

Multi-Objective Optimization of a Two-stage Helical Gearbox with Double Gears in the First Stage using MARCOS

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ABSTRACT

This study demonstrates the solution of the Multi-Objective Optimization Problem (MOOP) of a two-stage helical gearbox with double gears at the first stage, following the MARCOS methodology. The goal of this work is to identify the most effective essential design factors to reduce the bottom area of the gearbox while maximizing its efficiency, which constituted a significant novel finding. For this purpose, three crucial design parameters were selected, the first stage gear ratio and the wheel face width (Xba) coefficients for the first and second stage. Furthermore, the Multi-Criteria Decision Making (MCDM) issue was chosen to be handled by the MARCOS method, and the weight criterion for solving the MOOP was determined by the MEREC method. The drawn conclusions are useful in developing a two-stage helical gearbox with double gears at the first stage by helping to identify the ideal values for the three important design parameters.

Keywords-MARCOS method; ENTROPY method; helical gearbox; gear ratio

I. INTRODUCTION

A gearbox is an essential part of a mechanical power transmission system. It is employed to amplify the torque and decrease the speed transmitted from the motor shaft to the working shaft. It is utilized in several applications, including automotive systems, hoisting apparatuses, agricultural machinery, and more. Therefore, numerous academics are actively pursuing the ideal gearbox design.

Authors in [1] examined four objective parameters, the lowest size, weight, tooth deflection, and maximum life of a spur gear pair, deploying the Modified Iterative Weighted Techebycheff (MIWT) method. This approach, however, faces the challenge of achieving convergence, which is contingent upon the initial sample vector. Additionally, the time required for the convergence of the solution is frequently excessive. Authors in [2] proposed a method for the construction and optimization of multi-spindle gear trains, utilizing a two-stage procedure for the optimized algorithm to be achieved. The

initial phase involved employing a direct search approach to exclude impracticable alternatives, whereas the subsequent phase entailed implementing a heuristic strategy. Authors in [3] conducted a study on determining the most efficient gear ratios for mechanical-driven systems by employing a three-stage bevel helical gearbox and a chain drive. The objective of the study was to decrease the cross-sectional area of the system. Furthermore, six input parameters were considered, which encompassed the overall system ratio, the permissible contact stress, the face width coefficients for both the bevel and helical gear sets, and the output torque. The study yielded results that allowed for the estimation of the impact of input characteristics on the optimal ratios. Additionally, equations were proposed to facilitate the calculation of the optimal gear ratios. Authors in [4] attempted to determine the most efficient gear ratios for mechanical drive systems including a two-stage helical gearbox with double gear sets at the first stage and a chain drive. The study aimed to minimize the system length, which has been selected as the objective function for the optimization problem. In addition, the study examined the input parameters, namely the overall system ratio, the wheel face width coefficients for the first and second stages, the permissible contact stress, and the output torque. The derived equations demonstrated that the optimal gear ratios could be accurately and readily computed.

Authors in [5] employed a customized adaptive random search technique to optimize the weight of the helical gear pair. The design variables taken into consideration were the gear module, helix angle, pinion teeth, and face width, while the restrictions involved the contact stress and tooth-bending strength. The constraint lies in the fact that the suggested approach was a non-deterministic random-search technique, and it is only efficient for a limited number of design variables. Authors in [6] explored the failure mode characteristics of spur gear pairs made from 20 regularly used gear materials, using both full-depth 20° and 25° pressure angles. The target function was the center distance, whereas the bending, pitting, interference, and scoring failure were regarded as restrictions. Authors in [7] conducted a study on optimizing the prediction of optimal partial ratios for three-step helical gearboxes with second-step double gear sets. The study aimed to achieve various objectives, such as minimizing the gearbox length, minimizing the gearbox cross-section dimension, and minimizing the mass of gears. It focused on the moment equilibrium of a mechanical system consisting of three gear units and their resistance conditions. Three optimization problems were investigated to determine the minimum length of the gearbox, the minimum cross-sectional dimension of the gearbox, and the minimum mass of the gears. Furthermore, the regression analysis technique was employed to discover explicit models for computing the partial ratios of the gearboxes. Authors in [8] carried out a study on determining the most efficient method for calculating the gear ratios of a two-stage helical reducer, aiming to decrease the surface area of the cross-section of the reducer. Based on that study's findings, two methods were proposed for calculating the most efficient gear ratios of a two-stage reducer.

