A Greedy Simulated Annealing-based Multiobjective Algorithm for the Minimum Weight Minimum Connected Dominating Set Problem

Hayet Dahmri
Mechatronics Laboratory (LMETR), Optics and Precision Mechanics Institute, University Setif 1 - Ferhat Abbas, Setif, 19000, Algeria | Department of Computer Science, University Setif 1 - Ferhat Abbas, Setif, 19000, Algeria
hayet.dahmri@univ-setif.dz (corresponding author)

Salim Bouamama
Mechatronics Laboratory (LMETR), Optics and Precision Mechanics Institute, University Setif 1 - Ferhat Abbas, Setif, 19000, Algeria | Department of Computer Science, University Setif 1 - Ferhat Abbas, Setif, 19000, Algeria
salim.bouamama@univ-setif.dz

Samir Balbal
Department of Computer Science, University Setif 1 - Ferhat Abbas, Setif, 19000, Algeria
samir.balbal@univ-setif.dz

Received: 8 July 2024 | Revised: 29 July 2024 | Accepted: 3 August 2024

ABSTRACT

The minimum connected dominating set problem is a well-known NP-hard combinatorial optimization problem in graph theory, with various fields of application including wireless sensor networks, optical networks, and systems biology. This paper presents an adaptive multiobjective simulated annealing approach to address a variant of the minimum connected dominating set problem known as the Minimum Weight Minimum Connected Dominating Set Problem. This approach combines a multiobjective Pareto set with a greedy simulated annealing algorithm to tackle the problem by simultaneously optimizing two objectives, namely the cardinality and the total weight of the connected dominating set. Experimental results compared to those obtained by current state-of-the-art approaches show the superiority of our method.

Keywords-greedy heuristic; minimum connected dominating set; minimum weight connected dominating set; Pareto optimality; simulated annealing

I. INTRODUCTION

The Minimum Connected Dominating Set Problem (MCDS) and its variants are well-known combinatorial optimization problems in graph theory with wide-ranging applications, particularly in fields like wireless network communications [1-3], optical networks [4, 5], and systems biology [6]. More details in this context can be found in [7, 8].

Given a simple undirected graph $G = (V, E)$, a dominating set D is a subset of V, such that each vertex in D is adjacent to at least one vertex from D and the subgraph induced by D is connected. The MCDS problem asks for a connected dominating set with minimum cardinality (size). If a positive weight is associated with each vertex of V, the Minimum Weight Connected Dominating Set (MWCDS) problem looks for a connected dominating set with minimum total weight. Indeed, both the MCDS and the MWCDS problems are classified as NP-hard [9]. Given their inherent difficulty and the significant potential advantages of solving them, considerable research efforts have been directed towards devising effective solution approaches. As an example of such approaches, authors in [10] introduced the first metaheuristic to deal with the MCDS problem. The latter combines tabu search and a simulated annealing algorithm. Authors in [11] proposed an Ant Colony Optimization (ACO) algorithm with greedy heuristics. Authors in [12] presented the Greedy Randomized

Adaptive Search Procedure (GRASP) that incorporates the tabu search as a local enhancement process. Authors in [13] developed a tabu search procedure (RSN-TS) based on a restricted swap-based neighborhood. They conducted a considerable number of experimental tests to show that RSN-TS outperforms GRASP and ACO both in terms of solution quality and computation time. Authors in [14] implemented two methods for solving the MCDS problem. The first one is a memetic algorithm and the second one is simulated annealing. The performance of both approaches when applied to the MCDS problem on common benchmark instances is better than ACO and GRASP but worse than RSN-TS. However, only a few researches have been carried out on the MWCDS problem. Hybrid Genetic Algorithm (HGA) and a Population-Based Iterated Greedy (PBIG) algorithm were proposed in [15]. In [16], a hybrid ACO approach was combined with the Reduced Variable Neighborhood Search (ACO-RVNS) to solve both MCDS and MWCDS and outperformed RSN-TS, PBIG, and HGA on the considered benchmark sets.

