An Analytical Solution of Piezoelectric Energy Harvesting from Vibrations in Steel-Concrete Composite Beams subjected to Moving Harmonic Load

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ABSTRACT

Steel–concrete composite beams are ubiquitous in construction, especially in bridge building. This paper addresses the harvesting of energy from a beam subjected to a moving harmonic load using analytical methods. The harvesting is performed by attaching a thin piezoelectric patch directly to the bottom surface of the steel beam. Based on the assumptions of the Euler–Bernoulli beam theory for the relationship between displacement and deformation, the differential equation for the vibration of a beam is derived using Hamiltonian principles. A theoretical formulation is presented for the problem of harvesting energy from a harmonic moving load on a simply supported beam. The dynamic responses are determined in exact form using analytical methods, and the energy harvested from the piezoelectric material layer is calculated. The influence of the speed of the load on the energy harvesting of the piezoelectric material layer is investigated in detail.

Keywords-piezoelectric energy harvesting; beam; harmonic moving load; analytical solution

I. INTRODUCTION

Steel–concrete composite structures have many advantages as load-bearing components, and have been studied by numerous scientists and engineers. Authors in [1] studied steel–concrete composite beams using elastic connections between the concrete layer and the steel beam. Authors in [2] studied the behavior of steel–concrete composite slabs that have waste plastic added to the concrete layer. Authors in [3] used high-order beam theory to find a closed-form solution for a steel–concrete composite beam. Authors in [4] computed the bending capacity of a composite beam with an ultra-high-performance concrete slab using a nonlinear material model.

Energy harvesting from the vibration of structures has attracted the attention of many engineers and scientists. A common method of harvesting electrical energy from structural vibrations is to use piezoelectric materials. Many structures that can vibrate under loads can be used for energy harvesting, such as rigid pavement, bridges, and railroad tracks. To calculate the energy that can be harvested, it is necessary to analyze the dynamic response of the structures. Many studies have been conducted on the dynamic response of structures using analytical, semi-analytical, or experimental methods. Authors in [5] investigated the dynamic response of a sandwich beam with a flexible porous core subjected to moving mass using an analytical Navier approach. Authors in [6] investigated the natural vibration frequency of porous functionally graded beams using the trigonometric shear deformation theory. Authors in [7] employed the analytical solution for the free vibration of carbon nanotube–reinforced beams resting on a viscoelastic Pasternak foundation. Authors in [8] used a finite element method to calculate the dynamic response of continuous sandwich beams subjected to moving loads. Authors in [9] developed the User-Defined Material Model (UMAT) subroutine in the ABAQUS software to compare one-dimensional and three-dimensional models for free vibrations of functionally graded material beams with nonuniform cross-sections. Authors in [10] developed a higher-order beam element model for the stochastic free vibration of a beam with a random elastic modulus field. Authors in [11] combined the transfer matrix method and the analog beam method to study the dynamic response of steel–concrete beams with partial interaction subjected to moving loads. Energy harvesting from vibrations in beam and plate structures has been studied using both experimental and theoretical methods Authors in [12] developed an improved method to harvest energy from bridge vibrations caused by moving vehicles. Authors in [13] developed an asymptotic homogenization technique to solve a piezoelectric composite beam. Authors in [14] used the Runge–Kutta method to solve equations of motion to study and calculate the energy that can be harvested from the vibrations of a beam with a layer of piezoelectric material. Authors in [15] used the Lax–Friedrichs scheme to solve a partial differential equation for the stochastic response of the output voltage of an acoustic black hole energy harvester. A three-dimensional finite element model for the dynamics of a piezoelectric plate with initial stress was developed in [16]. In [17], the damage to concrete beams has been assessed through experiments, which included the use of piezoelectric transducers bonded with the reinforcing steel. Authors in [18] studied the nonlinear dynamic response of a cantilever beam using a Galerkin method to calculate the energy harvested from a piezoelectric patch.