Authors in [9] employed a Genetic Algorithm (GA) to optimize the volume of a two-stage helical gear train. The goal function was augmented with the static and dynamic penalty

functions to address design restrictions, such as contact stress, bending stress, number of teeth on gear and pinion, module, and face width of gear. The findings obtained using GA were compared to those attained by deploying a deterministic design technique, while it was determined that GA outperformed the deterministic approach. Authors in [10] introduced two sophisticated optimization techniques, namely Particle Swarm Optimization (PSO) and Simulated Annealing (SA), with the aim of identifying the most optimal combination of design parameters that would result in the smallest weight of a spur gear train. The constraints were formulated according to AGMA standards to minimize the weight of a basic spur gear pair that involves mixed integers utilizing PSO and SA. Authors in [11] attempted to identify the most efficient gear ratios for mechanical-driven systems. They focused on employing a chain drive and a two-step helical gearbox, with the first step involving double gear sets to minimize the system's cross-sectional area. The study examined several input parameters, including the total system ratio, the wheel face width coefficients of both helical gear sets, the permissible contact stress, and the output torque. The results demonstrated that the optimal ratios can be achieved with a high degree of precision by employing the proposed models.

Authors in [12] tried to determine the most efficient partial transmission ratios for mechanical drive systems. The study centered on employing a chain drive and a three-step helical reducer to decrease the height of the system's cross-section. The optimization problem yielded equations that provide the optimal partial ratios of the chain drive and three phases of the reducer. Authors in [13] proposed a multi-objective optimization approach using GA to find the optimal module, shaft diameter, and rolling bearing for a single-stage spur gearbox. The problem was defined by considering gear volume, shaft diameter, and rolling bearing dimensions as the objective functions, while tooth root fracture and surface fatigue failure were treated as constraints. Authors in [14] performed a multi-objective optimization of a two-stage helical gearbox deploying the MARCOS approach, which involves measuring alternatives and rating them based on a compromise solution. The objective of the study was to identify the most favorable primary design parameters that would enhance gearbox efficiency while reducing gearbox volume.

The MCDM method has been successfully utilized in various domains to ascertain the optimal solution. For instance, identifying the most suitable input parameters in the milling process [15, 16], selecting the optimal airport [17], choosing the appropriate material for crankshaft production [18], in material selection [19], or ranking the top ten universities in Vietnam [20]. A significant number of studies have been performed on various aspects of MCDM. Authors in [21] conducted a study comparing two MCDM methods using three examples from various fields. Authors in [22] accomplished a study aimed at extending the application range of the Preference Selection Index (PSI) method by determining the suitable data normalization method to be used in combination with the PSI method. Authors in [23] introduced a novel methodology aimed at enhancing the efficiency of the Ranking Alternatives by Perimeter Similarity (RAPS) technique within the context of MCDM. Recently, several investigations have

been conducted on the application of the MCDM method to address the MOOP, and thus identify the main design factors for a two-stage helical gearbox [24], as well as for a two-stage helical gearbox with second-stage double gear sets [25, 26]. However, until now, there has been no research for a two-stage helical gearbox with two gear sets at the first stage. To solve the MOOP of a two-stage helical gearbox with double gears at the first stage, this study uses the MARCOS technique to handle the MCDM problem, and the ENTROPY method to estimate the criteria weights. Reducing the gearbox bottom area while boosting efficiency is the aim of the current study. The results of the analysis helped to determine some of the most important essential gearbox design parameters.