One should mention that all previous approaches have something in common: they optimize only a single objective function such as minimizing the size of the Connected Dominating Set (CDS) or minimizing its total weight for MCDS and MWCDS, respectively. To the best of our knowledge, the first approach in the literature that considered these two objectives together was presented in [17]. The authors of this study first defined the Minimum Weight Minimum Connected Dominating Set (MWMCDS) problem of which the aim is to minimize simultaneously the size and the total weight of the generated CDS. Then, they proposed a Multi-Objective Genetic Algorithm (MOGA) based on a scalarization model to deal with the MWMCDS problem. In the same context, authors in [18] proposed an improved Pareto genetic algorithm based on NSGA-II named I-NSGA-II to tackle the MWMCDS problem. A comparison of their approach against NSGA-II demonstrates its superiority.

In this work, we propose to solve the MWMCDS problem with the Multiobjective Greedy Simulated Annealing (MGSA) algorithm. The concept of Pareto optimality is applied to evaluate the multiobjective solutions and store the nondominated ones in the Pareto Front. Additionally, a new greedy heuristic is proposed to seed simulated annealing with a good initial solution as well as in generating neighbors.

II. PROBLEM STATEMENT

Let us consider a given simple undirected weighted graph $G = (V, E)$ where $V = \{1,2,\cdots,n\}$ represents the set of vertices and $E \subset V \times V$ represents the set of edges. Two vertices are said to be adjacent or neighbors if they are joined by an edge. The set of neighbors of v is denoted by $N(v) = \{u \in V | (v,u) \in E\}$. A subset $D \subseteq V$ is called a dominating set if each vertex $v \in V$ is either in D or adjacent to at least one vertex in D. If D is a dominating set and its induced subgraph G(D) is connected, then D is called a connected dominating set. Vertices in the dominating set are called the dominators. An MWMCDS problem consists of a simple undirected connected graph $G = (V, E, w)$ where $w: E \mapsto R^+$ is a weight function that assigns a positive weight value $w_{(v,u)}$ to each edge $(v,u) \in E$ of the graph. This problem can thus be expressed as:

minimize $\{F_1(D), F_2(D)\}$

subject to $\forall\, v \in V\backslash D: N(v) \cap D \neq \emptyset$

$D \subseteq V$

$G(D)$ is connected.

In the above definition, we look for a connected dominating set $D \subseteq V$ (a candidate solution) in which two objective functions are simultaneously minimized. Let $|D|$ represent the cardinally of D. The first objective function $F_1(D) := |D|$, named as the cardinality objective function, intends to minimize the size of the candidate solution whereas the second objective function $F_2(D)$ named as the weight objective function, intends to minimize its total weight. $F_2(D)$ is calculated as follows:

$$F_2(D) \coloneqq F_{2a} + F_{2b} \quad (1)$$

$$F_{2a} = \sum_{((u,v)\in E)\wedge(u\in D\wedge v\in D)} w_{(u,v)} \quad (2)$$

$$F_{2b} = \sum_{(u\in V\backslash D)} \min\{w_{(u,v)} | (u,v) \in E \wedge v \in D\} \quad (3)$$

III. MULTIOBJECTIVE GREEDY SIMULATED ANNEALING (MGSA) ALGORITHM FOR MWMCDS

In the following, we describe the main components of the developed MGSA algorithm with the aim of obtaining high-quality solutions to the MWMCDS problem. As mentioned before, MGSA is a multiobjective greedy simulated annealing algorithm based on the Pareto optimization technique.

