

Displacement Analysis of a Hydrostatic Spindle: An Experimental Investigation

Manh-Toan Nguyen

School of Mechanical Engineering, Hanoi University of Science and Technology, Vietnam
toan.nguyenmanh@hust.edu.vn

Van-Hung Pham

School of Mechanical Engineering, Hanoi University of Science and Technology, Vietnam
hung.phamvan@hust.edu.vn

Van-Thuc Tran

School of Mechanical Engineering, Hanoi University of Science and Technology, Vietnam
thuc.tranvan@hust.edu.vn

Tuan-Anh Bui

School of Mechanical Engineering, Hanoi University of Science and Technology, Vietnam
anh.buituan@hust.edu.vn (corresponding author)

Received: 24 June 2024 | Revised: 9 July 2024 | Accepted: 19 July 2024

Licensed under a CC-BY 4.0 license | Copyright (c) by the authors | DOI: <https://doi.org/10.48084/etasr.8219>

ABSTRACT

This paper presents an experimental investigation of the displacement characteristics of a hydrostatic spindle used in a medium-sized circular grinding machine. The study focuses on the influence of lubricant pressure and oil viscosity on spindle displacement under varying load conditions. Experiments were conducted using a specially designed testing bench, which allowed precise control of pump pressure and oil viscosity. The results demonstrate a significant impact of both parameters on spindle displacement. An increase in oil viscosity leads to a decrease in displacement, with the optimal viscosity identified as 2 mPa.s. Similarly, higher pump pressure results in reduced displacement, with 5 MPa being the optimal pressure for the system studied. These findings highlight the importance of carefully selecting lubricant properties and operating conditions to achieve optimal machining accuracy and stability. This research provides valuable insights into the understanding and optimization of hydrostatic bearing systems in precision machining applications.

Keywords-oil pressure; oil viscosity; hydrostatic spindle bearing; spindle displacement

I. INTRODUCTION

A precise machining process, such as grinding, is critical in defining dimensional accuracy and surface roughness. Therefore, understanding how the machine and grinding parameters affect surface quality is vital in mechanical processing. This study aims to quickly stabilize the wheel axis post-startup during grinding. The focus is on the design and manufacture of a hydrostatic spindle bearing integrated into the grinding machine to determine the characteristics of the hydrostatic bearing and the feasible parameters for the grinding machine spindle.

Several studies have been conducted on hydrostatic bearings and their applications in various industries. In the realm of machine tools, research has been carried out to incorporate hydrostatic bearings into machines and examine their characteristics for precision machining, particularly in

grinding machines. In [1], the thermal characteristics of grinding machines using hydrostatic bearings were investigated. This study highlighted the impact of thermal deformation on the machining quality of CNC grinding machines that incorporated a hydrostatic bearing. The results showed that the thermal deformation of the grinding machine spindle was influenced by the temperature of the hydrostatic bearing. This correlation can be utilized to evaluate the thermal distortion properties of the grinding machine. In [2], a design approach and tools were proposed for the examination of hydrostatic sliding boards using capillary techniques in high-precision grinding machinery. This work contributed significantly to understanding and improving hydrostatic slide systems in precision machining applications, as it focused on improving the performance and reliability of these systems, which are integral to the operation of high-precision grinding machines.

In [3], a comprehensive study was conducted on the impact of both static and dynamic loads on hydrostatic bearings, particularly when alterations were made to the lubricant pressure and viscosity within the bearing. The research involved a detailed examination of the Reynolds equation and associated boundary conditions to understand changes in various parameters related to hydrostatic bearings, such as temperature distribution, modifications in oil viscosity, and radial load. Simulation results showed that an increase in the oil viscosity of the lubricant within the bearing, especially in the moving pads, led to a decrease in wear and tear, thus improving the bearing lifetime. In [4], the importance of incorporating a hydrostatic bearing into the spindle configuration of machinery that requires high accuracy in mechanical processing was highlighted. The analysis further suggested that replacing conventional roller bearings with a spindle designed with a hydrostatic bearing could lead to cost-effectiveness for low- and medium-speed machines. The study in [5] aimed to optimize the oil film thickness on the V-25 vertical lathe machine fitted with a hydrostatic spindle bearing. A simulation program was used to adjust parameters, such as rotational speed, oil viscosity, and stiffness, with different film thicknesses.

In [6], the potential to enhance the rigidity of spindle units in machine tools was explored, significantly contributing to the understanding and improvement of hydrostatic bearing systems in precision machining applications. This study also explored the benefits of this phenomenon across various spindle units in different machine tools. In [7], the eccentricity ratio contributing to film thickness, rigidity, and deformation of the spindle system was explored. In [8], a thermo-mechanical error model was presented for a hydrostatic spindle used in high-precision machine tools. The results showed that the thermal effect significantly influences the variation in spindle stiffness. In [9], a numerical and experimental method was introduced to choose parameters and construct a hydrostatic spindle unit. This module was designed to replace the ball-bearing spindle in the vertical machining apparatus operating at 800 rpm. In [10], the creation and development of testing platforms was investigated. These platforms were utilized to evaluate a hydrostatic bearing implemented in a linearly moving spindle. This arrangement facilitated tests aiming to minimize vibration, determine the rigidity of the hydrostatic bearing, and improve machining precision. In [11], a hydrostatic bearing was designed for a spindle milling machine, considering dynamic factors. The machining tests revealed a correlation between the spindle configuration and the dynamic variables, including rigidity.

