Monitoring of a Low-Order Even Radial Vibrational Circumferential Mode in a Round Hollow Cylinder

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Received: 30 July 2024 | Revised: 3 September 2024 | Accepted: 8 September 2024

 DOI: https://doi.org/10.48084/etasr.8515

ABSTRACT

This paper presents a nondestructive testing method for assessing the structural integrity of cylindrical elements by monitoring a range of radial vibrational modes, with a specific emphasis on the so-called ovalling mode. The method involves exciting the cylinder with a single vibration source in the radial direction and measuring the response using two vibration sensors positioned diametrically on the cylinder's surface. The ovalling mode was extracted from the frequency response by adding the in-phase signals recorded by the sensors. Experiments conducted on a PVC pipe showed good agreement between the measured resonance frequency of the ovalling mode and its predicted value, calculated using the theory for thin cylindrical shells and Finite Element Method simulations. This research is an investigation into the potential and reliability of this nondestructive technique for detecting corrosion and strength-weakening defects in concrete building elements, steel pillars, and columns. The extent of the strength reduction can be determined by analyzing the change in the resonance frequency of the ovalling mode.

Keywords-non-destructive testing; vibrations; hollow cylinder; circumferential mode; strength monitoring

I. INTRODUCTION

Cylindrical construction elements, generally termed columns or pillars, are either solid or hollow, made of concrete, steel, or any other solid material, and have a prevailing appearance in buildings, piers, and bridges. They are used as supporting elements for other building parts such as slabs and beams. The cross-section of these supporting columns is typically rectangular when they are concealed in buildings, but when they are made visible and for a more pleasant aesthetic appearance, their shape may be that of cylinders with a circular section. The maintenance and steady control of the strength characteristics of these building elements are important because of their direct implications regarding the integrity of the entire structure.

A wide range of non-destructive methods and techniques exist in civil and structural engineering, which are used to monitor the integrity of structures, as well as the strength condition of the materials that these building elements are made of. Concrete is the most widely used construction material, and many of these techniques have been specifically designed for concrete. Non-destructive methods have also been originally developed for other building materials, such as steel and wood, and later adapted to concrete, or vice versa. Moreover, certain specific methods have been developed by considering the shape of the structural element to be tested rather than only the material of its manufacture. The purpose of a nondestructive technique is to implement a method that enables the characterization of the state of soundness of a structure or its material without deteriorating them during the manufacturing phase and their use, or that can be deployed as part of their preservation and maintenance processes.

Damage to the structure and the aging of its building materials are known to reduce the former's stiffness. This fact is equally valid for the case of the integrality of the structure, as is the case of its separate constituent elements. A reduction in the stiffness of the material causes the structure to shift the natural frequencies of its vibrational modes towards lower values. One of the earliest reported attempts to establish a correlation between the extent of damage to a structure and the shift in its natural frequencies was [1]. The authors studied the effect of debonding in an element of a composite material on the vibration frequency. The technique was implemented on quartz particle-filled resin specimens and involved measuring the change in the dynamic modulus and the damping of the specimens as a result of the damage imparted to them.

Since then, vibration-based techniques have been utilized in various contexts for monitoring building structures. In the field of structural health monitoring, various methods have been recently proposed and reviewed, with their applications having been emphasized on composite structures [2] or slender structures [3]. Vibration-based damage identification techniques, which are easy to implement and relatively inexpensive, have also been employed to monitor the structural health of bridges [4]. Hence, assuming that damage is represented by a change in stiffness, numerous vibrational analysis methods have been proposed for detecting and locating defects in structures for structural health monitoring [5, 6].

Considering the analysis of specific elements isolated from complex structures, environmental factors have been found to have less influence on the shape of vibrational modes than on the proper frequency of vibration [7]. In certain cases, the mode shape can contribute to determining the damage location [8]. If damage results in a change in the mode shape (which, for instance, can be due to extensive damage or damage occurring in the vicinity of the site where the test on the element is conducted), a technique based on artificial neural networks has been proposed for locating the damage within the element. This method relies on comparing the mode shape as a result of the damage to that of the sound element, while several sensors may be required to increase the accuracy of the analysis [9]. More recently a review has been carried out on the potential advantages that may be gained from the use of chirplet transforms in the processing and analysis of ultrasonic signals for structural health monitoring [10].