II. OPTIMIZATION PROBLEM

A. Calculation of the Bottom Area of the Gearbox

The bottom area A_b of the gearbox is calculated by:

$$A_b = (L \cdot B) \quad (1)$$

where L and B are determined by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \quad (2)$$

$$B = b_{w1} + b_{w2} + 2 \cdot \delta \quad (3)$$

In the above Equations, $\delta=7/10$ mm [27], b_{wi} , d_{w1i} , and d_{w2i} are the width of the gear, the pitch diameter of the pinion and the gear of the i^{th} stage ($i=1/2$), which are found by:

$$b_{wi} = X_{bai} \cdot a_{wi} \quad (4)$$

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (5)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (6)$$

where X_{bai} and a_{wi} ($i=1/2$) denote the wheel face width coefficient and the gearbox center distance of stage i , a_{wi} is determined by [27]:

$$a_{wi} = k_a (u_i + 1) \sqrt[3]{T_{1i} k_{H\beta} / ([AS_i]^2 u_i X_{bai})} \quad (7)$$

In which, T_{1i} ($i=1/2$) is the torque on the pinion of the i^{th} stage, determined by:

$$T_{11} = T_{out} / (2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3) \quad (8)$$

$$T_{12} = T_{out} / (u_2 \cdot \eta_{hg} \cdot \eta_{be}^2) \quad (9)$$

B. Calculation of Gearbox Efficiency

The gearbox efficiency (%) is determined by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (10)$$

where P_l denotes the total power loss in the gearbox, which can be found by [28]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{z0} \quad (11)$$

In (11), P_{lg} , P_{lb} , P_{ls} , and P_{z0} denote the power loss in the gearings, in bearings, in seals, and in the idle motion. These components are determined, as in [29].

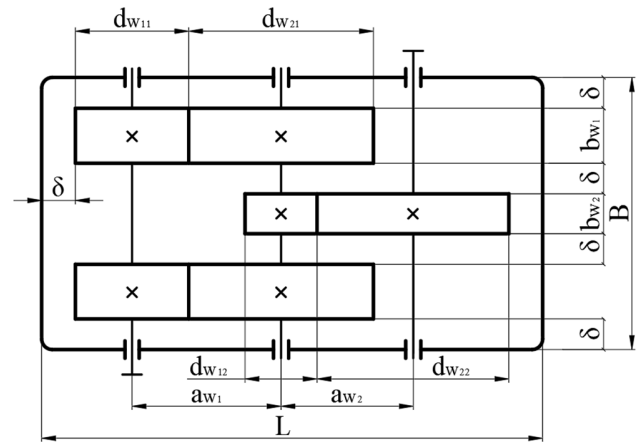


Fig. 1. For determination of gearbox bottom area.

C. Objectives and Constraints

1) Objectives

In this work, the MOOP has two single objectives:

- Minimizing gearbox bottom area:

$$\min f_1(X) = A_b \quad (12)$$

- Maximizing gearbox efficiency:

$$\min f_2(X) = \eta_{gb} \quad (13)$$

It is allowed for X to be the vector that duplicates the variables in the design. The five primary design elements that constitute a two-stage helical gearbox are u_1 , X_{ba1} , X_{ba2} , $AS1$, and $AS2$ [27]. Additionally, research discloses a relationship between the highest values and the ideal values of $AS1$ and $AS2$ [30]. Thus, u_1 , X_{ba1} , and X_{ba2} —the three primary design features - were used as variables in this work's optimization issue defined by:

$$X = \{u_1, X_{ba1}, X_{ba2}\} \quad (14)$$

2) Constraints

For the i^{th} stage of this gearbox $i=1/2$, $u_i=1/9$, and $X_{ba_i} = 0.25/0.4$ [27]. Therefore, there are two constraints in the MOOP:

$$1 \leq u_i \leq 9 \quad (15)$$

$$0.25 \leq X_{ba_i} \leq 0.4 \quad (16)$$

III. METHODOLOGY

1) Method for Solving MOOP

The study aims to achieve two objectives: reducing the gearbox bottom area and increasing its efficiency. The three crucial design elements that provide input for this study are shown in Table I. Moreover, the MOOP was solved using the methodology described in [17]. In Figure 2, the process for performing this task is depicted. It consists of two separate phases: As can be seen in Table I, the first stage involves addressing the single-objective optimization problem by reducing the differences between the input variables. Nevertheless, choosing the most ideal primary design variables

is the goal of the following step, which attempts to solve the multi-objective optimization issue. The smaller difference between the two levels of the input components is taken into consideration when reevaluating the MARCOS issue if the levels of the variables at the first stage do not differ by less than 0.02.