In addition to the operating mode and oil selection, the lubrication and cooling methods of the spindle unit are also crucial. These factors significantly influence the working quality of the spindle unit and, consequently, the machining quality [12, 13]. Proper lubrication reduces friction between the moving parts of the spindle assembly, minimizing wear and tear, and prolonging its lifespan. Meanwhile, effective cooling prevents overheating, which can cause thermal deformation and negatively affect the precision and stability of the machining process. The viscosity of the oil can be influenced by factors such as temperature and additives. It is important to

note that oil viscosity directly impacts the performance of hydrostatic bearing. Higher-viscosity oils can provide better lubrication and reduce friction, but they may also increase operating temperature and energy consumption. On the other hand, lower-viscosity oils may not provide sufficient lubrication, leading to increased wear and tear. Therefore, finding the right balance is key. Temperature plays a significant role in determining oil viscosity. As the temperature increases, the viscosity of the oil decreases, and vice versa. Hence, it is essential to consider the operating temperature when selecting an oil for a hydrostatic bearing.

Additives are often added to oils to enhance their properties [14, 15]. For instance, anti-wear additives can help reduce the wear and tear on the bearing surfaces, while viscosity index improvers can help maintain the oil viscosity over a wide temperature range. However, it is important to carefully select these additives as they can also alter the oil's viscosity and other properties.

However, there is a lack of studies on the impact of oil viscosity and pump pressure on the displacement of the hydrostatic spindle incorporated in medium-sized circular grinding machines. This study investigates the influence of oil viscosity and pump pressure on the displacement of hydrostatic spindle bearings based on experimental data.

II. HYDROSTATIC BEARING PARAMETERS

By replacing the hydrodynamic bearings with a hydrostatic bearing, the 3K12 grinding machine can achieve higher levels of accuracy and stability, meeting the demands of modern industrial requirements. The dimensions of the shaft and bearing case were machined within the range of $\Phi 70_{-0.029}^{+0.01}$ and $\Phi 70_0^{+0.03}$, respectively. The design of the hydrostatic spindle unit involved careful consideration of various factors to ensure optimal performance and stability. The key design parameters were:

- Clearance: The largest clearance (h_{0max}) was determined to be 59 μm , while the smallest clearance (h_{0min}) was 10 μm . This range allows for sufficient oil flow to maintain a stable lubricating film while minimizing friction and wear.
- Oil recesses: The bearing incorporates four oil recesses strategically positioned to provide adequate pressure distribution and support the shaft. The dimensions of the recesses, including their length and effective area, were calculated to ensure optimal performance under varying load conditions.
- Bearing geometry: The bearing length, recess length, and shaft diameter were chosen as 56 mm, 28 mm, and 70 mm, respectively. These dimensions were determined based on the specific requirements of the grinding machine and the load-bearing capacity needed for the spindle.
- Equilibrium force equation: The equilibrium force equation, considering the weight of the shaft, external load, the effective area of the oil recess, and eccentricity, ensures that the forces acting on the system are balanced. This balance is crucial for maintaining the shaft's position and minimizing vibration during operation.

The selection of oil recess pressure and viscosity plays a critical role in determining the performance and stability of the hydrostatic spindle. These parameters were carefully chosen based on the following considerations:

- Oil recess pressure (p_r): The oil recess pressure was selected within the range of 1-5 MN/m², considering the capabilities of manufacturing technology. The ratio of oil chamber to pump pressure ($\beta = p_r/p_s$) was kept within the recommended range of 0.4-0.7, as suggested in [7, 16]. Based on these considerations, pump pressures of 3, 4, and 5 MN/m² were chosen for the hydrostatic bearing.
- Oil viscosity: In general, lower oil viscosity is preferred for hydrostatic lubrication to improve the cooling effect of fluid flow. Oil viscosities of 1, 1.5, and 2 mPa.s were selected to investigate their influence on the displacement of the hydrostatic spindle.

These selections ensure that the hydrostatic bearing operates within the recommended pressure and viscosity ranges, promoting optimal performance and stability. The chosen oil viscosities also allow for investigating their impact on the stiffness of the spindle, providing valuable insights for further optimization.

III. EXPERIMENTAL SETUP

Assessing the operational capacity of the hydrostatic grinding spindle unit requires the development of dedicated experimental equipment. A key performance indicator for spindle units is the spindle displacement of the hydrostatic bearing, which directly influences machining accuracy and stability. To evaluate this, a displacement measurement system was specifically designed for the grinding machine, as shown in Figure 1.