A. *Greedy Heuristic*

A feasible solution is greedily constructed as follows. Assume a given empty solution S. Initially, all vertices in V are colored WHITE. Then, the color of the vertex chosen to be included in the solution S becomes BLACK and the color of their neighbors becomes GRAY. We repeatedly select a vertex from GRAY vertices to be included in S. Besides, its color becomes BLACK whereas the color of their WHITE neighbors will be changed to GRAY. The process is continued until no WHITE vertices are left. The selection of vertices is based on a combined score that considers both weight and cardinality as given in (4).

$$S(v) = \alpha \cdot GD(v)' + (1-\alpha) \cdot GW(v)' \quad (4)$$

where α is a parameter that allows adjusting the importance given to each objective. A higher α value emphasizes weight reduction, whereas a lower value prioritizes minimizing cardinality. Here α is set to 0.5. $GD(v)'$ and $GW(v)'$ are the normalized values of $GD(v)$ and $GW(v)$, respectively. $GD(v)$ and $GW(v)$ are described below.

Let V^{cand} denote the set V if all vertices are WHITE, or the set of GRAY vertices if there is at least one GRAY vertex in V, and $ds(v)$ represents the number of WHITE neighbors of vertex v. GD chooses vertex v having the greatest number of WHITE neighbors. GD is then calculated as:

$$GD(v) = \{d_s(v) \mid v \in V^{cand}\} \quad (5)$$

$GW(v)$ represents the weight to be added to F_2 upon the inclusion of vertex v in the solution ($GW(v)$ may have a positive or negative value). According to GW, the best vertex is the one with the lowest value of GW. GW is given in (6).

$$GW(v) \coloneqq$$

$$wt_{1(v)} - \sum_{(v\in V^{\text{cand}} \wedge u\in V^{\text{cand}} | (v,u)\in E)} wt_{2(v,u)} \qquad (6)$$

$$wt_{1(v)} = \sum_{((u,v)\in E, u\in D \wedge v\in V^{\text{cand}})} w_{(u,v)}$$

$$wt_{2_{1(u,u')}} =$$

$$\min\{w_{(u,u')} | (u,u')\in E, u\in V^{\text{cand}} \wedge u'\in D\}$$

if $wt_{2\_1(u,u')} > w_{(v,u)}$: $wt_{2(v,u)} = wt_{2\_1(u,u')} - w_{(v,u)}$

else $wt_{2(v,u)} = 0$

Employing this greedy heuristic when solving the MWMCDS problem improves the quality of solutions and reduces computational effort. This is achieved through the efficient balanced consideration of both objectives via a combined score and the adaptive vertex selection, which dynamically focuses on relevant candidates (GRAY vertices).

IV. MULTIOBJECTIVE SIMULATED ANNEALING FRAMEWORK

Simulated Annealing (SA) [19] is a well-known metaheuristic approach that has been applied successfully to a large number of combinatorial optimization problems [20-22]. SA is both a single-solution-based and exploitation-oriented algorithm. The proposed algorithm named MGSA is a multiobjective SA that starts with a good initial solution based on the greedy heuristic previously defined. The neighbors are generated either greedily or randomly with equal probability. Moreover, MGSA uses useful methods for changing temperature and for accepting solutions. After a maximum number of iterations $maxIter$, the approximate Pareto set will be returned and then improved by eliminating the redundant vertices. The pseudocode of the MGSA algorithm follows.

Algorithm: MGSA for the MWMCDS problem
1. Input: A problem instance (G, V, E, w), and parameters: maximum number of iterations maxIter, initial temperatures T_{01} and T_{02}, predetermined numbers of iterations used in the annealing schedule N_1 and N_2. The number of acceptances N_A, number of iterations to be executed before the first return to base N_{B0}, and base return parameter r_B.
2. $T_1 \leftarrow T_{01}$
3. $T_2 \leftarrow T_{02}$
4. generate greedily the initial solution S_0
5. $archive \leftarrow S_0$
6. S $\leftarrow S_0$
7. for i = 0 to $maxIter$ do
8. generate the neighbor $S_{new} = N(s)$ greedily or randomly with equal probability
9. if (S_{new} is not dominated by any solution from $archive$) then
10. $archive \leftarrow archive \cup \{S_{new}\}$
11. S $\leftarrow S_{new}$
12. else if (S_{new} verify the acceptance probability p) then
13. S $\leftarrow S_{new}$
14. end if
15. periodically, return to base based on N_1, N_{Bi}, and r_B
16. periodically, reduce T_1 and T_2 based on N_1, N_2, and N_A
17. end for
18. remove redundant vertices from $archive$
19. Output: $archive$ that represents an approximate Pareto set

