Homogeneous and Stratified Liquid-Liquid Flow Effect of a Viscosity Reducer:

I. Comparison in parallel plates for heavy crude

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Abstract—**Production of heavy crude oil in Mexico, and worldwide, is increasing which has led to the application of different methods to reduce viscosity or to enhance transport through stratified flow to continue using the existing infrastructures. In this context, injecting a viscosity improver that does not mix completely with the crude, establishes a liquid-liquid stratified flow. On the basis of a parallel plates model, comparing the increase of flow that occurs in the one-phase case which assumes a complete mixture between the crude and the viscosity improver against another stratified liquid-liquid (no mixing between the oil and compared improver); it was found that in both cases there is a flow increase for the same pressure drop with a maximum for the case in which the flow improver is between the plates and the crude.**

*Keywords- **stratified flow; velocity profile; heavy oil***

I. INTRODUCTION

Transporting heavy oil has gained significant interest due to high viscosity and operational problems such as high pumping costs, heavy fractions deposition and other phase changes [1-3]. The study of velocity profiles is very important because these effects are directly related to the pumping power and transport costs [4], as well as the design of monitoring pipeline [5]. Although frequently emulsions are used to improve pipeline transport [6, 7] and viscosity reducers are injected [8], few studies have examined the effects related to the shape of the injected fluid and dosing of products in the crude oil. Most of the time the research focuses on pump dimensioning [9, 10] limiting the capacity to use the same infrastructure for heavier fluids.

It is important to consider the mechanism that determines the transport of a compound injected to a fluid to reduce its viscosity. This can determine the effectiveness of its action and interaction effects that are set at the molecular level and that modify the rheological properties. [11-14]. In this regard, it has been found that the injection of a liquid into the inner periphery of a pipeline can substantially improve the flow of transported fluid [15, 16]. Various numerical methods [18, 19] and theoretical simulation models have been proposed to explain this effect [17]. Studies have also been carried out focusing on the involvement of surface tension exerted by the flow improvers and leading to an oil phase dispersion increase and the consequent influence in viscosity [20-22]. However, there are not papers reporting the effects of injection of other liquids on flow enhancement, as in the case of highly viscous fluids such as heavy and extra heavy crude, that generally maintain a laminar flow. A liquid-liquid stratified flow can be established if mechanical mixing between the oil and the chemical is not presented, as when oil extraction is performed by the reservoir natural pressure, or the flow improver is injected at a later point or away from the pump discharge.

The aim of this work is to model theoretically the behavior of a system consisting of crude oil and flow improver moving between two parallel plates considering three different conditions: i) complete mixing between the two fluids, so that we have a single-phase system of lower viscosity, ii) no mixing between the crude oil and the flow improver, establishing a stratified system with only one interface between them iii) establishing a stratified flow where one of the fluids is in the center and the other in the duct walls, creating two interphases.

II. Mathematical model

The mathematical model used to predict the velocity profiles and the total flow related with the volume fraction of the flow improver is based on the equations of continuity and change in cylindrical coordinates, in all cases considering i) Newtonian flow with constant density and viscosity, ii) isothermal system, iii) laminar flow regime and iv) steady state. Schemes corresponding to the cases to be considered are shown in Figures 1-3.

The distance x between the plates is to be considered much less than the width Y, so that one can assume that the momentum transport occurs preferentially in the x direction, while the liquid moves in the direction z. To quantify the effect of the flow improver on total flow is determined by the relationship:

$$q=\frac{Q_{12}}{Q_1} \text{ and } Q_{12}=Q_1+Q_2 \tag{1}$$

where the subscript 1 refers to the crude and the subscript 2 refers to the flow improver. Q_{12} represents the homogenous mix of the two substances. Q_1 represents the flow of oil; q allows to quantify the flow increase because of the flow improver injection, for the same pressure drop.