This paper investigates the harvesting of piezoelectric energy from the dynamics of concrete composite steel beams subjected to moving loads. The general Hamiltonian principle according to the assumptions of Euler–Bernoulli beam theory is applied to deduce the governing equations. An analytical method is used to calculate the piezoelectric energy harvested in the beam.

II. THEORETICAL APPROACH FOR THE DYNAMICS OF A STEEL–CONCRETE COMPOSITE BEAM SUBJECTED TO HARMONIC MOVING LOAD

The subject of this study is a steel–concrete composite beam subjected to harmonic moving load as shown in Figure 1. Using the classical plate theory, the displacement fields at an arbitrary point (x, y, z) in the plate are given by:

$$\begin{aligned} U(x,z,t) &= u(x,z,t) - z\frac{\partial w}{\partial x} \\ W(x,z,t) &= w(x,t) \end{aligned} \tag{1}$$

where u and w are the displacement components on the mid-plane. The strain energy of a beam is:

$$\begin{aligned} U &= \underbrace{\frac{1}{2}\int_0^L \int_A \left\{ E_s \left[y \frac{\partial^2 w_0}{\partial x^2} \right]^2 \right\} dx}_{\text{Steel}} \\ &+ \underbrace{\frac{1}{2}\int_0^{L_c} \int_A \left\{ E_{\text{Con}} \left[y \frac{\partial^2 w_0}{\partial x^2} \right]^2 \right\} dx}_{\text{Concrete}} \end{aligned} \tag{2}$$

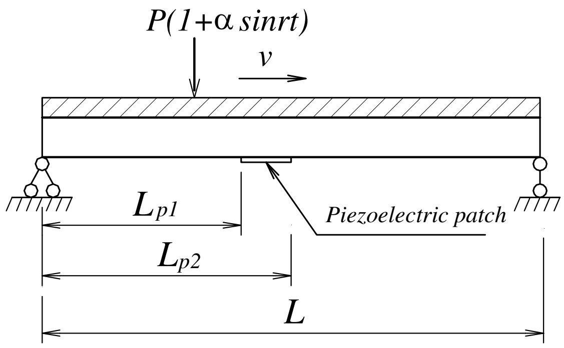


Fig. 1. Steel–concrete composite beam.

The governing differential equation of a beam subjected to a harmonic moving load can be expressed using the Hamilton principle:

$$\begin{aligned} (EI)_{\text{eq}} \frac{\partial^2 w(x,t)}{\partial x^2} + m\frac{\partial^2 w(x,t)}{\partial t^2} \\ = P\left[1 + \alpha \sin\theta(rt)\right]\delta(x - vt). \end{aligned} \tag{3}$$

The displacement of the beam is expressed as:

$$w(x,t) = \sum_{i=0}^{\infty} W_i(x) T_i(t) \tag{4}$$

where W_i is the eigenfunction for the i^{th} mode shape with simple boundary conditions:

$$W_i\ (x) = \sin(k_i\, x) = \sin\left(i\frac{\pi}{l}x\right) \tag{5}$$

Substituting (4) into (3) yields:

$$\begin{aligned} EI_i (k_i)^4 T\int_0^l (W_i)^2\, dx + m\ddot{T}_i \int_0^l (W_i)^2\, dx \\ = \int_0^l W_i P\left[1 + \alpha\sin\theta(rt)\right]\delta(x - vt)\, dx. \end{aligned} \tag{6}$$

This can be transformed into the following simple form:

$$\begin{aligned} \ddot{T}_i + \frac{(EI)_{eq}(k_i)^4}{m} T_i = \frac{2P}{ml}\sin(\Omega_i t) \\ + \frac{\alpha P}{ml}\cos\left[(\Omega_i + r)t\right] + \frac{\alpha P}{ml}\cos\left[(\Omega_i - r)t\right] \end{aligned} \tag{7}$$