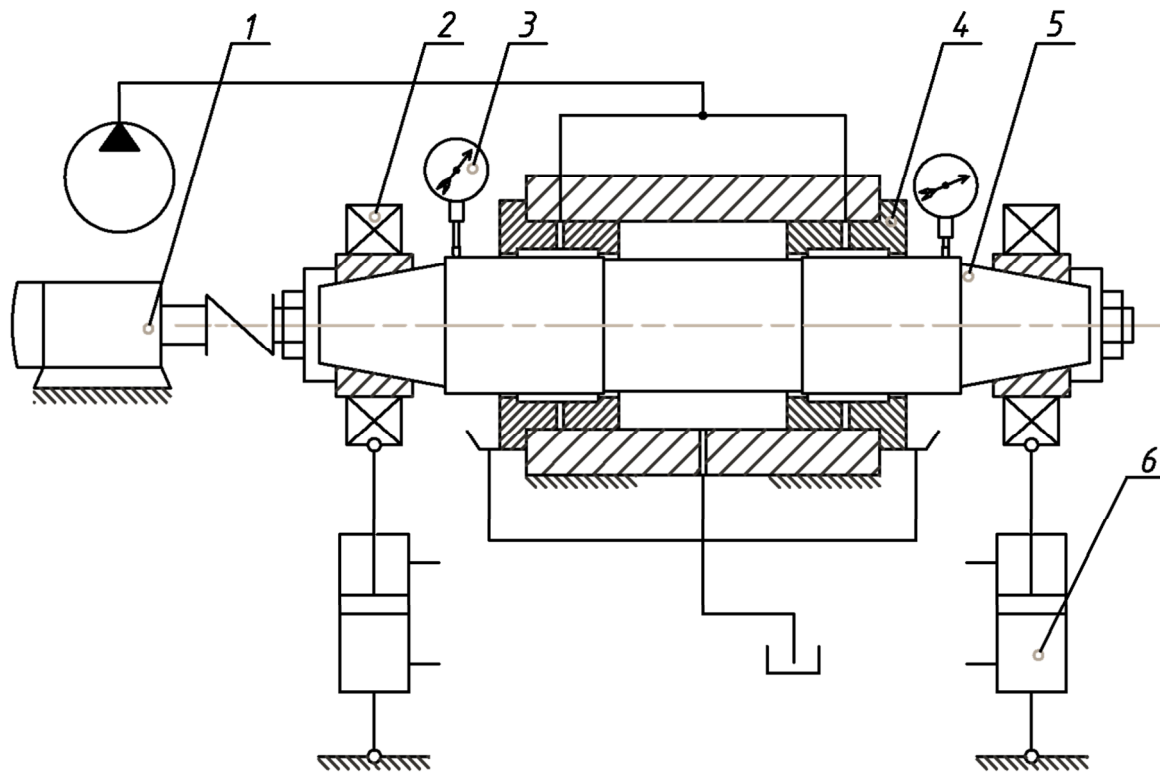


Fig. 1. Experimental layout of hydrostatic spindle displacement measurement: 1-motor, 2-journal bearing, 3-radial indicator, 4-bearing bush case, 5-spindle, 6-pneumatic cylinder.

The system incorporates several key features to simulate real-world operating conditions. Pump pressure can be adjusted within a range of 3-5 MPa, allowing investigation of its impact on spindle displacement. The special load-generating system was set up using two pneumatic cylinders to generate radial forces at both ends of the spindle, simulating the load experienced during actual machining. Radial force is precisely controlled by adjusting the pressure in pneumatic cylinders. Two radial indicators accurately measure the spindle displacement under the applied load, providing data to assess the hydrostatic bearing performance. Three separate compressed air sources were used to deliver designed loads of

500, 1000, and 1500 N, to ensure precise and repeatable load application during testing. Finally, the experiment utilizes oil viscosities of 1, 1.5, and 2 mPa.s to investigate their influence on spindle displacement. This comprehensive measurement system provides a reliable platform for evaluating the displacement of the hydrostatic spindle unit under various operating conditions and oil viscosities. The data collected will be instrumental in optimizing the performance and stability of the spindle for high-precision grinding applications.

IV. RESULTS AND DISCUSSION

The experiment was conducted meticulously according to a standardized procedure: (i) Hydrostatic pressure initiation: Hydrostatic pressure was pumped into the bearing; (ii) Pressure stabilization: A one-minute waiting period ensured the stability of oil pressure within the system; (iii) Load application: A load was applied to the spindle, and its displacement was measured; (iv) Repetition: Each working condition was tested nine times and the spindle displacement was determined by averaging these nine measurements.