II. VIBRATION ANALYSIS

The vibration analysis refers to the procedure of monitoring the vibration signatures recorded from a structure or structural element and analyzing that information. Structural vibration testing and analysis is vital for the advance of many industries, including building construction, aerospace, vehicle manufacturing, wood, power generation, defense, and transportation, to name only the most important fields. Common applications of vibration analysis include determining the health condition of structures and identifying and suppressing unwanted vibrations to improve the quality of a product or prevent its deterioration.

Structural vibrations can be measured using transducers that convert vibration motion into electrical signals. Time-domain analysis refers to the analysis of the signal recorded by a sensor as a function of time. In the past, this analysis was performed using an oscilloscope, but currently, a computer-like dynamic signal analyzer is deployed to acquire the signal and present it on a monitor. The analysis of the electrical signal enables the comprehension of the vibration nature to identify faults in structures or in the operation of machines. Signal analysis is generally divided into time and frequency domains, and each domain provides a different visualization and understanding of the nature of vibration. The plot of the vibration of the amplitude levels provides information on the characteristic parameters describing the behavior of the structure. This most often involves measuring the peak levels of vibration, determining its period, and evaluating its decay rate.

A Fourier Transform (FT) of the time signal converts it into the frequency domain, and the analysis of the obtained transfer function provides valuable information regarding structural vibrations, such as the frequency and damping of the proper modes of vibration. Existing digital techniques process FT and other complex theoretical operations quickly and efficiently. The study of FT is conducted on a real-time basis by dynamic signal analyzers that are incorporated in the most modern acoustical and vibrational tests and measurement systems.

III. MATERIALS AND METHODS

A. Materials

For the present study, a solid cylinder composed of hard Polyvinyl Chloride (PVC) was used. The cylinder had a height of 1.00 m, an inner diameter of 79.0 mm, and a thickness of 5.0 mm. From the weight and geometrical measurements of the cylinder, the density of PVC was estimated to be ρ = 1320 kg/m^3. The measurement of the propagation of the stress waves along the axis of the cylinder resulted in the calculation of the Modulus of Elasticity (MoE) E = 42.0 MPa. The measurement was performed on the Impulse Response (IR) (more details in Section IIIB3) from which the Transfer Function (TF) was processed. The first peak in the TF occurred at frequency f for which the length l of the cylinder corresponded to half a wavelength, that is, with free–free boundary conditions for the propagation of longitudinal waves. The speed of the wave propagation from which the MoE was evaluated according to the equation $E = 4\rho f^2 l^2$ was then calculated.

B. Method

1) Equipment

A typical mechanical vibration measurement equipment was utilized for the experiments. The source of the mechanical vibrations was a force generator consisting of an electrodynamic mini-shaker that induced vibrations to the tested object. This shaker was held on a tripod with its vibrating cone connected to a steel rod (a stinger) that was attached to the middle of the cylinder and oriented towards the radial direction. To sense the response of the cylinder to the vibration excitation, two identical vibration sensors were attached to its surface. These consisted of similar uniaxial piezoelectric accelerometers with a sensitivity of 10 pC/ms^{-2} and a working frequency range in the band between 0.1–4800 Hz. The accelerometers were attached at two diametrically opposed positions on the cylinder, and their responses were added to enhance the vibration amplitude of the ovalling mode because of the in-phase responses of this specific mode at these positions.

2) Measurement Method

The IR of the tested object was the quantity of interest from the experimental measurements. The TF, which is the frequency response, can be then obtained from the IR through a simple (digital) FT. The IR is defined as the signal obtained from the system under test as a response to an excitation signal that is, ideally, of infinite amplitude and zero-time duration, a so-called Dirac pulse. Thus, the measurement method deployed in the present context was borrowed from room acoustics, where the IR was acquired by submitting the tested object to an excitation signal with the characteristics of "random" noise. The latter, if being of appropriate frequency content and energy level, allows for the IR to be obtained merely by executing a series of simple digital operations on the input signal and the system's response to it.