A. Archiving and Acceptance

In the archiving procedure, if the new solution is dominated by any members of the archive, it is not archived, otherwise it is archived and the archive is updated by removing the dominated solutions. All archived solutions are accepted. If a solution is not archived, then it is accepted with a probability given by:

$$p = \prod_{i=1}^{2} \exp\left(-\frac{[f_i(s_{n+1}) - f_i(s_n)]}{T_i}\right) \qquad (7)$$

Thus, the overall acceptance probability is the product of individual acceptance probabilities for each objective, and therefore, each objective is assigned an associated temperature, T_i, which obviates the need to scale the objectives carefully with respect to each other, as long as suitable temperatures can be determined automatically, as described in the next section.

B. Annealing Schedule

In the presented algorithm, we adopted the annealing schedule proposed in [23]. Initially, all temperatures are initialized to large values, hence, all feasible solutions are accepted. A statistical record is maintained for each observed objective function value. After a pre-determined number of iterations, N_1, the temperatures T_1 and T_2 are set equal to the standard deviation, σ_i, of the accepted values of F_1 and F_2, respectively, i.e., $T_i = \sigma_i$. After reaching either a specified number of iterations, N_2, or a certain number of acceptances, N_A, the temperatures are lowered according to:

$$T_i' = \alpha_i T_i \qquad (8)$$

where T_i' denotes the updated temperature, and α_i is computed using the formulation proposed in [24]:

$$\alpha_i = \max\left(0.5, \exp\left[-\frac{0.7T_i}{\sigma_i}\right]\right) \qquad (9)$$

In this expression, σ_i represents the standard deviation of F_i values for the accepted solutions at temperature T_i. Subsequently, both counters for N_2 and N_A are reset to zero.

C. Return to Base

In order to completely expose the trade-off between objectives, the periodic random selection of a solution from the archive, from which to recommence the search, is done as follows: Following the initiation of the search process, the activation of a return-to-base occurs once the fundamental aspects of the trade-off between objectives are established. It is prudent for this activation to coincide with the initial reduction in temperatures, specifically after N_1 iterations. Thereafter, the rate of return is naturally heightened to enhance the exploration within the trade-off. The number of iterations N_{Bi} to be executed before the i^{th} return-to-base after the start of the search is given by:

$$N_{Bi} = r_B N_{Bi-1}, \quad i = 2, 3, 4 \ldots . \tag{10}$$

where r_B is a parameter ranging from 0 to 1 which determines the frequency of return. Naturally, N_{Bi} cannot decrease indefinitely, and thus a lower bound for N_{Bi} is established to ensure $N_{Bi} \geq 10$.

V. EXPERIMENTAL EVALUATION

The proposed MGSA algorithm was implemented using C++ language. The experimental results were obtained on a PC with an Intel Core i5-1135G7 2.40GHz processor and 8.0 GB of memory. To benchmark our algorithm, we utilized two distinct datasets. The first one comprises instances originally proposed in [17], featuring undirected edge-weighted graphs. The second set, as suggested in [15], presents undirected vertex-weighted graphs. Given that the MWMCDS problem necessitates an edge-weighted graph, we derived edge weights by averaging the weights of their endpoints. We partitioned the second dataset into two categories: small and medium instances which include $\{10,25,50,100,200,250\}$ vertices, and large instances which contain $\{500,750,1000\}$ vertices. The number of edges (m) is varied for each vertex count (n) to observe the influence of vertex connectivity on the outcomes. Typically, each graph configuration entails 10 instances.

