Determining the Best Design Factors of a Two-stage Helical Gearbox with Two Gear Sets in the First Stage to Increase Efficiency and Reduce Volume using the SAW Method

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ABSTRACT

This study describes the outcomes of employing the Simple Additive Weighting (SAW) approach to address the Multi-Objective Optimization Problem (MOOP) of a two-stage helical gearbox with two gear sets at the first stage. Its objective is to determine the key design variables that can reduce the volume of the gearbox while simultaneously maximizing its efficiency. For this investigation, three key design parameters were selected, namely the coefficients of the wheel face width of the first and second stages (X_{ba1} and X_{ba2}), and the gear ratio of the first stage u_1. In addition, the SAW technique was deployed to deal with the problem of Multi-Criteria Decision Making (MCDM), while the Method based on the Removal Effects of Criteria (MEREC) was employed to determine the weight criterion for addressing the MOOP. The obtained results are valuable for defining the optimal values for three primary design factors, which are essential for the development of a two-stage helical gearbox with two gear sets at the first stage.

Keywords- SAW method; MEREC; helical gearbox; gear ratio

I. INTRODUCTION

A gearbox is a vital part of a mechanical power system, since its operation involves increasing the torque and decreasing the speed sent from the motor shaft to the working shaft. There is a wide range of gearbox types, such as worm gearboxes, planetary gearboxes, bevel gearboxes, and helical gearboxes. Helical gearboxes are the most popular kind due to their affordability, ease of use, and simple design. The rationale behind the academic research conducted on this field is related to optimizing the helical gearbox. Authors in [1] evaluated the failure mode behavior of spur gear pairs made from 20 regularly used gear materials. They specifically investigated gear pairs with a full depth of 20° and 25° pressure angles. The target function of the study was to minimize the center distance while ensuring that the bending, pitting, interference, and scoring failure were kept within acceptable limits as restrictions. Authors in [2] performed a study on determining the most efficient gear ratios for mechanical-driven systems. The study focused on employing a chain drive and a two-step helical gearbox, with the first phase encompassing double gear sets to decrease the system's cross-sectional area. The study examined several input parameters, including the total system ratio, the wheel face width coefficients of both helical gear sets, the permissible contact stress, and the output torque. The results demonstrated that the optimal ratios can be achieved with a high level of precision by utilizing proposed models. Authors in [3] attempted to determine the most efficient gear ratios for a two-stage helical reducer that utilizes double gear sets at the first stage. A study was conducted to develop an optimization problem aimed at minimizing the cross-sectional area of the gearbox. The study assessed the impact of input parameters, such as the total reducer ratio, the wheel face width coefficient, the allowed contact stress, and the output torque, on the ideal gear ratios. Furthermore, equations for identifying the most favorable gear ratios were presented. Authors in [4] utilized a Genetic Algorithm (GA) to optimize the volume of a two-stage helical gear train. The objective function was modified by incorporating static and dynamic penalty functions to address design restrictions such as contact stress, bending stress, number of teeth on gear and pinion, module, and face width of the gear. The findings obtained using a GA were compared to those attained following a deterministic design technique, and it was found that the GA outperformed the deterministic approach. Authors in [5] conducted a research to identify the most effective partial transmission ratios for mechanical drive systems. They utilized a V-belt drive and a three-stage helical reducer to lower the size of the system's cross-section. The optimization problem yielded results that led to the formulation of equations for determining the ideal partial ratios of the V-belt drive and the three stages of the reducer. Authors in [6] deployed two sophisticated optimization methods, Particle Swarm Optimization (PSO) and Simulated Annealing (SA), to determine the ideal combination of design parameters that would result in the smallest weight of a spur gear train. The constraints were formulated according to AGMA standards, while PSO and SA were utilized to minimize the weight of a basic spur gear pair, which involved mixed integers, utilizing PSO and SA.