The solutions of (7) include the solutions of the homogeneous equation:

$$T_{i0} = c_{1i} \sin(\omega_i t + \varphi_i) \tag{8}$$

and the individual solutions as follows:

$$T_{i1} = c_{2i} \sin(\Omega_i t) + c_{3i} \cos\left[(\Omega_i + r)t\right] + \\ + c_{4i} \cos\left[(\Omega_i - r)t\right]. \tag{9}$$

The general solution of (7) is:

$$T_i = T_{i0} + T_{i1} = c_{1i} \sin(\omega_i t + \varphi_i) + c_{2i} \sin(\Omega_i t) \\ + c_{3i} \cos\left[(\Omega_i + r)t\right] + c_{4i} \cos\left[(\Omega_i - r)t\right] \tag{10}$$

III. PIEZOELECTRIC ENERGY HARVESTING

The voltage obtained from the piezoelectric pad across the resistive load can be described by the equation given by Erturk and Inman [19]:

$$C_p \frac{dV(t)}{dt} + \frac{V(t)}{R} = -e_{31} h_{pc} b \int_{L_1}^{L_2} \frac{\partial^3 w(x,t)}{\partial x^2 \partial t} dx \tag{11}$$

where C_p is the piezoceramic patch capacitance and e_{31} is the plane piezoelectric stress, with:

$$C_p = e_{33}^S b_p \frac{L_2 - L_1}{h_p} \tag{12}$$

Equation (11) can be transformed as follows:

$$\frac{dV(t)}{dt} + \frac{V(t)}{C_p R} = \\ = \frac{-e_{31} h_{pc} b}{C_p} \int_{L_1}^{L_2} \frac{\partial^3 \left\{ \sum_{i=0}^{\infty} W_i(x) T_i(t) \right\}}{\partial x^2 \partial t} dx \\ = \frac{-e_{31} h_{pc} b}{C_p} \sum_{i=0}^{\infty} \left\{ \left. \frac{dW_i(x)}{dx} \right|_{L_1}^{L_2} \frac{dT_i(t)}{dt} \right\} = \sum_{i=0}^{\infty} \left\{ \psi_i \frac{dT_i(t)}{dt} \right\} \tag{13}$$

where:

$$\psi_i = \frac{-e_{31} h_{pc} b_p}{C_p} \left. \frac{dW_i(x)}{dx} \right|_{L_1}^{L_2} \tag{14}$$

The first-order differential equation in (13) has the solution:

$$V(t) = e^{-\frac{t}{C_p R}} \sum_{i=0}^{\infty} \left\{ \int \psi_i \frac{dT_i(t)}{dt} e^{\frac{t}{C_p R}} dt \right\} \\ = e^{-\frac{t}{C_p R}} \sum_{i=0}^{\infty} \left\{ \int \psi_i \begin{Bmatrix} c_{1i}\omega_i \sin(t + \varphi_i) + c_{2i}\Omega_i \sin(\Omega_i t) + \\ + c_{3i}(\Omega_i + r)\cos\left[(\Omega_i + r)t\right] \\ + c_{4i}(\Omega_i - r)\cos\left[(\Omega_i - r)t\right] \end{Bmatrix} e^{\frac{t}{C_p R}} dt \right\} \tag{1}$$

The harvested power due to beam vibration is:

$$P(t) = \frac{1}{R}\left[e^{-\frac{t}{C_p R}} \sum_{i=0}^{\infty} \left\{ \int \psi_i \frac{dT_i(t)}{dt} e^{\frac{t}{C_p R}} dt \right\} \right]^2 \\ = \frac{1}{R}\left[e^{-\frac{t}{C_p R}} \sum_{i=0}^{\infty} \left\{ \int \psi_i \begin{Bmatrix} c_{1i}\omega_i \sin(t + \varphi_i) + c_{2i}\Omega_i \sin(\Omega_i t) + \\ + c_{3i}(\Omega_i + r)\cos\left[(\Omega_i + r)t\right] \\ + c_{4i}(\Omega_i - r)\cos\left[(\Omega_i - r)t\right] \end{Bmatrix} e^{\frac{t}{C_p R}} dt \right\} \right]^2 \tag{16}$$

IV. NUMERICAL EXAMPLES

In this section, numerical examples are provided for a simply supported steel–concrete composite beam with the following dimensions and material properties: length L = 15 m, concrete modulus of elasticity $E_c = 30$ GPa, and steel modulus of elasticity $E_s = 200$ GPa. The moving harmonic load has parameters P = 50 kN, a = 0.1, and r = 20 r/s.