TABLE I. DISPLACEMENT OF THE SPINDLE UNIT

Pump pressure (MPa)	Load (N)	Oil viscosity (mPa.s)					
		1		1.5		2	
		Mean (μm)	SD (μm)	Mean (μm)	SD (μm)	Mean (μm)	SD (μm)
3	500	3.33	0.50	3.00	0.00	2.78	0.44
	1000	6.22	0.44	5.78	0.44	5.33	0.50
	1500	9.44	0.53	8.78	0.44	8.33	0.50
4	500	2.56	0.53	2.33	0.50	2.11	0.33
	1000	4.78	0.44	4.22	0.44	3.89	0.60
	1500	7.22	0.44	6.44	0.53	5.89	0.33
5	500	2.00	0.00	1.80	0.44	1.66	0.50
	1000	3.80	0.44	3.40	0.53	3.22	0.44
	1500	5.90	0.33	5.60	0.53	5.10	0.33

The experimental results, shown in Table I, demonstrate that the hydrostatic spindle's displacements are influenced by the load, viscosity, and pump pressure:

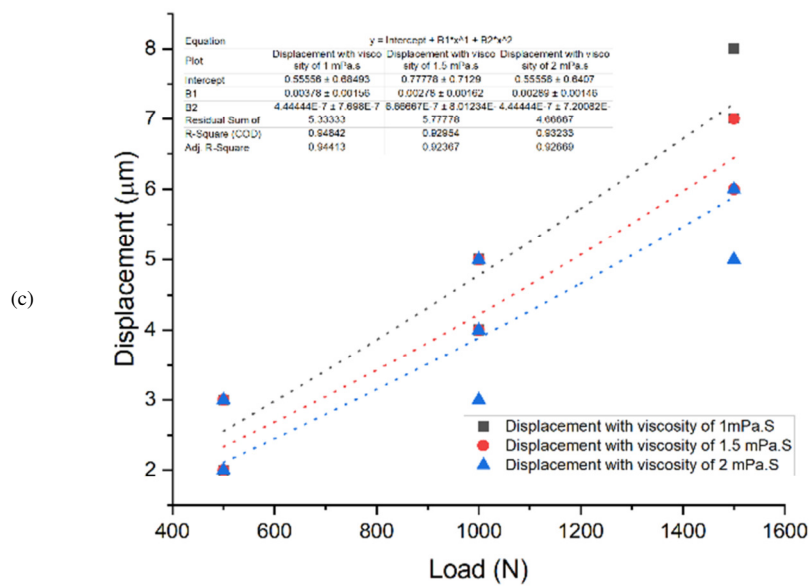
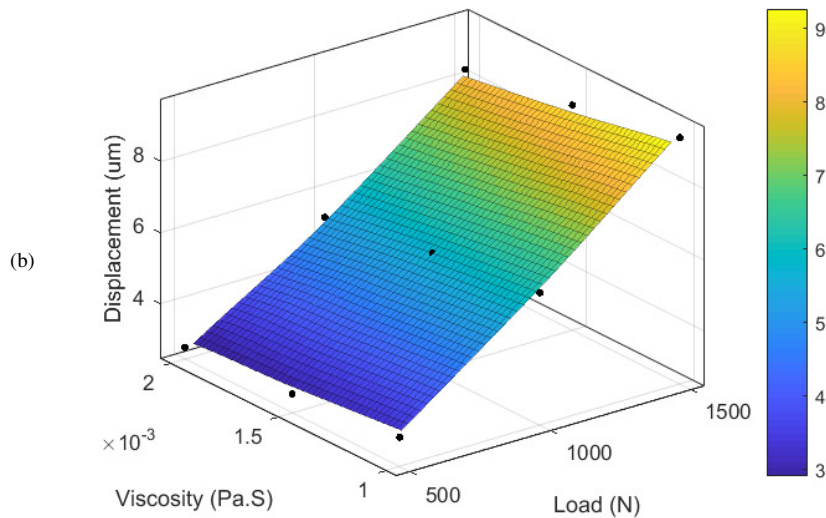
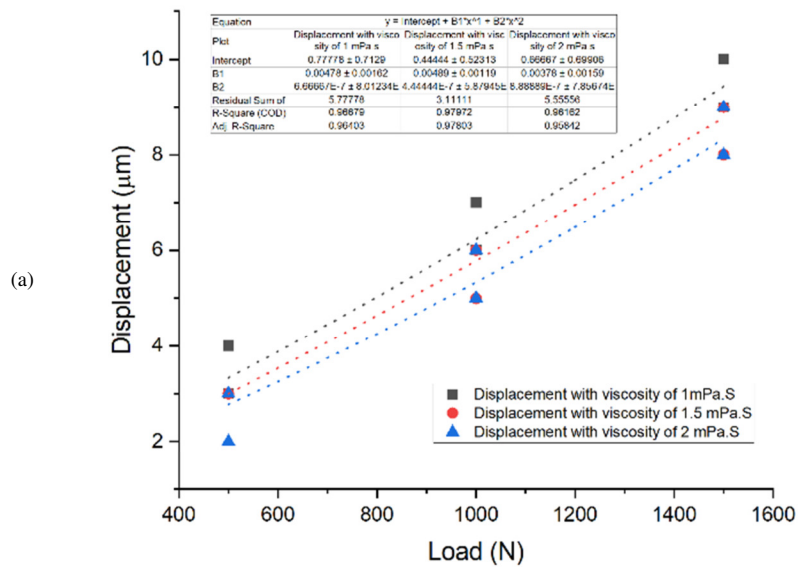
- Oil viscosity: The displacement tends to decrease as the oil viscosity increases, reaching its minimum value at a viscosity of 2 mPa.s. This is attributed to the increased damping effect of the higher-viscosity oil, which effectively reduces the spindle's displacement.
- Pump pressure: As the pump pressure increases, the displacement of the spindle underload decreases. Within the experimental range, the lowest displacement was observed at a pump pressure of 5 MPa. This phenomenon can be attributed to the higher pressure generating a greater force within the oil chambers. This force effectively counteracts the external load, improving the system's stiffness and stability. Consequently, higher pump pressures result in reduced spindle displacement and improved resistance to deformation under load.