In more details, if a system (in this study's case the test object) is causal, i.e. a system with no response expected from it prior to applying to it an input signal, and if it is characterized by an impulse response $g(t)$ (the response of the system when an ideal Dirac pulse signal $\delta(t)$ is applied to it), then applying an arbitrary excitation signal $s(t)$ to it will result in a response $r(t)$ expressed as:

$$r(t) = \int_{-\infty}^{+\infty} s(\tau) g(t-\tau) d\tau \quad (1)$$

An operation known as the convolution between two functions s and g is commonly expressed as $r(t) = s(t) \cdot g(t)$. In the frequency domain, that is, when operating an FT on (1), this results in the product of the spectra $S(f)$ and $G(f)$ of $s(t)$ and $g(t)$, respectively:

$$R(f) = S(f) \cdot G(f) \quad (2)$$

The TF of the test object $G(f)$ is equal to $R(f)/S(f)$, provided that $S(f)$ is not equal to zero for any frequency within the frequency range of the input signal. The measurement procedure is schematically shown in Figure 1.

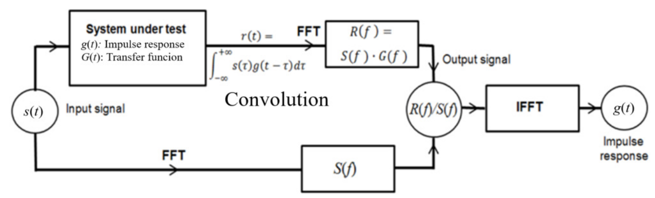

Fig. 1. Illustration of the testing method for acquiring the IR of a causal system using a Fast Fourier Transform (FFT) and Inverse FFT (IFFT).

This can be achieved through the proper choice of the excitation signal, and for this purpose, white noise with a flat power spectrum is of particular interest [11]. Measurements must be also performed in large numbers to arrive at a reliable result for the TF of the system or its IR, which is the IFFT of the TF. The requirement of averaging over a large number, ideally an infinite number, of measurements may be relaxed using random signals as test signals [12]. As it may be technically difficult to generate perfectly random signals, a particular type of pseudo-random signal based on the Maximum Length Sequence (MLS) concept is generated. These signals contain periodic series of pulses of well-defined amplitude and pattern, as well as their time width and spacing. For systems containing too high background noise, MLS signals do not guarantee the required signal-to-noise ratio to extract the representative IR of the investigated system, and the recourse is instead converted to sine-sweep signals, which are signals swept with adequate speed from the lowest frequency to the highest frequency of the continuous spectrum of interest [13]. Thus, the experimenter could control the frequency bandwidth of the excitation signal by limiting it to cover the range containing the frequencies of the proper modes of vibration of the system under study. The measurement procedure complies with the ISO-3382 standard for room acoustics, which is used to determine the reverberation time from a reverse time-integration of the IR (the latter is also used for calculating several other room acoustical parameters for the psychoacoustical quality assessment of closed spaces, such as auditoria and concert halls).

In this study, sine sweep signals were utilized to enhance the form of the IR. In the actual measurement system, the frequency sweep was set in the range of 20–20000 Hz, with a time length of 5.000 ms for an expected IR of 1.000 ms. The sampling rate was set to 44100 Hz prior to the FFT operation, which for an FFT with a size of 8192, leads to a frequency resolution of 5.4 Hz for the frequency spectrum. Equivalently this results in a time resolution of 0.12 ms for the IR. The level of the input signal to the vibration source was controlled so as not to cause damage to the electrodynamic shaker. The measurement system also executed power spectrum compensation, specifically over the low-frequency range of the sine-sweep signal, to improve the signal-to-noise ratio in the case of existing background noise. The measurement system sets this compensation automatically after the first pilot measurement without altering the measurement setup and conditions afterwards.

IV. RESULTS

Figure 2 illustrates the setup used for the measurements conducted on a PVC tube to identify the vibrational ovalling mode. Measurements were carried out on five pipes with similar geometrical characteristics: a length of 100 cm, an outer diameter of 168 mm, and a thickness of 5.0 mm. The frequencies of the ovalling mode were 786, 798, 804, 804, and 810 Hz.