The used parameters for the MGSA algorithm are the following: the maximum number of iterations $MaxIter$ is defined as 1000, the initial temperatures are set to $T1 = T2 = 1000$, the predetermined number of iterations used in the annealing schedule are $N_{T1} = 200$ and $N_{T2} = 100$, the number of acceptances is $N_A = 10$, the number of iterations to be executed before the first return to base is $N_{B0} = 50$, with a base return parameter of $r_B = 0.9$.

The performance of the proposed method was compared against current state-of-the-art approaches, namely MOGA [17], MCDS [25], and I-NSGA-II [18]. In the first dataset, each instance consists of a simple undirected edge-weighted graph modeling a data transfer system where every vertex transfers data at instant t with probability P_t, and this data can be dropped with probability P_d. The distance traveled by the data in the transfer is represented indirectly by the weight, which is expressed by the energy consumed during the travel. Thus, the energy consumed in the case of successful transfer is equal to the distance traveled by the data, and in the case of failed transfer, it is equal to half of the distance traveled. The energy consumed in the transfer of all data in the network is represented by the energy consumption. To measure the performance in the second dataset, we used the hypervolume indicator [26], which consists of calculating the volume of the dominated portion of the objective space. The higher the HVI value, the more the convergence and diversity of the solutions. In addition, the execution time and the approximate Pareto fronts are given.

A. First Dataset Comparison

The performance of MGSA against MOGA and MCDS with respect to energy consumption for 100 instances of data transfer is shown in Figure 1. It is evident that MGSA consumes the least amount of energy in all cases. Figure 2 represents the number of dominator vertices produced by MGSA, MOGA, and MCDS for different networks. It can be seen that MGSA outperforms MCDS in 8 out of 10 cases, whereas yielding identical results in the remaining two cases. When compared to MOGA, MGSA demonstrates equivalent performance in all cases except for instances where n = 80, n = 90, and n = 100, where MGSA exhibits superior performance.

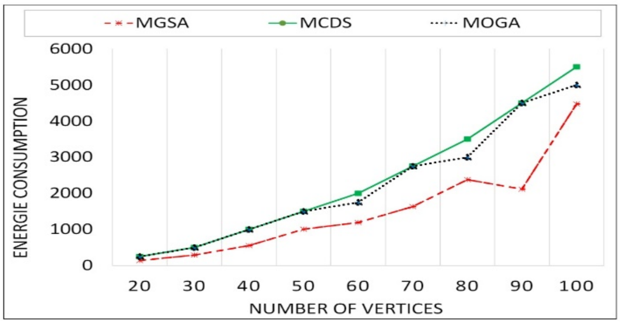


Fig. 1. Energy consumption in MGSA, MOGA, and MCDS.

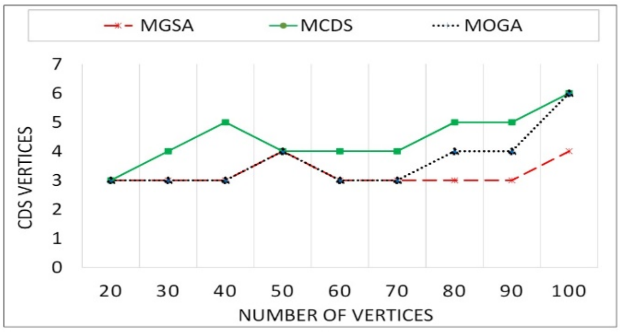


Fig. 2. Size of CDS in MGSA, MOGA, and MCDS.

B. Second Dataset Comparison

The comparative analysis between the MGSA and I-NSGA-II algorithms across small and medium instances (Table I) reveals that MGSA outperforms I-NSGA-II in terms of execution time for all instances. Considering HVI, MGSA is better than I-NSGA-II in 10 out of 17 instances.

Examining Table II, the evaluation of MGSA and I-NSGA-II in large instances highlights that MGSA maintains its superiority in time efficiency in all cases and 12 out of 15 cases regarding HVI.