Authors in [7] introduced a multi-objective optimization approach, which employs the GA to determine the most suitable module, shaft diameter, and rolling bearing for a single-stage spur gearbox. The problem was defined by utilizing gear volume, shaft diameter, and rolling bearing dimensions as the objective functions, while tooth root fracture and surface fatigue failure were considered as constraints. Authors in [8] utilized the Taguchi and Grey Relation Analysis (GRA) techniques to examine the MOOP of constructing a two-stage helical gearbox. This study selected two objectives, achieving the minimum gearbox bulk and maximizing gearbox efficiency. The study's findings were used to ascertain the optimal values for the five fundamental design components involved in constructing a two-stage helical gearbox. In [9], the same methods were also adopted to reduce the cross-sectional area of the gearbox and enhance its efficiency. The techniques described in [10] were used to optimize a two-stage helical gearbox with second-stage double gear sets to enhance its efficiency and reduce gearbox mass. Authors in [11] followed the TOPSIS technique to solve the MOOP of a two-stage helical gearbox. This project aims to reduce the cross-sectional area of the gearbox and enhance its efficiency.

Authors in [12] conducted a research to identify the most effective primary design parameters for reducing the cross-sectional area of a two-stage helical gearbox. This study considered five key design characteristics of the gearbox for their optimal values to be determined. These factors included the gear ratio of the first stage, the coefficient of wheel face width for stages 1 and 2, and the permissible contact stress for stages 1 and 2. The study's conclusions also demonstrated optimal values for these factors. Authors in [13] conducted a multi-objective optimization of a two-stage helical gearbox utilizing the SAW technique. The objective of the study was to enhance the efficiency of the gearbox while minimizing its cross-sectional area. Authors in [14] conducted a study on optimizing the prediction of optimal partial ratios for three-step helical gearboxes with second-step double gear sets. The study aimed to achieve various objectives, such as minimizing gearbox length, minimizing gearbox cross-section dimension, and minimizing gear mass. The study focused on the moment equilibrium of a mechanical system consisting of three gear units and their regular resistance condition. Three optimization tasks were conducted to determine the minimal gearbox length, minimal gearbox cross-section dimension, and minimal mass of gears. Furthermore, the regression analysis technique was employed to discover explicit models for computing the partial ratios of the gearboxes. Authors in [15] performed a study on identifying the most efficient method for calculating gear ratios in a two-stage helical reducer. The objective was to decrease the cross-section area of the reducer. Based on the findings of that study, two methods were proposed for calculating the most efficient gear ratios of a two-stage reducer.

The MCDM method has been employed across multiple fields. Its utilization involved determining the most appropriate input parameters to identify the optimal design factors for a two-stage helical gearbox design [16], selecting the ideal airport [17], or assessing the ranking of universities [18]. Numerous studies have been conducted on different aspects of the MCDM. Authors in [19] employed the Preference Selection Index (PSI) approach to identify the optimal input elements in the external grinding of SCM steel. Authors in [20] utilized three MCDM methods: Magnitude of the Area for the Ranking of Alternatives (MARA), Root Assessment Method (RAM), and Proximity Indexed Value (PIV) for material selection. Authors in [21] used the PIPRECIA and modified FUCA methods for lathe selection. Authors in [22] adopted three methodologies: the RAM, PSI, and Simple Ranking Process (SRP) to evaluate the financial health of several banks in Vietnam. The current study employs the SAW strategy to address the MOOP of a two-stage helical gearbox with two gear sets at the first stage. Additionally, the MEREC is utilized to calculate the weights of the criteria. The aim is to decrease the volume of the gearbox while simultaneously increasing its efficiency. The obtained results enabled the identification of several major gearbox design parameters.

II. OPTIMIZATION PROBLEM

A. Calculating Gearbox Volume

As can be seen in Figure 1, the gearbox volume V_{gb} is calculated by:

$$V_{gb} = L \cdot B \cdot H \tag{1}$$

where L, B and H are computed by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \tag{2}$$

$$B = 2 \cdot b_{w1} + b_{w2} + 4 \cdot \delta \tag{3}$$

$$H = \max(d_{w21}; d_{w22}) + 8.5 \cdot \delta \tag{4}$$

where δ=7/10 mm [23], b_{wi}, d_{w1i}, d_{w2i} represent the gear with the pitch diameter of the pinion and the gear of the ith stage (i=1/2), which are found by:

$$b_{wi} = X_{bai} \cdot a_{wi} \tag{1}$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \tag{6}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \tag{7}$$

where X_{bai} and a_{wi} (i=1/2) are the wheel face width coefficient and the center distance of stage I. a_{wi} can be calculated by [23]:

$$a_{wi} = k_a(u_i + 1)\sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \tag{2}$$

where T_{1i} (i=1/2) denotes the torque on the pinion of the ith stage, which is determined by:

$$T_{11} = T_{out}/\left(2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \tag{9}$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \tag{10}$$