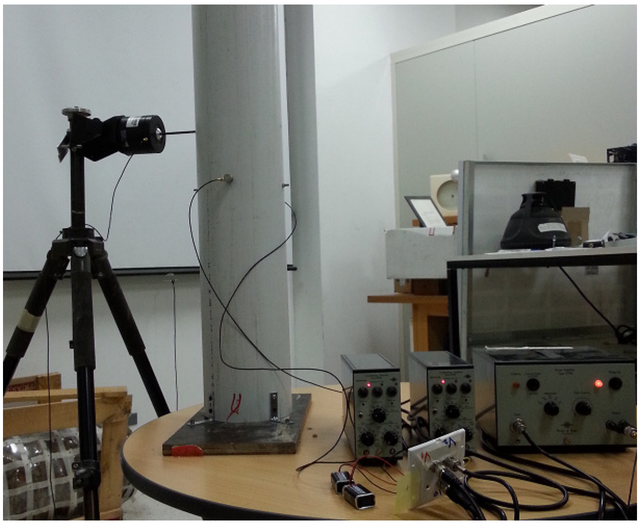

Fig. 2. The experimental setup used for the measurements on the cylinder.

Figure 3 depicts the attachment of the vibration sensors on the surface of the cylinder at two diametrically opposed positions, and the addition of the two recorded signals to enhance the presence of the ovalling mode from the amplitude of the frequency response of the tube to a broadband excitation signal. The bending modes could be traced in the TF plot by connecting a single vibration sensor to record the vibrations of the cylinder surface. Figure 4 portrays a schematic representation of the wiring of the electrical components utilized to obtain the electronic adder. The cost of the ensemble of components entering the adder circuit amounts to less than 2.0 USD.

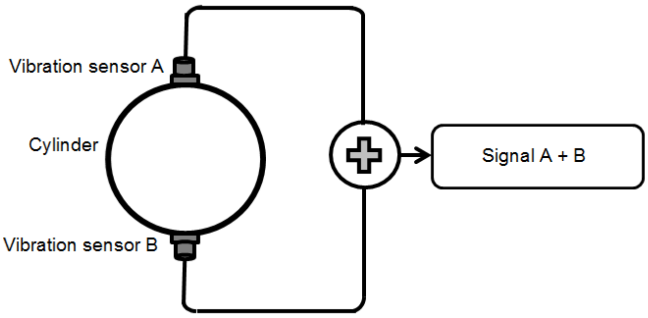

Fig. 3. Illustration of the procedure for enhancing the presence of the ovalling mode from the vibrational frequency response of a hollow circular cylinder.

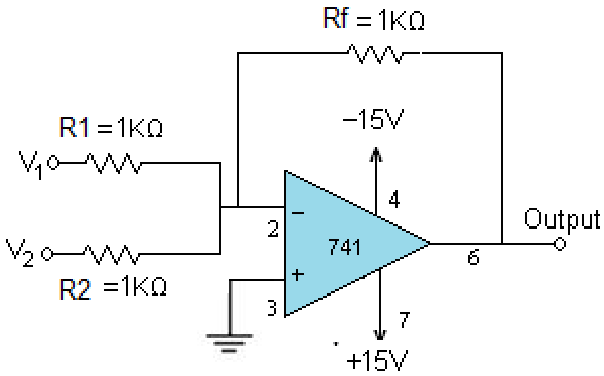

Fig. 4. Electronic wiring schematics of the electronic adder used for summing the signals from the accelerometers mounted diametrically opposite the surface of the cylinder. The electronic components consisted of a simple operational amplifier of 741 μA and resistors of common values [14].

After processing IR through a digital FTT, a graph representing the amplitude of the frequency spectrum was obtained, as showcased in Figure 5. The ovalling mode exhibited a clear peak at 794 Hz in the graph of the amplitude of the frequency spectrum. A cylinder with a smaller diameter or stiffer material is expected to display extensional cross-sectional modes with higher frequencies. Moreover, the slenderer the cylinder, the lesser is the expected effect of the vibration reflections at the cylinder ends on the cross-sectional extensional modes.

The curves in Figure 5 are presented in both the complete (blue) and smoothed (red) forms. The low level of the frequency response amplitude in the low frequency range is attributed to the limited performance of the excitation shaker, which reaches its maximum output at around 60 Hz and maintains thereafter an almost flat response of up to around 5.0 kHz. At the other limit of high frequencies, the lower level of the frequency response level is mainly due to the accelerometer's cut-off frequency being slightly above 4.0 kHz, and to a lesser degree to the lower output of the mini-shaker being above 5.0 kHz. The plots show clear peaks, the most pronounced of which are attributed to the resonances of the extensional modes. In the case of the hollow PVC pipe, the frequency of the ovalling mode was 798 Hz, whereas the peak at the lower frequency (156 Hz) could be attributed to either the first flexural mode or to the resonance frequency of the mass-spring system constituted by the mass of the tube and the spring system of the moving part of the electrodynamic shaker.