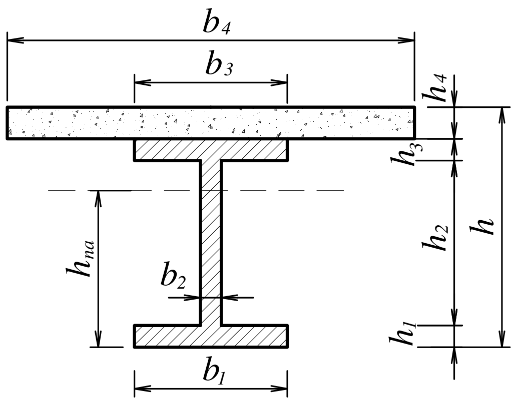


Fig. 2. Cross-section of the beam.

The dimensions of the cross-section of beam are:

$h = 1.2$ m, $h_1 = 0.02$ m, $h_3 = 0.02$ m, $h_4 = 0.15$ m,
$b_1 = 0.3$ m, $b_2 = 0.014$ m, $b_3 = 0.3$ m, $b_4 = 2.2$ m.

The piezoelectric material layer is bonded to the bottom flag of the steel beam using lead zirconate titanate (PZT-5A) piezoelectric material [20] with the properties listed in Table I. The moving load on the beam is investigated at three speeds: v = 10 m/s, 20 m/s, and 30 m/s.

TABLE I. PROPERTIES OF THE PIEZOELECTRIC MATERIAL

Description	Parameter	Numerical value
Plane piezoelectric stress constant	e_{31}	-16 C/m^2
Permittivity component	e_{33}^S	9.57 nF/m
Width of piezoceramic patch	b_p	0.05 m
Height of piezoceramic patch	h_p	0.0002 m

Figure 3 illustrates the displacement at the midpoint of the beam for various speeds of the moving load, showing that the maximum displacement does not increase significantly with the load speed. However, at lower speeds, due to the influence of the harmonic load component, the beam oscillates more rapidly around the static equilibrium position.

(a)
(b)
(c)

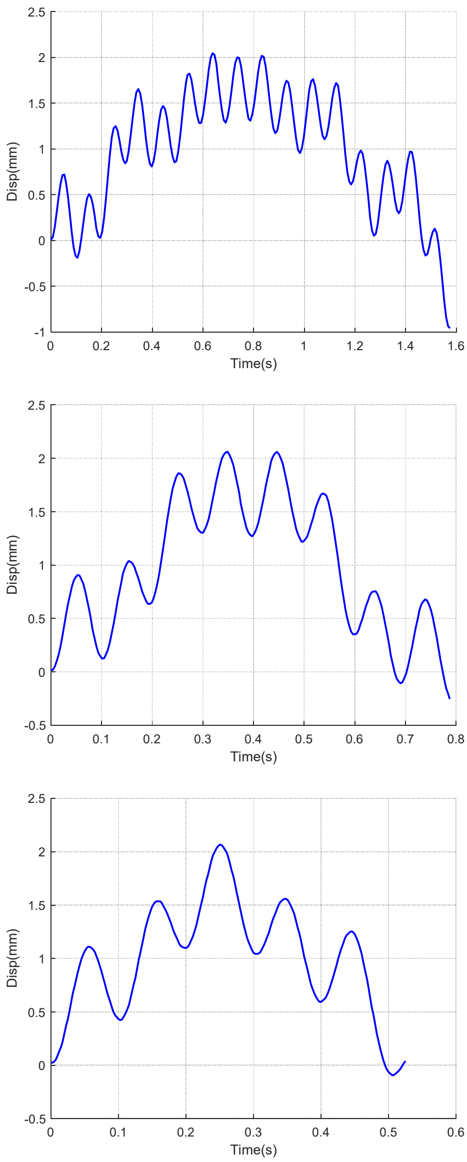


Fig. 3. Displacement at the midpoint of the beam subjected to a moving load at speed: (a) 10 m/s, (b) 20 m/s, and (c) 30 m/s.