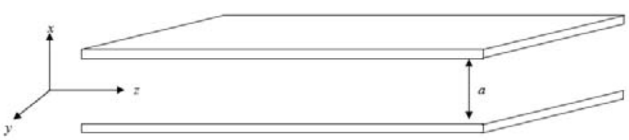

Fig. 1. Case A: single-phase flow consisting of the mixture of oil and the flow improver.

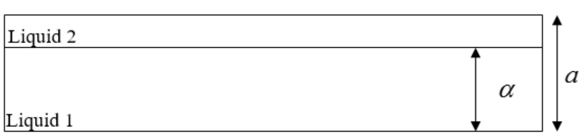

Fig. 2. Case B: no mixing between the oil crude and the flow improver, so that the system behaves as a two-phase stratified flow with a single interface between the fluids

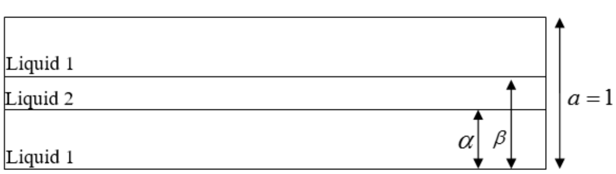

Fig. 3. Case C: one of the fluids is injected in the center, such that stratified flow is developed with two interfaces between the fluids

A. *Case A*

When a complete mix is produced between crude oil and the flow improver, either via a mixing tank or an injection prior to pumping, blend viscosity decreases. Many empirical equations exist to describe this effect. The ones proposed by Lederer:

$$\ln\mu_{12}=g_2\ln\frac{\mu_2}{\mu_1}+1 \tag{2}$$

$$g_2=\frac{\sigma y_2}{\sigma\left(1\text{-}y_2\right)+y_2} \tag{3}$$

$$\sigma=0.5\text{-}0.004T$$

and Kendall and Monroe:

$$\mu_{12}^{\frac{1}{3}} = x_1\mu_1^{\frac{1}{3}} + x_2\mu_2^{\frac{1}{3}} \tag{4}$$

respectively, are classical results, which can be used for mix of oil and improver flows [23, 24]. In this paper, (2) is used in order to describe the system behavior when a complete mix is produced because of experimental results showed this is one of the higher correlation mixing rule [25].

From the considerations set, we obtained that the velocity profile established in this system is described by the partial differential equation:

$$-\frac{\partial p}{\partial z}=\mu\frac{d^2v}{dx^2} \tag{5}$$

where p is pressure and v is the fluid velocity. Defining the dimensionless parameter γ:

$$\gamma=\frac{x}{\alpha} \tag{6}$$

and:

$$\Phi=\alpha^2\frac{\partial p}{\partial z} \tag{7}$$

equation (5) is written in the form:

$$-\Phi=\mu\frac{d^2v}{d\gamma^2} \tag{8}$$

with the boundary conditions:

$$\begin{aligned} v(0)&=0\\ v(1)&=0 \end{aligned} \tag{9}$$

Differential equation (8) has an analytical solution, and considering (9), velocity profile is obtained:

$$v=\frac{1}{2}\frac{\Phi}{\mu}\left(\gamma\text{-}\gamma^2\right) \tag{10}$$

From the velocity profile, volumetric flow is obtained for the single-phase case $Q=\int_0^Y\int_0^1\left(\frac{1}{2}\frac{\Phi}{\mu}\left(\gamma\text{-}\gamma^2\right)\right)d\gamma dy$ and therefore:

$$Q=\frac{1}{12}\frac{\Phi}{\mu}Y \qquad (11)$$

Taking into account equations (1), (2) y (11) yields:

$$q=\frac{\mu_1}{\exp\left(\frac{(0.5-0.004T)y_2}{(1-y_2)(1-(0.5-0.004T))+y_2}\ln\frac{\mu_2}{\mu_1}+1\right)} \qquad (12)$$