Figures 3 and 4 depict the approximate Pareto fronts derived from MGSA and I-NSGA-II methods for small and medium instances, as well as large instances, respectively. It is clearly seen that MGSA consistently outperforms I-NSGA-II in all cases, except for instances (10, 20) and (25, 100) where the results are incomparable. The previous results showcase the ability of MGSA to deliver solutions quickly while taking their quality into account.

TABLE I. RESULTS OF MGSA AND I-NSGA-II FOR SMALL AND MEDIUM INSTANCES- BEST RESULTS IN BOLD

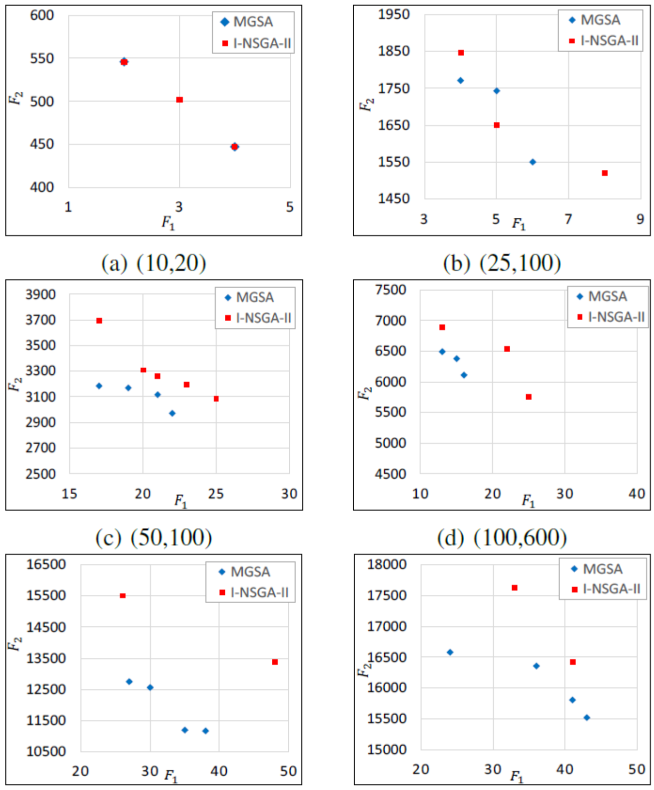
n	*m*	MGSA		I-NSGA-II	
		Time (s)	HVI	Time (s)	HVI
10	20	**0.113**	5768	19.992	**6067**
	40	**0.0786**	18498	19.643	**19762**
25	100	**0.358**	126434	20.964	**126476**
	250	**0.366**	**386878**	18.214	381058
50	100	**0.649**	125287	25.281	**125806**
	500	**0.974**	**1.493E+06**	23.290	1.487E+06
100	200	**1.962**	**494505**	31.076	476289
	600	**2.390**	**3.157E+06**	31.520	3.156E+06
	1000	**7.184**	5.840E+06	26.896	**5.843E+06**
200	400	**8.773**	**1.954E+06**	125.586	1.846E+06
	1200	**16.069**	**1.263E+07**	50.662	1.248E+07
	2000	**9.847**	2.286E+07	36.313	**2.312E+07**
250	500	**9.095**	**2.994E+06**	134.257	2.855E+06
	1000	**11.697**	**1.114E+07**	103.257	1.077E+07
	1500	**11.627**	**1.937E+07**	95.514	1.904E+07
	2000	**13.923**	**2.854E+07**	51.075	2.824E+07
	2500	**16.968**	3.761E+07	47.800	**3.792E+07**
Average		**6.593**	**8.750E+06**	50.667	8.700E+06