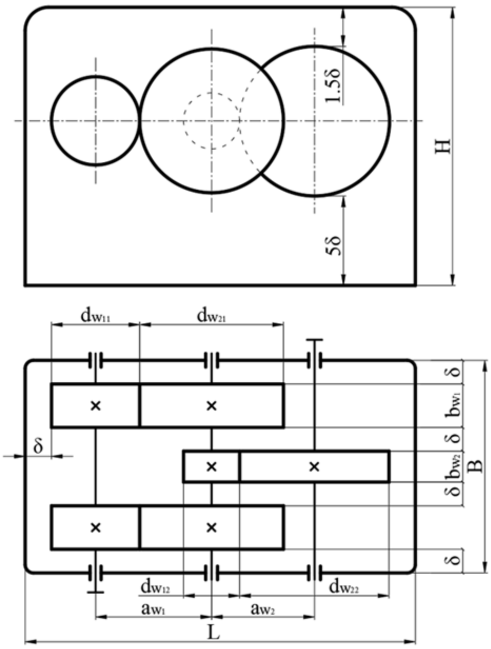

Fig. 1. Finding gearbox volume.

B. Calculating Gearbox Efficiency

The gearbox efficiency (%) can be determined by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{11}$$

where P_l is the total power loss, which can be found by [24]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{12}$$

In (12), P_{lg}, P_{lb}, P_{ls}, and P_{Z0} represent the power loss in the gearings, bearings, seals, and the idle motion. These components are determined as in [13].

C. Objectives and Constraints

1) Objectives

The MOOP in the present study has two single objectives, minimizing gearbox volume and maximizing gearbox efficiency:

$$\min f_1(X) = V_{gb} \tag{13}$$

$$\min f_2(X) = \eta_{gb} \tag{14}$$

where X denotes the vector in the design that duplicates the variables. A two-stage helical gearbox with a first stage has two gear sets comprising five fundamental design components u_1, X_{ba1}, X_{ba2}, AS_1, and AS_2 [13]. Furthermore, the findings reveal a link between the maximum and optimal values of AS_1 and AS_2 [13]. Consequently, the three primary design features - u_1, X_{ba1}, and X_{ba2} - were employed as variables in the optimization problem of this study. It is thus currently distributed:

$$X = \{u_1, Xba_1, Xba_2\} \tag{15}$$

2) Constraints

For the gearbox, u_i=1/9 and $Xba_i = 0.25/0.4$ (i=1/2) [13]. Therefore, for the MOOP, there are the two following constraints:

$$1 \leq u_i \leq 9 \tag{16}$$

$$0.25 \leq Xba_i \leq 0.4 \tag{17}$$

III. METHODOLOGY

A. Method for Solving MOOP

The purpose of the current study is to enhance gearbox efficiency and reduce its volume. Table I indicates the three fundamental design elements which are the investigation's input. Moreover, the methodology detailed in [13] was used for solving the MOOP. The process for executing this activity is depicted in Figure 2. There are two separate stages in this process. Minimizing the differences among the input variables is the initial step in addressing the single-objective optimization issue, as demonstrated in Table I. Nonetheless, the subsequent step aims to solve the MOOP by identifying the optimal primary design factors. If the difference between the levels of the variables is less than 0.02, the smaller difference between the two levels of the input components will be used to conduct the SAW approach/by the SAW approach.

TABLE I. INPUT PARAMETERS

Parameter	Minimal value	Maximal value
u_1	1	9
X_{ba1}	0.25	0.4
X_{ba2}	0.25	0.4

B. Method to Solve MCDM

The MCDM problem was resolved with the SAW approach. To accurately implement this strategy, it is essential to meticulously monitor the subsequent processes [25]:

- Step 1: Create the first decision-making matrix:

$$X = \begin{array}{ccccc} & C_1 & C_2 & \cdots & C_n \\ A_1 & y_{11} & y_{12} & \cdots & y_{1n} \\ A_2 & y_{21} & xy_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ A_m & y_{m1} & y_{m2} & \cdots & y_{mn} \end{array} \qquad (18)$$

where m and n are option and criterion numbers.