B. Case B

The system of differential equations describing spatial velocity profile of a stratified flow with a single interface between the fluids is given by:

$$-\Phi=\mu_1\frac{d^2v_1}{d\gamma^2} \text{ for } 0<\gamma<\alpha \qquad (13)$$

$$-\Phi=\mu_2\frac{d^2v_2}{d\gamma^2} \text{ for } \alpha<\gamma<1 \qquad (14)$$

Near of the wallsm it is considered that velocity is zero, therefore:

$$\begin{aligned} v_1(0)&=0 \\ v_2(1)&=0 \end{aligned} \qquad (15.a)$$

The others boundary conditions are stablished taken into account the continuity of velocity and shear stress on interface α, in such way that:

$$\begin{aligned} v_1(\alpha)&=v_2(\alpha) \\ \tau_1(\alpha)&=\tau_2(\alpha) \end{aligned} \qquad (15.b)$$

where τ is fluid shear tensor, defined:

$$\tau=-\mu\frac{dv}{dx} \qquad (16)$$

The differential equations (13) and (14) have, in this case, the solution:

$$v_1(\gamma)=C_{22}+C_{16}\gamma-\tfrac{1}{2}\Phi\frac{\gamma^2}{\mu_1} \quad \text{for } 0<\gamma<\alpha \qquad (17)$$

$$v_2(\gamma)=C_{26}+C_{15}\gamma-\tfrac{1}{2}\Phi\frac{\gamma^2}{\mu_2} \quad \text{for } \alpha<\gamma<1 \qquad (18)$$

where:

$$\begin{aligned} C_{22}&=0 \\ C_{15}&=\frac{\Phi}{2\mu_2}\frac{\left(\mu_1-\alpha^2(\mu_1-\mu_2)\right)}{\mu_1-\alpha(\mu_1-\mu_2)} \\ C_{16}&=\frac{\Phi}{2\mu_1}\frac{\left(\mu_1-\alpha^2(\mu_1-\mu_2)\right)}{\mu_1-\alpha(\mu_1-\mu_2)} \\ C_{26}&=\frac{\Phi}{2\mu_2}\frac{(\mu_1-\mu_2)\alpha(\alpha-1)}{\mu_1-\alpha(\mu_1-\mu_2)} \end{aligned} \qquad (19)$$

To find the α value that corresponds to the interphase position, the flow values are determined as a function of α:

$$Q_1=\frac{1}{6}Y\Phi\frac{\alpha^2}{\mu_1}\left(\frac{3}{2}\frac{\left(\mu_1-\alpha^2(\mu_1-\mu_2)\right)}{\mu_1-\alpha(\mu_1-\mu_2)}-\alpha\right) \qquad (20)$$

$$Q_2=\frac{1}{6}\frac{Y\Phi}{\mu_2}(1-\alpha)\left(\frac{3}{2}(1+\alpha)\frac{\left(\mu_1-\alpha^2(\mu_1-\mu_2)\right)}{\left(\mu_1-\alpha(\mu_1-\mu_2)\right)}+3\frac{(\mu_1-\mu_2)\alpha(\alpha-1)}{\left(\mu_1-\alpha(\mu_1-\mu_2)\right)}-\left(1+\alpha+\alpha^2\right)\right) \qquad (21)$$

and from defining the volume fraction of the injected flow improver $y_2 = Q_2/Q_{12}$ relative to the total two-phase flow, α is cleared from the equation:

$$\frac{y_2}{1-y_2}=\mu_1(\alpha-1)^2\frac{\left(\mu_1-2\alpha\mu_1+4\alpha\mu_2+\alpha^2\mu_1-\alpha^2\mu_2\right)}{\alpha^4\mu_2^2-\mu_1\alpha^4\mu_2-2\mu_1\alpha^3\mu_2+3\mu_1\alpha^2\mu_2} \qquad (22)$$

