2	0.8666	0.9987	0.4707	0.4563	0.4707	0.456	0.9694	30
3	0.8377	0.9976	0.4550	0.4558	0.4550	0.456	1.0018	20
4	0.8142	0.9966	0.4422	0.4553	0.4422	0.455	1.0297	10
5	0.7949	0.9957	0.4317	0.4549	0.4317	0.455	1.0537	5
6	0.9283	0.9804	0.5042	0.4479	0.5042	0.448	0.8885	75
…								
30	0.7941	0.9952	0.4313	0.4547	0.4313	0.455	1.0543	3
31	0.9275	0.9799	0.5037	0.4477	0.5037	0.448	0.8888	71
32	0.8886	0.9786	0.4826	0.4471	0.4826	0.447	0.9265	52
…								
50	0.8624	0.9008	0.4684	0.4116	0.4684	0.412	0.8787	82
51	0.9021	0.9991	0.4899	0.4565	0.4899	0.456	0.9317	47
52	0.8658	0.9978	0.4702	0.4559	0.4702	0.456	0.9694	28
79	0.8130	0.9953	0.4416	0.4548	0.4416	0.455	1.0299	7
80	0.7934	0.9945	0.4309	0.4544	0.4309	0.454	1.0545	1
81	0.9268	0.9788	0.5033	0.4472	0.5033	0.447	0.8885	73
…								
123	0.9160	0.8999	0.4975	0.4112	0.4975	0.411	0.8265	110
124	0.8859	0.8990	0.4811	0.4108	0.4811	0.411	0.8537	95
125	0.8612	0.89781	0.4677	0.4102	0.4677	0.41	0.877	85

TABLE V. OPTIMAL MAIN DESIGN FACTORS

No.	u_{gb}				
	5	10	15	20	25
u_1	2.95	4.13	5.27	5.90	6.38
X_{ba1}	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.4	0.4	0.4	0.4	0.4

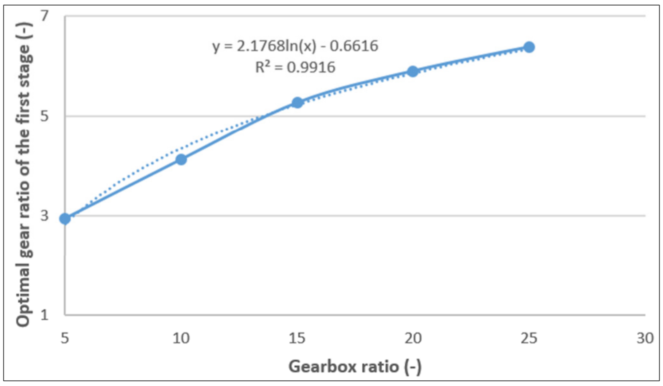

Fig. 6. Relation between optimal values of u_1 and u_{gb}.

VI. CONCLUSIONS

This paper presents the findings of a study that employed the Evaluation by an Area-based Method of Ranking (EAMR) technique to examine the Multi-Objective Optimization Problem (MOOP) in a two-stage helical gearbox comprising two gears in the initial stage. The primary objective of this study is to identify the optimal fundamental design elements that enhance gearbox efficiency while simultaneously reducing the gearbox's volume. To achieve this goal, three essential elements of design, namely u1, Xba1, and Xba2, were selected. Furthermore, the EAMR approach was used to address the Multi-Criteria Decision-Making (MCDM) problem, while the entropy method was employed to calculate the weights of the criteria. The following conclusions were derived from the study.

- The EAMR methodology has been proven to be an effective means of resolving the MOOP, thereby identifying the optimal primary design factors for a two-stage helical gearbox comprising two gear sets in the initial stage.
- In regard to the principal design parameters, two distinct objectives were evaluated: maximizing the efficiency of the gearbox and minimizing its volume.
- Equation (30) and Table V can be employed to determine the three optimal main design factors for the gearbox, as evidenced by the study's findings.
- Equation (30), with an R^2 value of 0.9916, demonstrated an exceptional alignment between the experimental data and the proposed model of u_1, thereby confirming their dependability.

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