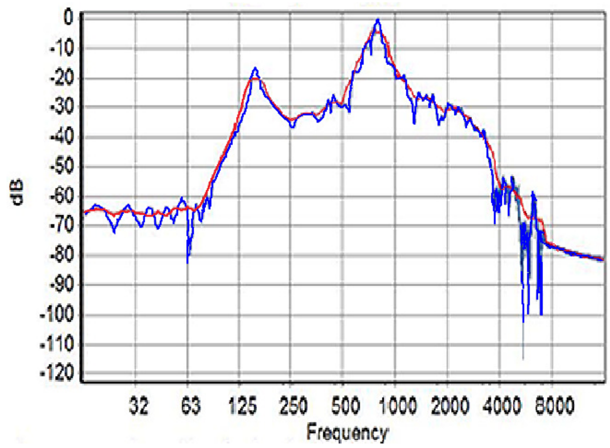

Fig. 5. The transfer function, that is, the FTT of the IR, was applied to the sum of the signals recorded from two diametrically opposed vibration sensors attached to the surface of the PVC pipe. Measurement system: ODEON®.

V. DISCUSSION

Owing to their importance in construction, aeronautics, and submarine technologies, circular cylindrical shells have long been a subject of interest for theorists and experimentalists. Thus, the problem of vibrations in shells has witnessed continuous progress in the development of analytical models and numerical methods for predicting and simulating their behavior. A compilation of the various theories and approaches proposed for this effect has resulted in a substantial review of more than 20 years of extensive research worldwide and documented in almost 1.000 publications that have been summarized and presented in [15]. This monograph continues to serve as an essential reference for researchers in the field of shell vibration. More than half of the references included in that review dealt with circular cylindrical shells. From [15] and as reported earlier in [16] and [17], considering the dimensionless thickness factor k for the cylindrical shell:

$$k = \frac{h^2}{12R^2} \quad (3)$$

where h is the thickness of the shell and R is its average radius. The nondimensional natural frequency Ω is evaluated according to:

$$\Omega^2 = \frac{1}{2}\left[\{1+n^2+k(1-n^2)^2\} \pm \sqrt{\{1+n^2+k(1-n^2)^2\}^2 - 4kn^2(1-n^2)^2}\right] \quad (4)$$

where n is an integer that takes values of 0, 1, 2, …, and represents the order of the different modes of vibration. The upper sign in ± corresponds to external pressure, whereas the lower sign corresponds to internal pressure.

Angular frequency ω is expressed as:

$$\omega = \sqrt{\frac{E\Omega^2}{\rho R^2(1-\nu^2)}} \quad (5)$$

From this, the frequency of the proper mode of vibration with order n is expressed as:

$$f = \frac{\omega}{2\pi} \quad (6)$$

where ρ denotes the density of the material, E is its MoE, and ν is Poisson's ratio (considered here for simplicity, $\nu = 0.3$).

Calculations using (3)-(6) as well as the geometrical and physical data of the experimental hollow PVC cylinder in the present study predict an eigenfrequency value for the $n = 2$ circumference mode at 821 Hz, which is considered acceptable for an approximation given the fact that the theory applies to considerably thinner shells (in [15], tables for comparing values from various theories are provided for the value of the parameter R/h up to 500, whereas in the present study, this ratio is only slightly less than 8).

To further support the theoretical analysis and experimental validation, this study is concluded by performing a numerical simulation. The behavior of the circular cylindrical shell simulated locally on the plane normal to its axis and its vibration as a whole body are manifested in Figure 6.

Figure 6 shows the results of implementing the COMSOL–MATLAB Finite Element Method (FEM) software to simulate the response of a cylinder with the geometrical and physical characteristics of the cylinder used in the experimental part of the present study.

The shape of the ovalling mode in the cylinder's cross-sectional plane is more of an oval slightly crammed at the antipodes, giving it more resemblance to a broad in-the-middle 8-figure. Moreover, the periodic vibration pattern of the cylindrical shell along its axis warrants further investigation.