After determining the value of the position of the interface for a fraction of the injected liquid ratio, relationship between the total biphasic flow and single phase flow of crude oil is determined:

$$\begin{aligned} q=&2\mu\frac{\alpha^2}{\mu_1}\left(\frac{3}{2}\frac{\left(\mu_1-\alpha^2(\mu_1-\mu_2)\right)}{\mu_1-\alpha(\mu_1-\mu_2)}-\alpha\right) \\ &+2\mu\frac{(1-\alpha)}{\mu_2}\left(\frac{3}{2}(1+\alpha)\frac{\left(\mu_1-\alpha^2(\mu_1-\mu_2)\right)}{\left(\mu_1-\alpha(\mu_1-\mu_2)\right)}+3\frac{(\mu_1-\mu_2)\alpha(\alpha-1)}{\left(\mu_1-\alpha(\mu_1-\mu_2)\right)}-\left(1+\alpha+\alpha^2\right)\right) \end{aligned} \qquad (23)$$

C. Case C

Following the same methodology as in Case B, but considering two interfaces among fluids, the velocity profile that is set for two-phase flow, where one fluid is in contact with both plates, (Figure 3) is obtained:

$$v_A(\gamma)=C_{22}+C_{16}\gamma-\tfrac{1}{2}\Phi\frac{\gamma^2}{\mu_A} \text{ for } 0<\gamma<\alpha \qquad (24)$$

$$v_A(\gamma)=C_{22}+C_{16}\gamma-\tfrac{1}{2}\Phi\frac{\gamma^2}{\mu_A} \text{ for } \beta<\gamma<1 \qquad (25)$$

$$v_B(\gamma)=C_{26}+C_{15}\gamma-\tfrac{1}{2}\Phi\frac{\gamma^2}{\mu_B} \text{ for } \alpha<\gamma<\beta \qquad (26)$$

where:

$$\begin{aligned} C_{22}&=0 \\ C_{15}&=\frac{\Phi}{2}\frac{\left(\mu_B+(\mu_A-\mu_B)(\alpha+\beta)\right)}{\mu_A\mu_B} \\ C_{16}&=\frac{1}{2}\frac{\Phi}{\mu_A} \\ C_{26}&=-\frac{\Phi}{2}\frac{\alpha\beta(\mu_A-\mu_B)}{\mu_A\mu_B} \end{aligned} \qquad (27)$$

In this case there may be two possibilities. First is that the flow improver moves inside (in the centre) and the crude is in contact with both plates. In which case we obtain:

$$\frac{y_2}{1-y_2}=\frac{\frac{1}{\mu_2}(\beta-\alpha)\left(3\mu_2(\alpha+\beta)-2\alpha\beta\mu_1+(\mu_1-3\mu_2)\left(\alpha^2+\beta^2\right)\right)}{\left((\beta-1)^2(2\beta+1)-\alpha^2(2\alpha-3)\right)} \tag{28}$$

$$q=\left((\beta-1)^2(2\beta+1)-\alpha^2(2\alpha-3)\right)+\frac{1}{\mu_2}(\beta-\alpha)\left(3\mu_2(\alpha+\beta)-2\alpha\beta\mu_1+(\mu_1-3\mu_2)\left(\alpha^2+\beta^2\right)\right) \tag{29}$$

second case is if oil flows through the inside and the flow improver is in contact with both plates:

$$\frac{y_2}{1-y_2}=\mu_1\frac{\left((\beta-1)^2(2\beta+1)-\alpha^2(2\alpha-3)\right)}{(\beta-\alpha)\left(3\mu_1(\alpha+\beta)-\left(\beta^2+\alpha^2\right)(3\mu_1-\mu_2)-2\alpha\beta\mu_2\right)} \tag{30}$$

$$q=\frac{\mu_1}{\mu_2}\frac{1}{\mu_1}(\beta-\alpha)\left(3\mu_1(\alpha+\beta)-\left(\beta^2+\alpha^2\right)(3\mu_1-\mu_2)-2\alpha\beta\mu_2\right)+\frac{\mu_1}{\mu_2}\left((\beta-1)^2(2\beta+1)-\alpha^2(2\alpha-3)\right) \tag{31}$$