- Determine the normalized matrix by:

$$n_{ij} = \frac{r_{ij}}{\max r_{ij}} \qquad (19)$$

$$n_{ij} = \frac{\min r_{ij}}{r_{ij}} \qquad (20)$$

Note that (19) is used for the gearbox efficiency objective, and (20) for the gearbox volume.

- Calculate the preference value for each alternative:

$$V_i = \sum_{j=1}^{n} w_j \cdot n_{ij} \qquad (3)$$

- Rank the alternative's order by maximizing V_i.

IV. METHOD TO FIND CRITERIA WEIGHTS

In this work, the criteria weights for the MCDM problem were found by deploying MEREC. To use this method, the following steps must be taken [26]:

1. Generate the initial matrix following the first step of the SAW method.
2. After normalizing the matrix, calculate the values of its elements by:

- For the gearbox efficiency objective:

$$h_{ij} = \frac{\min x_{ij}}{x_{ij}} \qquad (21)$$

- For the gearbox volume objective:

$$h_{ij} = \frac{x_{ij}}{\max x_{ij}} \qquad (22)$$

3. Determine the effectiveness of the S_i options by:

$$S_i = \ln\left[1 + \left(\frac{1}{n}\sum_j \left|\ln(h_{ij})\right|\right)\right] \qquad (23)$$

4. Compute the efficiency of the ith option S'_{ij} by:

$$S'_{ij} = \mathrm{Ln}\left[1 + \left(\frac{1}{n}\sum_{k,k\neq j} \left|\ln(h_{ij})\right|\right)\right] \qquad (24)$$

5. Find the removal effect of the jth criterion E_j:

$$E_j = \sum_i \left|S'_{ij} - S_i\right| \qquad (25)$$

6. Determine the criteria's weight by:

$$w_j = \frac{E_j}{\sum_k E_k} \qquad (26)$$

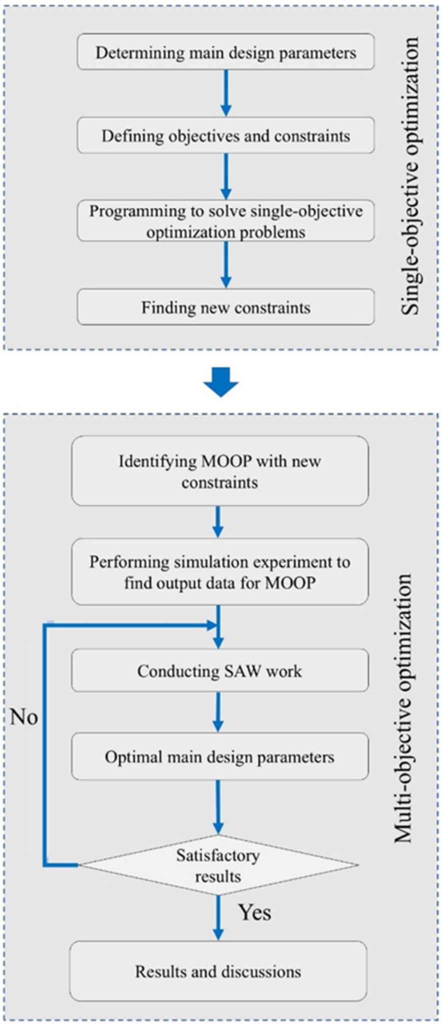

Fig. 2. The procedure for solving MOOP.

V. SINGLE-OBJECTIVE OPTIMIZATION

This study employed a direct search method for the optimization of a singular target. Additionally, a MATLAB computational application was employed to evaluate two separate single-objective problems: optimizing η_{gb} and reducing V_{gb}. The program's results are concisely expressed in the following observations. Figure 3 depicts the link between η_{gb} and u_1. It represents a particular value of u_1 at which η_{gb} attains its maximum, signifying the optimal value. Figure 4 displays the correlation between the variable u_1 and the variable V_{gb}. The optimal value of u_1, as shown in Figure 3, corresponds to the minimal value of V_{gb}. Also, Figure 5 portrays the relation between the optimal values of u_1 and u_{gb}.

The constraints for the variable u_1 were established according to the optimal values of u_1, as indicated in Table II.