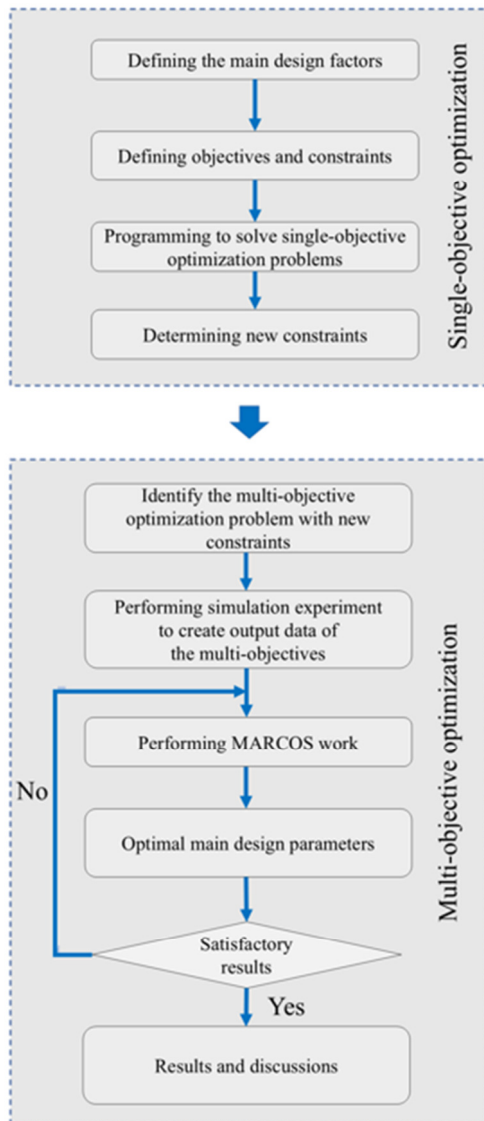


Fig. 2. The procedure for solving MOOP.

TABLE I. INPUT PARAMETERS

Parameter	Minimal value	Maximal value
u_i	1	9
X_{ba1}	0.25	0.4
X_{ba2}	0.25	0.4

2) Methodology for Solving MCDM

The MCDM issue was resolved by using the MARCOS approach. It is essential that close attention be paid to the following steps to use this strategy properly [31]:

1. Build the first decision-making matrix:

$$X = \begin{bmatrix} X_{11} & \dots & X_{1n} \\ X_{21} & \dots & X_{2n} \\ \vdots & \dots & \vdots \\ X_{m1} & \dots & X_{mn} \end{bmatrix} \tag{17}$$

where m and n represent the number and the criteria numbers.

2. Extend the scope of the decision-making matrix by presenting an optimal solution (AI) and a suboptimal solution (AAI):

$$X = \begin{matrix} AAI & \begin{bmatrix} X_{aa1} & \dots & X_{aan} \\ X_{11} & \dots & X_{1n} \\ X_{21} & \dots & X_{2n} \\ \vdots & \dots & \vdots \\ X_{m1} & \dots & X_{mn} \\ X_{ai1} & \dots & X_{ain} \end{bmatrix} \end{matrix} \tag{18}$$

where $AAI = \min(x_{ij})$ and $AI = \max(x_{ij})$ are used for the gearbox efficiency target, $AAI = \max(x_{ij})$ and $AI = \min(x_{ij})$ for the gearbox bottom area, $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$.

3. Normalize the extended first matrix X. The normalized matrix $N = [n_{ij}]_{m \times n}$ can be found by:

$$u_{ij} = x_{AI} / x_{ij} \tag{19}$$

$$u_{ij} = x_{ij} / x_{AI} \tag{20}$$

Equation (19) is used for the gearbox bottom area target, and (20) is applied for the gearbox efficiency.

4. Calculate the weighted normalized matrix $C = [c_{ij}]_{m \times n}$ by:

$$c_{ij} = u_{ij} \cdot w_j \tag{21}$$

where w_j is the weight coefficient of criterion j.