These findings align with theoretical understanding and confirm the importance of optimizing both oil viscosity and pump pressure to achieve optimal performance of the hydrostatic spindle unit. The determined optimal pressure value of 5 MPa in these experiments provides valuable guidance for future applications of this technology in high-precision machining. To further assess the impact of oil viscosity and pump pressure on the spindle displacement, calculations were made based on the experimental data. These results, which correspond to the loads, pump pressure, and oil viscosity used in the experimental study, are also presented in Table I. This analysis provides a more comprehensive understanding of the relationships between these parameters and their influence on the spindle's displacement. The results of this study contribute significantly to the understanding and improvement of

hydrostatic bearing systems in precision machining applications. By optimizing the operating parameters, the performance and stability of the hydrostatic spindle unit can be significantly enhanced, leading to improved machining accuracy and productivity. To analyze the displacement of the hydrostatic spindle under various operating conditions, experimental regression functions were determined based on empirical data. Spindle displacement increases correspondingly with the increase in load across all pump pressures and oil viscosities. Therefore, an experimental regression function can be chosen in linear or nonlinear form. A second-degree polynomial function was selected, followed by an analysis of the fit between the regression equation and the experimental data. In addition, these regression functions allow for a deeper understanding of the relationship between operational parameters and spindle displacement. Using a second-degree polynomial, the nonlinear characteristics of the system can be captured more accurately. This approach not only enhances the precision of the predictive model but also provides insight into the influence of each variable. The validity of the regression model was verified through statistical tests and residual analysis to ensure that it reliably represents the experimental observations. This comprehensive analysis helps optimize the design and control of hydrostatic spindles in practical applications. The results of the experimental regression equations that describe the relationship between the displacement of the hydrostatic spindle and various load conditions for pump pressures of 3, 4, and 5 MPa are shown in Tables II, III, and IV, respectively. It can be observed that a second-degree polynomial regression function demonstrates an appropriate fit for this relationship. This is evidenced by the adjusted R^2 values, which exceed 90% in all cases, indicating a high degree of correlation and reliability. Consequently, the relationship can be accurately modeled as a second-degree polynomial. Furthermore, the robustness of this polynomial model is supported by its ability to capture the nonlinear behavior of spindle displacement under varying load conditions and pump pressures. This high level of fit suggests that the regression model can reliably predict spindle displacement within the examined range of conditions. These results are also visually shown in Figure 2, providing a clear representation of the data trends and the model's accuracy. The second-degree polynomial regression model not only fits the empirical data well but also enhances our understanding of the dynamic behavior of hydrostatic spindles under different operational scenarios. These findings are crucial for optimizing spindle design and improving operational performance in practical applications.

TABLE II. REGRESSION EQUATION FOR THE DISPLACEMENT OF THE HYDROSTATIC SPINDLE WITH PUMP PRESSURE OF 3 MPa

Equation	$y = C + B*x + A*x^2$		
Viscosity (mPa.s)	1	1.5	2
C	0.778 ± 0.713	0.444 ± 0.523	0.667 ± 0.699
B	0.00478 ± 0.00162	0.00489 ± 0.00119	0.00378 ± 0.00159
A	$6.66667E-7 \pm 8.01234E-7$	$4.44444E-7 \pm 5.87945E-7$	$8.88889E-7 \pm 7.85674E-7$
Adj. R^2	0.96403	0.97803	0.95842