III. RESULTS AND DISCUSSION

The results obtained with the model were based on a crude with viscosity μ_1 = 35 Pa.s and a flow improver with viscosity μ_2 = 0.007 Pa.s. To make the study, the velocity profile was analyzed considering Φ=1 for a flow improver volume fraction of 0.03. Results predicted by the model are shown in Figures 4.a, 4.b, 4.c y 4.d. It is appreciated that when there is mixing between the two liquids (Case A) the speed of transport is increased significantly compared with that achieved in the absence of flow improver. If there is no mixing, we see a well-defined interface between the two flows, where the speed reached in the liquid layer corresponding to the flow improver is significantly higher than the phase of crude oil (Case B), but is lower than that reached with complete mixing. When the flow improver is injected in the center (Case C.1), even when the speed is increased, this increase is less pronounced than the corresponding liquid-liquid stratified flow with a single interface, while a further increase occurs when the flow improver is in contact with both plates.

As shown in Figure 5, the model predicts different behavior between the volume fraction and flow improvement is achieved for the same pressure drop depending on the form of injection. Thus very significant increases are achieved when the enhancer is in contact with the walls. This apparently is because it has significantly lower viscosity than crude, so that the shear stress on the wall of the plates decreases, which is manifested in increased flow for the same pressure drop. In correspondence with this, the smallest increase in flow is achieved for the case where the crude oil is in contact with both plates. However, this two interfaces model between fluids has two limiting factors: the first is that there is no explicit criteria established by the asymmetry of the same (distance from each of the interfaces), and it only has a flow ratio to determine the height of the interface, so that the asymmetry must be defined to make other predictions.

The second limitation is that it is necessary to define under what conditions interfaces can be established because in horizontal ducts, the density difference between both flows favors the formation of a stratification. When Cases A and B are compared, it can be seen that in both cases there is an increased flow. Although this increase is more marked when mixing, it is still advisable to inject the flow improver before pumping. The results in terms of flow profiles are consistent with those obtained in [16, 26-27]. The maximum possible flow is the flow improver and oil crude substituted in (1), such that q=5000; it can be seen that the better effect is around 95%.