5. Find the utility of alternatives K_i^- and K_i^+ by:

$$K_i^- = S_i / S_{AAI} \tag{22}$$

$$K_i^+ = S_i / S_{AI} \tag{23}$$

where S_i is calculated by:

$$S_i = \sum_{j=1}^m c_{ij} \tag{24}$$

6. Determine the utility function $f(K_i)$ of the options by:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}} \tag{25}$$

where $f(K_i^-)$ is the utility function associated with the anti-ideal solution and $f(K_i^+)$ is the utility function correlated with the ideal solution. These functions can be found by:

$$f(K_i^-) = K_i^+ / (K_i^+ + K_i^-) \tag{26}$$

$$f(K_i^+) = K_i^- / (K_i^+ + K_i^-) \tag{27}$$

7. Rank the alternative's order by maximizing $f(K_i)$:

3) Method for Finding Criteria Weights

The Entropy approach has been used to figure out the weight criterion for the current study. The following actions are taken when employing this method [32]:

1. Identify the indicator's normalized values by:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^m x_{ij}^2} \quad (28)$$

2. Determine the Entropy for each indicator by:

$$me_j = - \sum_{i=1}^m [p_{ij} \times \ln(p_{ij})] - \left(1 - \sum_{i=1}^m p_{ij}\right) \times \ln\left(1 - \sum_{i=1}^m p_{ij}\right) \quad (29)$$

3. Find the weight of each indicator by:

$$w_j = \frac{1 - me_j}{\sum_{j=1}^m (1 - me_j)} \quad (30)$$

IV. SINGLE-OBJECTIVE OPTIMIZATION

In this work, the direct search strategy was employed to optimize a single objective. In addition, a Matlab computational application was used to assess two separate single-objective problems: increasing η_{gb} and reducing A_b . The program's discoveries are summarized in the following observations. The association between η_{gb} and u_1 is shown in Figure 3. It is evident that there is a particular value of u_1 at which η_{gb} attains its highest value, indicating an optimal value. Figure 4 depicts the correlation between u_1 and A_b . The optimal value of u_1 , as shown in Figure 3, corresponds to the minimum value of A_b . Moreover, Figure 5 illustrates the connection between the optimal values of u_1 and the u_{gb} . The new constraints for the variable u_1 were established according to the optimal values of u_1 , as displayed in Table II.

TABLE II. NEW RESTRICTIONS OF U1

u _{gb}	u ₁	
	Lower limit	Upper limit
5	1.59	1.98
10	2.47	2.68
15	3.00	3.44
20	3.77	4.11
25	4.27	4.75
30	4.67	5.35

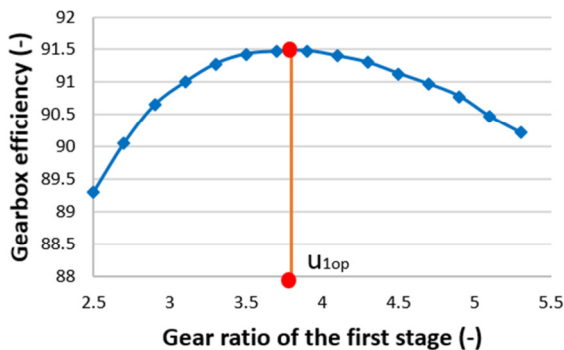


Fig. 3. Relation between u_1 and η_{gb} .

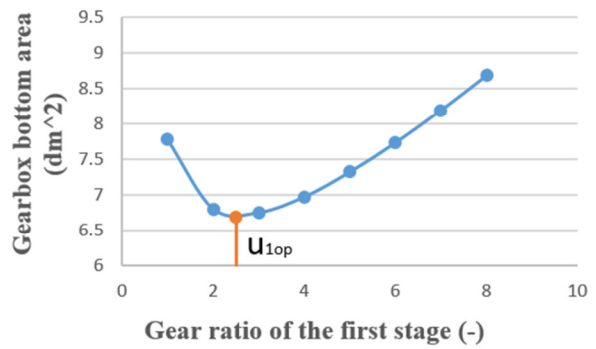


Fig. 4. Relation between u_1 and A_b .