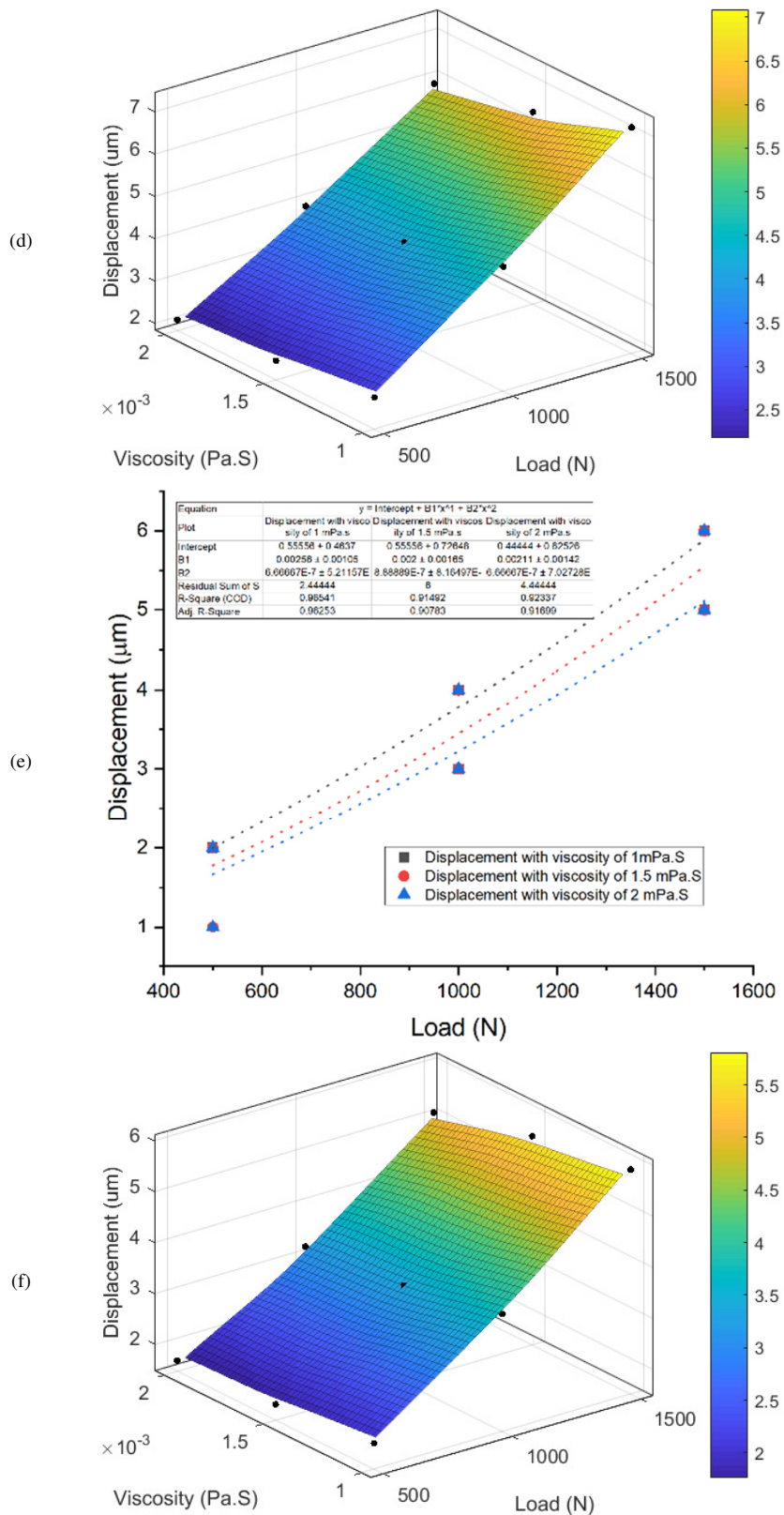


Fig. 2. Displacement of hydrostatic spindle unit with pump pressure: (a, b) 3 MPa, (c, d) 4 MPa, (e, f) 5 Mpa.

TABLE I. REGRESSION EQUATION FOR THE DISPLACEMENT OF THE HYDROSTATIC SPINDLE WITH PUMP PRESSURE OF 4 MPa

Equation	$y = C + B*x + A*x^2$		
Viscosity (mPa.s)	1	1.5	2
C	0.556 ± 0.685	0.778 ± 0.713	0.556 ± 0.641
B	0.00378 ± 0.00156	0.00278 ± 0.00162	0.00289 ± 0.00146
A	4.44444E-7 ± 7.698E-7	6.66667E-7 ± 8.01234E-7	4.44444E-7 ± 7.20082E-7
Adj. R ²	0.94413	0.92367	0.92669

TABLE II. EMPIRICAL REGRESSION EQUATION FOR THE DISPLACEMENT OF THE HYDROSTATIC SPINDLE WITH PUMP PRESSURE OF 5 MPa

Equation	$y = C + B*x + A*x^2$		
Viscosity (mPa.s)	1	1.5	2
C	0.556 ± 0.464	0.556 ± 0.726	0.444 ± 0.625
B	0.00256 ± 0.00105	0.002 ± 0.00165	0.00211 ± 0.00142
A	6.66667E-7 ± 5.21157E-7	8.88889E-7 ± 8.16497E-7	6.66667E-7 ± 7.02728E-7
Adj. R ²	0.96253	0.90783	0.91699

From Figure 2, it can be observed that:

- Decreasing displacement with increasing pressure and viscosity: Across all loads tested, the spindle displacement decreases proportionally with the increase in pump pressure and oil viscosity. This can be explained by the increased stiffness of the hydrostatic bearing system at higher pressures and viscosities. Higher pressure generates a greater force within the oil chambers, counterbalancing the load and enhancing the system's stiffness. Similarly, higher-viscosity oil provides greater damping, effectively reducing spindle vibration and displacement.
- Peak displacement at 3 MPa and 1 mPa.s: At a pump pressure of 3 MPa and an oil viscosity of 1 mPa.s, the spindle displacement reaches a peak value of approximately 9.4 μm with a load of 1500 N. This emphasizes the importance of optimizing these parameters to achieve the highest stiffness under specific working conditions.
- Proportional decrease with increasing viscosity: Displacement tends to decrease proportionally with an increase in oil viscosity across all load ranges and pump pressures. This further highlights the importance of oil viscosity in enhancing spindle stability and precision.
- Increasing displacement with increasing load: For a given oil viscosity, the displacement tends to increase with increasing load. This is expected as the load directly influences the deformation of the hydrostatic bearing. However, the rate of increase in displacement with load decreases as the oil viscosity increases, demonstrating the effectiveness of higher viscosity oil in mitigating load-induced deformation.