TABLE II. RESULTS OF MGSA AND I-NSGA-II FOR LARGE INSTANCES- BEST RESULTS IN BOLD

n	*m*	MGSA		I-NSGA-II	
		Time (s)	HVI	Time (s)	HVI
500	1000	**28.838**	**1.165E+07**	565.842	1.160E+07
	2000	**36.759**	**4.530E+07**	646.827	4.371E+07
	3000	**96.697**	**7.762E+07**	301.842	7.548E+07
	4000	**46.037**	1.154E+08	232.733	**1.157E+08**
	5000	**44.169**	**1.447E+08**	180.837	1.439E+08
750	1500	**75.306**	**2.727E+07**	1541.980	2.517E+07
	3000	**88.287**	**9.526E+07**	1587.260	9.485E+07
	4500	**100.016**	**1.774E+08**	1084.457	1.738E+08
	6000	**114.328**	2.501E+08	711.109	**2.556E+08**
	7500	**108.594**	**3.273E+08**	354.330	3.264E+08
1000	2000	**102.763**	**4.622E+07**	3060.541	4.578E+07
	4000	**163.637**	**1.749E+08**	2754.252	1.722E+08
	6000	**152.561**	**3.116E+08**	2082.110	3.087E+08
	8000	**145.614**	**4.419E+08**	1103.110	4.395E+08
	10000	**200.758**	5.815E+08	749.328	**5.855E+08**
Average		**100.291**	**1.885E+08**	1130.437	1.879E+08

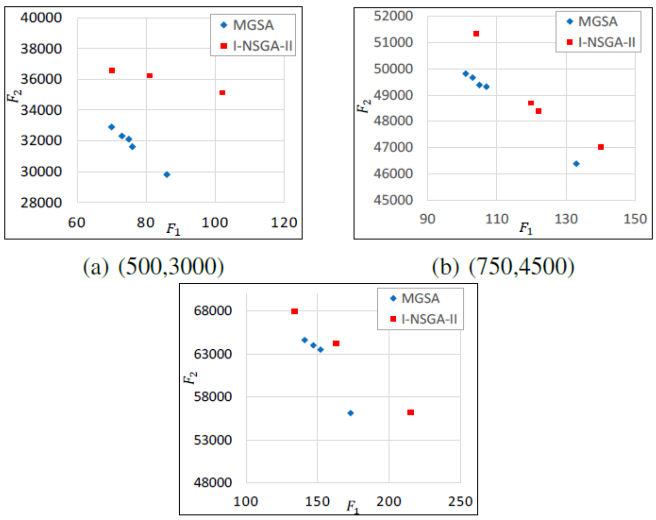
VI. CONCLUSION

In this work, a novel method called Multi-Objective Greedy Simulated Annealing (MGSA) is introduced for tackling the Minimum Weight Minimum Connected Dominating Set (MWMCDS) problem. This problem involves minimizing two objectives simultaneously: the weight and the size of the connected dominating set. MGSA integrates a simulated annealing algorithm, which is seeded by a combined greedy heuristic that considers both the weight and the cardinality reduction.



(a) (10,20) (b) (25,100) (c) (50,100) (d) (100,600) (e) (200,1200) (f) (250,1500)

Fig. 3. Approximate Pareto fronts produced by MGSA and I-NSGA-II for small and medium instances.



(a) (500,3000) (b) (750,4500) (c) (1000,6000)

Fig. 4. Approximate Pareto fronts produced by MGSA and I-NSGA-II for large instances.

The performance of the proposed algorithm was compared with the existing state-of-the-art techniques MOGA, MCDS, and I-NSGA-II. The comparison was based on energy consumption and the size of CDS in the data transfer system for the first dataset, and on hypervolume indicator values, runtime, and approximate Pareto fronts for the second dataset. The obtained results demonstrate the effectiveness of our algorithm.

For future research, we intend to enhance MGSA by developing more advanced greedy heuristics and using multiple greedy heuristics simultaneously. Moreover, we aim to apply MGSA to other bi-objective problems, particularly those similar to MWMCDS, such as the bi-objective minimum spanning tree, bi-objective shortest path, and bi-objective facility location problems. We plan also to extend the applicability of MGSA to problems with more than two objectives.

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