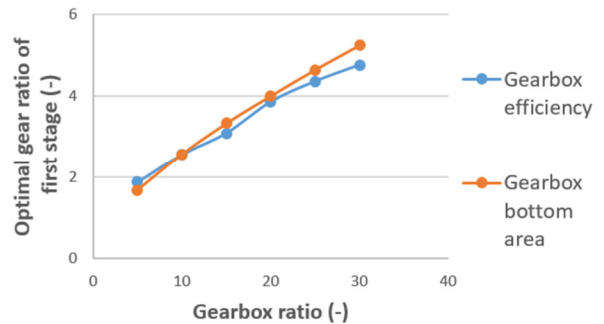


Fig. 5. Relation between u_{gb} and optimal values of u_1 .

V. MULTI-OBJECTIVE OPTIMIZATION

A computer program has been created to perform simulation experiments. The investigation examined the values of u_{gb} , which varied from 5 to 35 in increments of 5. Below, the resolutions for the issue with $u_{gb}=15$ are provided. The gearbox ratio previously specified was utilized for 125 initial testing cycles, as described in Methodology. The experiment submits its output data, specifically the efficiency of the gearbox bottom area, to MARCOS as input parameters for solving the MOOP. This procedure continues until the discrepancy between two levels of each variable becomes smaller than 0.02. Table III presents the primary design parameters and output responses for the fifth and final iteration of the MARCOS experiment, with a u_{gb} value of 35. The weight criterion was established using the Entropy approach, as explained above, in the subsequent manner. The values of p_{ij} are standardized using (28). The entropy value for each indicator m_{e_j} was calculated using (29). The value of the weight criterion, w_j , was determined by utilizing (30). The weights allocated to A_b and η_{gb} were calculated as 0.6200 and 0.3800, respectively. The methodology provides guidance on the optimal utilization of the MARCOS technique to address the MCDM challenge. To be more explicit, the following components are included. By applying (18), the ideal solution (AI) and the suboptimal solution (AAI) are calculated. To find the normalized values of u_{ij} , (19) is used for A_b and (20) for η_{gb} in the subsequent phases. The data were standardized using (21), which considered the weight of the c_{ij} . Moreover, the coefficients K_i^- and K_i^+ were obtained from (22) and (23). Equations (26) and (27) were used to compute the values of $f(K_i^-)$ and $f(K_i^+)$. The values of $f(K_i)$ were calculated using (25) in the end. The

results of the option ranking and parameter computation following the MARCOS approach for the final iteration of MARCOS analysis are portrayed in Table IV. As observed, option 26 is the optimal choice among all the available alternatives. The ideal values for the critical design elements are $u_1=3.17$, $X_{ba1}=0.25$, and $X_{ba2}=0.25$, as demonstrated in Table V.

TABLE III. MAIN DESIGN FACTORS AND OUTPUT RESULTS FOR $U_{GB}=15$

u_{gb}	u_1	X_{ba1}	X_{ba2}	A_b (dm ²)	η_{gb} (%)
1.00	3.15	0.25	0.25	6.76	93.06
2.00	3.15	0.25	0.29	6.78	93.27
3.00	3.15	0.25	0.33	6.81	93.29
4.00	3.15	0.25	0.36	6.85	93.31
5.00	3.15	0.25	0.40	6.89	93.37
6.00	3.15	0.29	0.25	6.92	92.14
-	-	-	-	-	-
25.00	3.15	0.40	0.40	7.40	88.81
26.00	3.17	0.25	0.25	6.75	93.05
27.00	3.17	0.25	0.29	6.77	93.27
-	-	-	-	-	-
50.00	3.17	0.40	0.40	7.40	88.80
51.00	3.19	0.25	0.25	6.75	93.04
52.00	3.19	0.25	0.29	6.77	93.26
-	-	-	-	-	-
100.00	3.21	0.40	0.40	7.40	88.77
101.00	3.23	0.25	0.25	6.75	93.02
102.00	3.23	0.25	0.29	6.77	93.25
-	-	-	-	-	-
123.00	3.23	0.40	0.33	7.37	88.80
124.00	3.23	0.40	0.36	7.38	88.76
125.00	3.23	0.40	0.40	7.40	88.75