These findings suggest that users may need to adjust the load within an appropriate range to achieve the highest stiffness under specific working conditions. In addition, selecting the

appropriate oil viscosity based on the desired operating pressure and load requirements is crucial to optimize spindle performance. These observations underscore the significant role of pump pressure and oil viscosity in determining the performance of hydrostatic spindles. The inverse relationship between displacement and both pump pressure and oil viscosity indicates that optimizing these parameters can enhance spindle stability and precision. This knowledge is crucial for designing and operating hydrostatic spindle units in high-precision engineering applications.

The detailed analysis provided by the interpolation models in Figures 2(b), (d), and (f) offers a robust predictive framework to anticipate spindle behavior under varying operational conditions. This can be invaluable to engineers and researchers seeking to optimize spindle performance and develop new and improved hydrostatic bearing systems. These results contribute significantly to the understanding and improvement of hydrostatic bearing systems in precision machining applications. By highlighting the critical parameters that influence spindle displacement, this study provides valuable insight that can inform design and operational strategies to achieve optimal performance in high-precision engineering contexts. This knowledge paves the way for further advances in the field of precision machining, enabling the production of high-quality components with greater accuracy and efficiency.

The experimental findings reveal a complex interaction between pump pressure, load, and spindle displacement. While the overall trend shows a reduction in displacement with increasing pump pressure for each oil viscosity, this change is modulated by the load applied to the spindle. In more detail, it can be observed that:

- Decreasing displacement with increasing pressure: The observed decrease in displacement with increasing pump pressure is consistent with theoretical expectations. Higher pressure generates a greater force within the oil chambers, counterbalancing the load and enhancing the system's stiffness. This leads to a more stable spindle with reduced displacement under load.
- Load-dependent displacement: The change in displacement with pressure is not uniform across all load levels. The experiments reveal a minimum displacement at a load of 500 N, followed by a slight increase within the load range of 1000 to 1500 N. This behavior can be attributed to two primary factors: (i) Hydrostatic spindle upgrade: The hydrostatic spindle is an upgrade from a hydrodynamic one, retaining the original bush case housing but incorporating newly designed bush cases. These new bush cases undergo elastic deformation during assembly, which can vary around the circumference. This uneven deformation can influence the stiffness of the oil film and contribute to the observed load-dependent displacement behavior. (ii) System deformation under load: Under heavy load, the system, particularly the copper bronze bush case, experiences deformation. This deformation reduces the stiffness of the oil film and the spindle unit, leading to a slight increase in displacement at higher loads.

- Optimal load range: The experimental results indicate an optimal load range for this hydrostatic spindle bearing between 500 and 1000 N. Within this range, the spindle displacement is minimally affected by the load. These findings align well with the practical operating conditions of circular grinding machines.

These findings underscore the importance of adjusting the load according to the working conditions to achieve optimal system stiffness. Users should consider both pump pressure and load when optimizing spindle performance for specific machining tasks. These observations highlight the critical influence of pump pressure and load on the performance of hydrostatic spindles. The relationship between displacement and these parameters underscores the need for precise control and optimization to maintain spindle stability and enhance machining precision. Detailed analysis of spindle behavior under varying conditions provides valuable insight that can guide the design and operational strategies for hydrostatic bearing systems. This research contributes significantly to the understanding and improvement of hydrostatic bearing systems in precision machining applications. By elucidating the key factors that affect spindle displacement, this study offers practical recommendations for achieving optimal performance in high-precision engineering contexts. This knowledge paves the way for further advances in the field of precision machining, enabling the production of high-quality components with greater accuracy and efficiency.

V. CONCLUSION

This experimental study provided valuable insights into the displacement characteristics of a hydrostatic spindle integrated into a circular grinding machine. The key findings are: (i) Spindle displacement decreases with increasing oil viscosity, with the minimum displacement observed at an oil viscosity of 2 mPa.s; (ii) Higher pump pressures lead to reduced spindle displacement, with the optimal pressure determined to be 5 Mpa; (iii) Empirical regression models were developed to capture the nonlinear relationship between load, oil viscosity, pump pressure, and spindle displacement, enabling accurate prediction and optimization of the system's performance. The results of this study contribute significantly to the understanding and improvement of hydrostatic bearing systems in precision machining applications, particularly for grinding machines, where maintaining dimensional accuracy and surface finish are crucial for product quality. The identified optimal oil viscosity and pump pressure values can be used to guide the design and operation of hydrostatic spindles in similar grinding machine applications, leading to enhanced machining precision and productivity.

ACKNOWLEDGMENT

This research was funded by the Hanoi University of Science and Technology, grant number T2021-PC-034.