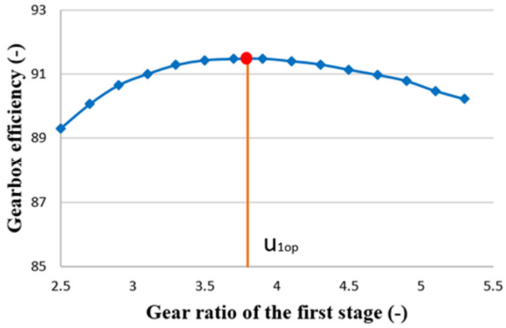

Fig. 3. Relation between u_1 and η_{gb}.

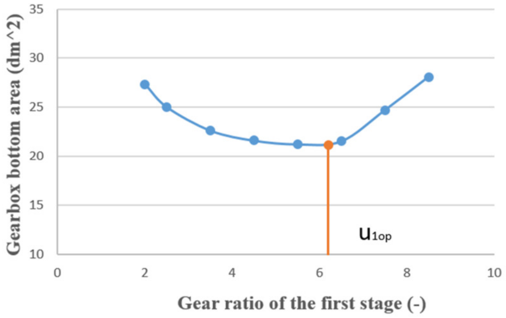

Fig. 4. Relation between u_1 and V_{gb}.

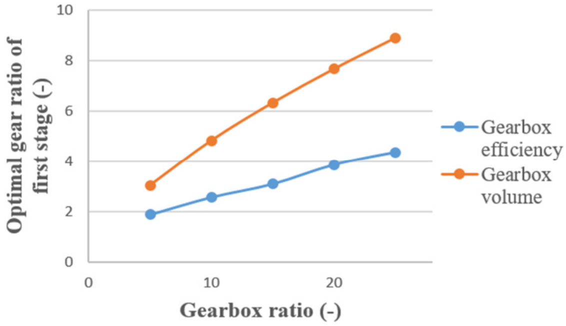

Fig. 5. Relation between u_{gb} and optimal gear ratio of the first stage.

TABLE II. NEW CONSTRAINTS OF U_1

u_{gb}	u_1	
	Lower limit	Upper limit
5	1.78	3.14
10	2.47	4.93
15	3	6.43
20	3.77	7.77
25	4.27	9

VI. MULTI-OBJECTIVE OPTIMIZATION

A simulation experiment was conducted to solve the MOOP. The input parameters of the experiment were the main design parameters of the gearbox (Table I). The experiment aimed to determine the values of η_{gb} and V_{gb} to set the input parameters for the MCDM problem. A computer program has been created to perform simulation experiments. The investigation examined the values for u_{gb}, which varied from 5 to 25 in increments of 5. The subsequent solutions deal with the issue where u_{gb} equals 10. The previously chosen gearbox ratio was employed for 125 initial testing cycles, as detailed in Section III. The experiment will provide SAW with output data, specifically the gearbox volume efficiency, to serve as input parameters for addressing the MOOP. This method will continue until the gap between the two levels of each variable is below 0.02. Table III outlines the primary design elements and output responses for the fifth and final iteration of the SAW project, involving an u_{gb} value of 10. The criteria weights were established with the MEREC, as outlined in Section III.C, in the subsequent manner. The values h_j were normalized utilizing (22) and (23). The values of S_i and S_{ij}' were calculated using (24) and (25). The effect of eliminating the condition was determined using (26). The weights for the criteria, w_j, were calculated via (27). Section III.B offers guidelines for the effective application of the SAW method in solving issues related to MCDM. The approach began by computing the decision-making matrices utilizing (18). The original matrix must be normalized utilizing (19) and (20). The calculation of Vi is thereafter executed using (21). Ultimately, the options were arranged to ensure that the solution with the maximum benefit possesses the highest Vi. The results of the option ranking and parameter calculation employing the SAW method are displayed in Table IV (for the final iteration of the SAW process). Table IV indicates that option 30 is the most advantageous choice among all the alternatives. The optimal values for the essential design components are u_1=3.34, X_{ba1}=0.25, and X_{ba2}=0.4, as seen in Table V. Figure 8 illustrates the correlation between the optimal values of u_1 and u_{gb}. The provided regression equation, with an R² coefficient of determination of 0.9932, can be utilized to determine the ideal values of u_1:

$$u_1 = 2.1719 \cdot \ln\left(u_{gb}\right) - 0.643 \qquad (27)$$

TABLE III. MAIN DESIGN PARAMETERS AND OUTPUT RESULTS FOR U_{GB}=10

Trial.	u_1	X_{ba1}	X_{ba2}	V_{gb} (dm^3)	η_{gb} (%)
1	3.32	0.25	0.25	21.87	93.69
2	3.32	0.25	0.29	21.01	93.62
3	3.32	0.25	0.33	20.33	93.55
4	3.32	0.25	0.36	19.77	93.49
5	3.32	0.25	0.40	19.31	93.52
6	3.32	0.29	0.25	22.45	92.81
…					
29	3.34	0.25	0.36	19.75	93.46
30	3.34	0.25	0.40	19.29	93.50
31	3.34	0.29	0.25	22.43	92.79
…					
60	3.35	0.29	0.40	19.67	92.59
61	3.35	0.33	0.25	22.97	91.75
62	3.35	0.33	0.29	22.00	91.67
…					
123	3.38	0.40	0.33	22.09	89.20
124	3.38	0.40	0.36	21.39	89.14
125	3.38	0.40	0.40	20.81	89.19

TABLE IV. CALCULATED RESULTS AND RANKINGS OF OPTIONS FOR U_{GB}=10

Trial.	n_{ij}		V_i	Rank
	V_{gb}	η_{gb}		
1	0.8797	1.0000	0.9777	31.0000
2	0.9158	0.9993	0.9838	21.0000
3	0.9464	0.9985	0.9888	11.0000
4	0.9732	0.9979	0.9933	6.0000
5	0.9964	0.9982	0.9978	2.0000
6	0.8570	0.9906	0.9658	56.0000
…				
29	0.9742	0.9975	0.9932	8.0000
30	0.9974	0.9980	0.9979	1.0000
31	0.8578	0.9904	0.9658	57.0000
…				
60	0.9781	0.9883	0.9864	19.0000
61	0.8376	0.9793	0.9530	83.0000
62	0.8745	0.9784	0.9592	73.0000
…				
123	0.8710	0.9521	0.9370	110.0000
124	0.8995	0.9514	0.9418	105.0000
125	0.9246	0.9520	0.9469	95.0000

TABLE V. OPTIMAL MAIN DESIGN FACTORS

No.	u_{gb}				
	5	10	15	20	25
u_1	2.95	4.16	5.27	5.90	6.38
X_{ba1}	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.4	0.4	0.4	0.4	0.4

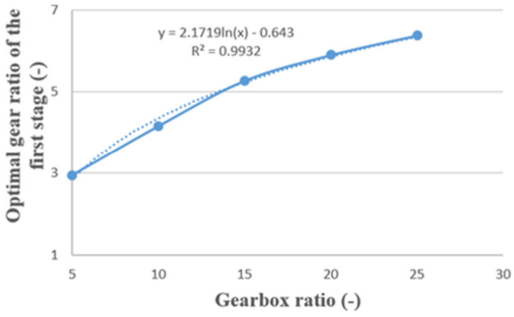

Fig. 6. Relation between optimal values of u_1 and u_{gb}.

VII. CONCLUSIONS

This paper describes the outcomes of a study applying the Simple Additive Weighting (SAW) approach to examine the Multi-Objective Optimization Problem (MOOP) in a two-stage helical gearbox having two gear sets at the initial stage. The main aim of the study is to find the best possible fundamental design elements that improve gearbox efficiency while simultaneously minimizing its volume. To accomplish this, three fundamental elements of design, specifically u_1, X_{ba1}, and X_{ba2}, were selected. The SAW methodology was utilized to deal with the Multi-Criteria Decision Making (MCDM) problem, while the Method based on the Removal Effects of Criteria (MEREC) was applied to determine the criteria weights. The study's findings contributed to the identification of the most advantageous essential design elements for a two-stage helical gearbox featuring two gears at the initial stage. The required data are found in Table V, and the relevant mathematical formula is shown in (27). The subsequent conclusions were derived from this work:

- The SAW method effectively resolved the MOOP to identify the best primary design factors for a two-stage helical gearbox with two gears at the initial stage.
- In this study, two primary objectives were examined: minimizing gearbox volume and maximizing gearbox efficiency.
- Table V and (27) facilitate the estimation of optimal primary design parameters for the gearbox, as indicated by the study's findings. The high degree of confidence and strong agreement of (27) with the experimental data is evidenced by its R^2 value of 0.9932.

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