TABLE IV. CALCULATED RESULTS AND RANKINGS OF OPTIONS BY MARCOS FOR $U_{GB}=15$

No.	K-	K+	f(K-)	f(K+)	f(Ki)	Rank
1	0.0100	0.0104	0.5102	0.4898	0.0068	5
2	0.0100	0.0104	0.5102	0.4898	0.0068	10
3	0.0099	0.0103	0.5102	0.4898	0.0068	15
4	0.0099	0.0103	0.5102	0.4898	0.0067	20
5	0.0099	0.0103	0.5102	0.4898	0.0067	24
6	0.0098	0.0102	0.5102	0.4898	0.0067	26
-	-	-	-	-	-	-
25	0.0093	0.0096	0.5102	0.4898	0.0063	121
26	0.0100	0.0104	0.5102	0.4898	0.0068	1
27	0.0100	0.0104	0.5102	0.4898	0.0068	6
-	-	-	-	-	-	-
50	0.0093	0.0096	0.5102	0.4898	0.0063	122
51	0.0100	0.0104	0.5102	0.4898	0.0068	2
52	0.0100	0.0104	0.5102	0.4898	0.0068	8
-	-	-	-	-	-	-
100	0.0093	0.0096	0.5102	0.4898	0.0063	123
101	0.0100	0.0104	0.5102	0.4898	0.0068	4
102	0.0100	0.0104	0.5102	0.4898	0.0068	9
-	-	-	-	-	-	-
123	0.0093	0.0097	0.5102	0.4898	0.0063	110
124	0.0093	0.0097	0.5102	0.4898	0.0063	115
125	0.0093	0.0096	0.5102	0.4898	0.0063	125

Figure 6 illustrates the correlation between the optimal values of u_1 and u_{gb} . The provided regression equation, with an R^2 coefficient of determination of 0.9895, can be utilized to obtain the ideal values of u_1 :

$$u_1 = 0.1563 \cdot u_{gb} + 1.1953 \quad (31)$$

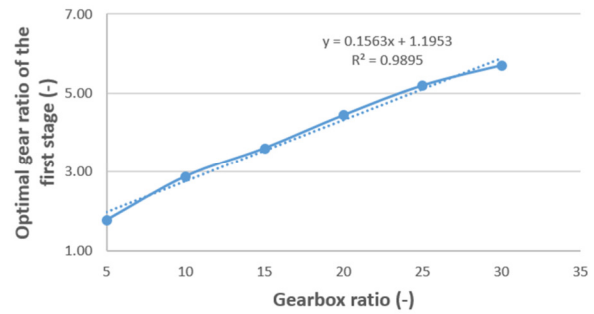


Fig. 6. Relation between optimal values of u_1 and u_{gb} .

TABLE V. OPTIMAL MAIN DESIGN PARAMETERS

No.	u_{gb}					
	5	10	15	20	25	30
u_1	1.78	2.88	3.60	4.44	5.19	5.70
X_{ba1}	0.25	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25

VI. CONCLUSIONS

This paper describes the findings of a study that utilized the MARCOS technique to address the MOOP of a two-stage helical gearbox with double gears at the first stage. The primary objective of the study was to determine the most effective fundamental design variables that improve gearbox efficiency while also minimizing the bottom area of the gearbox. To achieve this goal, three key design elements, specifically u_1 , X_{ba1} , and X_{ba2} , were selected. Furthermore, the MARCOS methodology was employed to deal with the issue of MCDM, while the Entropy method was utilized to compute the criteria weights. The following conclusions were drawn:

- Using the MARCOS approach to determine the optimal major design variables for a two-stage hydraulic gearbox with double gears at the first stage, the MOOP was successfully solved.
- Regarding the primary design parameters, two single objectives—the greatest gearbox efficiency and the smallest gearbox length—were evaluated.
- Equation (31) and Table V can be used to estimate the three best key design elements for the gearbox based on the study's findings. The former's high reliability and strong consistency with the experimental data are demonstrated by its R^2 value of 0.9895.

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