REFERENCES

- [1] B. S. Kim, G. T. Bae, G. N. Kim, H. M. Moon, J. P. Noh, and S. C. Huh, "A study on the thermal characteristics of the grinding machine applied hydrostatic bearing," *Transactions of the Canadian Society for Mechanical Engineering*, vol. 39, no. 3, pp. 717–728, Sep. 2015, <https://doi.org/10.1139/tcsme-2015-0057>.
- [2] H. C. Huang, S. J. Wei, and C. M. Chen, "Design of a Hydrostatic Slide Table for Precision Surface Grinding Machine," in *Proceedings of the 14th IFToMM World Congress*, Nov. 2015, pp. 510–513, <https://doi.org/10.6567/IFToMM.14TH.WC.PS18.008>.
- [3] V. Srinivasan, "Analysis of Static and Dynamic Load on Hydrostatic Bearing with Variable Viscosity and Pressure," *Indian Journal of Science and Technology*, vol. 6, pp. 4777–4782, 2013.
- [4] K. L. Wasson and N. Elmira, "A comparison of rolling element and hydrostatic bearing spindles for precision machine tool applications," in *ASPE summer topical meeting on precision bearings and spindles*, ASPE, 2007.
- [5] N. Doshi and M. Bambhania, "Optimization of Film Thickness for Hydrostatic Circular Pad Bearing Used in V-25 vertical Turning Machine," *International Journal on Mechanical Engineering and Robotics*, vol. 1, no. 1, pp. 118–121, 2013.
- [6] R. Przybyl, "Some Aspects of Application of the Hydrostatic Bearings in Machine Tools," *Mechanics and Mechanical Engineering*, vol. 12, no. 3, pp. 243–253, 2008.
- [7] D. Chen, J. Fan, and F. Zhang, "Dynamic and static characteristics of a hydrostatic spindle for machine tools," *Journal of Manufacturing Systems*, vol. 31, no. 1, pp. 26–33, Jan. 2012, <https://doi.org/10.1016/j.jmsy.2010.11.006>.
- [8] D. Chen, M. Bonis, F. Zhang, and S. Dong, "Thermal error of a hydrostatic spindle," *Precision Engineering*, vol. 35, no. 3, pp. 512–520, Jul. 2011, <https://doi.org/10.1016/j.precisioneng.2011.02.005>.
- [9] H. Qiang, L. Lili, R. Fengzhang, and V. Alex, "Numerical Simulation and Experimental Study of the Hydrostatic Spindle with Orifice Restrictors," *The Open Mechanical Engineering Journal*, vol. 10, no. 1, Apr. 2016, <https://doi.org/10.2174/1874155X01610010079>.
- [10] S. Uberti, G. Baronio, and D. Cambiaghi, "Study & Design of a Special Test Bench for Hydrostatic Spindle Housings," in *DS 60: Proceedings of DESIGN 2010, the 11th International Design Conference*, Dubrovnik, Croatia, 2010, pp. 1729–1740.
- [11] W. Chen, Y. Sun, Y. Liang, Q. Bai, P. Zhang, and H. Liu, "Hydrostatic spindle dynamic design system and its verification," *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, vol. 228, no. 1, pp. 149–155, Jan. 2014, <https://doi.org/10.1177/0954405413497006>.
- [12] D. D. Le and T. A. Bui, "Analyzing the Effects of Lubrication Techniques on CNC Spindle Bearing Heat: An Experimental Investigation," *Engineering, Technology & Applied Science Research*, vol. 13, no. 5, pp. 11581–11585, Oct. 2023, <https://doi.org/10.48084/etasr.6146>.
- [13] D. D. Le, V. H. Pham, and T. A. Bui, "Computational and Experimental Investigation of Thermal Generation in CNC Milling Machine Spindle Bearing with the Oil-Air Lubrication Method," *Engineering, Technology & Applied Science Research*, vol. 14, no. 1, pp. 12900–12905, Feb. 2024, <https://doi.org/10.48084/etasr.6603>.
- [14] T. A. Bui, D. D. Le, D. T. Tran, M. T. Nguyen, V. T. Tran, and N. T. Bui, "Analyzing the Impact of Fly Ash Additive Ratio on Lubricant Properties," *Engineering, Technology & Applied Science Research*, vol. 13, no. 5, pp. 11547–11554, Oct. 2023, <https://doi.org/10.48084/etasr.6114>.
- [15] B. Belahcene, A. Mansri, and A. Benmoussat, "Investigation on the Rheological Behavior of Multigrade Oil under the Effect of Organic and Inorganic Impurities," *Engineering, Technology & Applied Science Research*, vol. 4, no. 6, pp. 711–713, Dec. 2014, <https://doi.org/10.48084/etasr.513>.
- [16] V. H. Pham, M. T. Nguyen, and T. A. Bui, "Oil pressure and viscosity influence on stiffness of the hydrostatic spindle bearing of a medium-sized circular grinding machine," *International Journal of Modern Physics B*, vol. 34, no. 22n24, Sep. 2020, Art. no. 2040156, <https://doi.org/10.1142/S0217979220401566>.