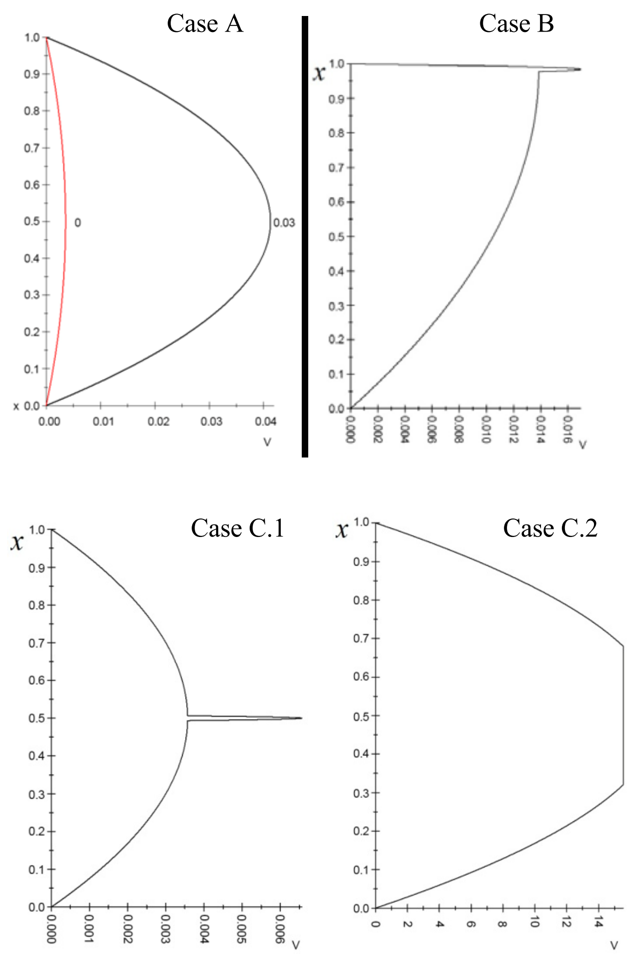

Fig. 4. Velocity profiles predicted by the model. Case A: complete mix with the fraction of flow improver as parameter. Case B: stratified flow with a single interface. Case C: stratified flow with two symmetrical interfaces. 1 the oil crude is located in contact with both plates; 2. The flow improver is in contact with both plates

IV. CONCLUSION

The injection of a flow improver always causes an increase of fluid flow for the same pressure drop, where the increased magnitude order, from lowest to highest, is: stratified flow with two interphases, which is crude oil that is in contact with the walls; stratified flow with an interface between the two flows; complete mixing between the two fluids with the establishment of a single phase flow of lower viscosity and two interfaces stratified flow where the flow improver is in contact with the plates. These results indicate that the way in which the injection of the flow improver and the existence of mechanical mixing with oil crude occur influences the overall enhancement effect. The model has as limitation that it cannot explicitly set the conditions under which the formation of two interfaces occurs, so that oil flows through the center, which may reasonably be expected if similar fluids densities are obtained. It is necessary to develop technologies that allow the injection to be made in the wall with subsequent increase in improved operating systems extraction and transportation of crude oil.

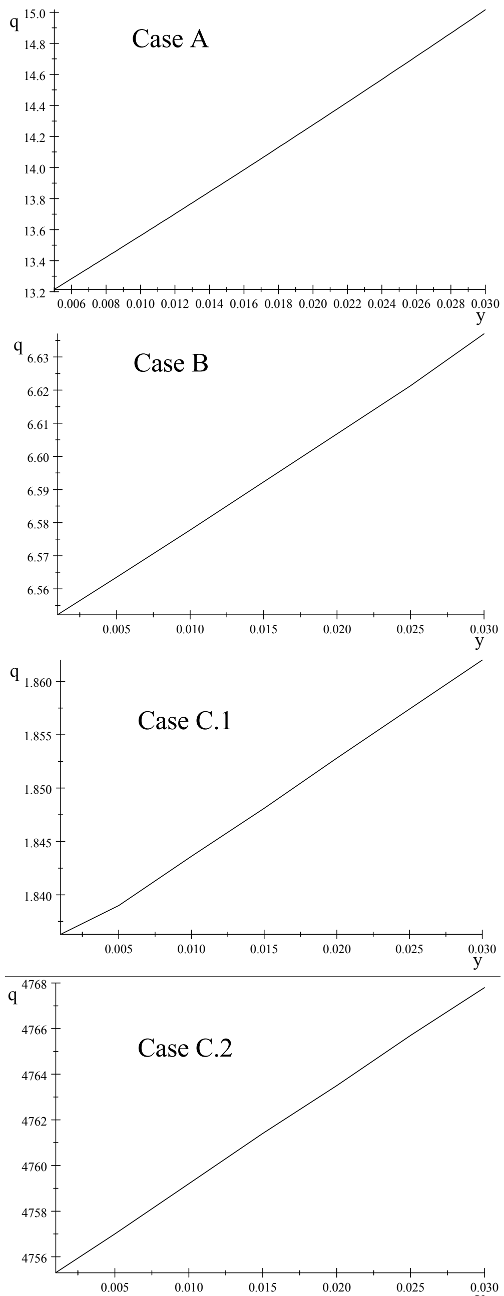

Fig. 5. Behavior of flow increase with respect to the volume fraction of improver

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