VI. CONCLUSIONS

Various techniques have been employed in the field of non-destructive testing of building materials to assess the strength of the building elements and to control structure integrity. Many of these techniques rely on measuring the penetration or reflection of electromagnetic or acoustic signals, whereas others measure the response of the material to impact or forced vibrations without considering the geometrical characteristics of the tested object. Furthermore, the implementation of these techniques in the field requires carrying heavy and sensitive equipment around most of the time as well as they necessitate trained personnel for their operation. In the present paper a new method is presented for investigating the stiffness of a hollow circular cylinder of solid material. Additionally, the uniform cross-sectional characteristics of the cylinder in terms of the diameter and wall thickness were also considered.

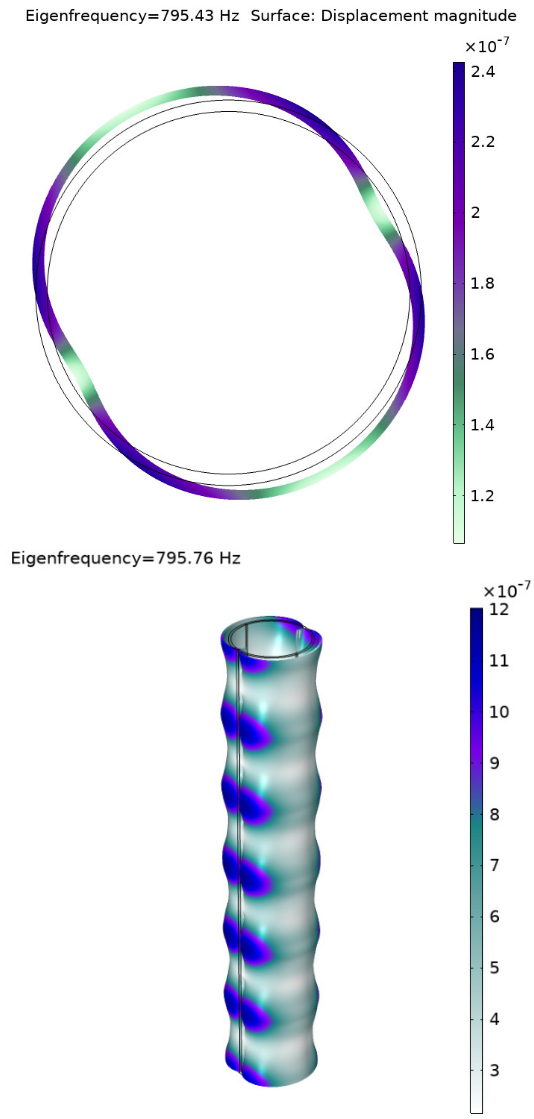

Fig. 6. Numerical simulation using the COMSOL–MATLAB code for the $n = 2$ circumferential mode in a finite-length hollow cylinder vibrating under free–free boundary conditions. Up: 2-dim case, cross-section displacement, with *fov* = 795 Hz, and Down: 3-dim case, overall cylinder behavior, with *fov* = 796 Hz.

The proposed method utilizes affordable equipment and free software, and is relatively simple because it is based on vibrations. This technique uses two similar vibration sensors to track the circumferential vibrational mode of order two on the cylinder, that is, the ovalling mode. The two sensors were attached at diametrically opposite positions on the cylinder, and the response of the ovalling mode was further enhanced by adding the responses recorded by the two sensors. Experiments were conducted on a PVC pipe, and good agreement was obtained between the values of the measured resonance frequency of the mode and its prediction, as calculated from the theory for thin cylindrical shells or from simulations following the Finite Element Method (FEM). This measurement method can find applications in the nondestructive testing of circular cylindrical hollow building elements, such as construction columns or bridge pillars, as well as pipes for oil and gas transportation. From the FEM simulations carried out in this study, it was found that the length of the cylinder has little influence on the action of the ovalling mode, provided the cylinder's length is several times its diameter. As the proper frequency of the ovalling mode is related to the stiffness of the material the cylinder is made of, a decrease in the Modulus of Elasticity (MoE) value, of the material due to corrosion or aging results in a lowering of the proper frequency of the ovalling mode.

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