Figure 4 shows the piezoelectric output voltage when a moving load travels on the beam at different speeds. The shape of the electric potential is similar to the displacement because it is proportional to the strain rate in the bottom fiber of the beam. Figure 5 illustrates the instantaneous power of the piezoelectric layer when the beam is subjected to a moving load at three different speeds, namely 10 m/s, 20 m/s, and 30 m/s. As the speed increases, the fluctuation in power decreases. However, the maximum power obtained across the three cases occurs at a speed of 20 m/s. Figure 6 clearly shows the general trend that as the speed increases, the oscillations of the beam around the equilibrium position grow in amplitude, increasing the energy harvested from the piezoelectric layer. In the specific calculation example here, the maximum power is achieved at a speed of 25 m/s, rather than the highest speed calculated.

(a)
(b)
(c)

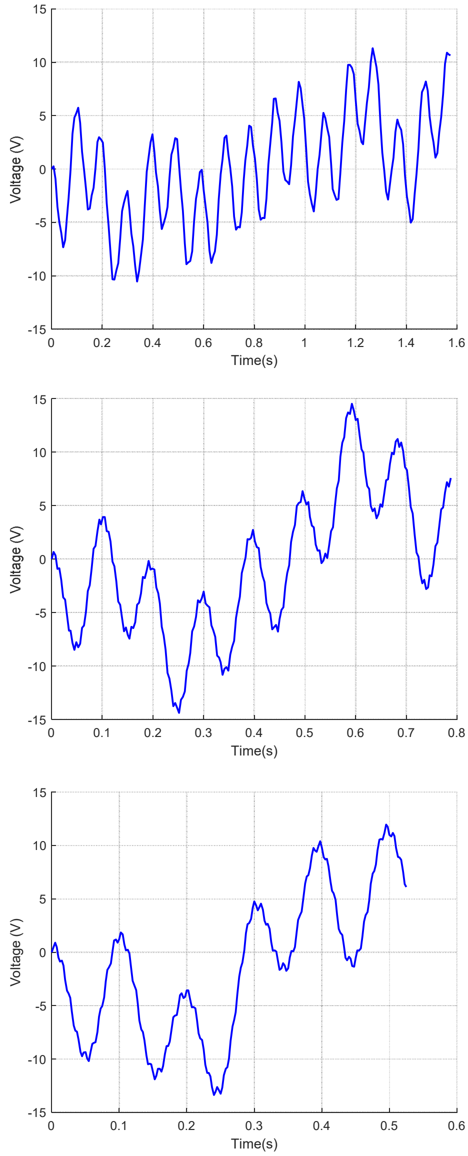
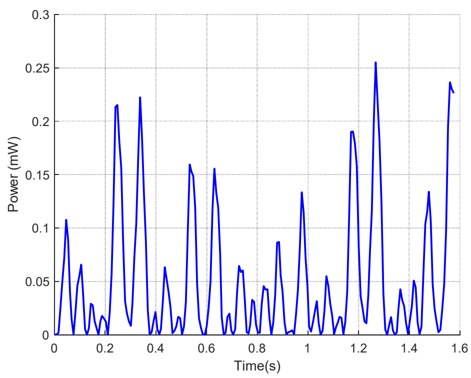


Fig. 4. Piezoelectric output voltage for a moving load at speed: (a) 10 m/s, (b) 20 m/s, and (c) 30 m/s.

(a)



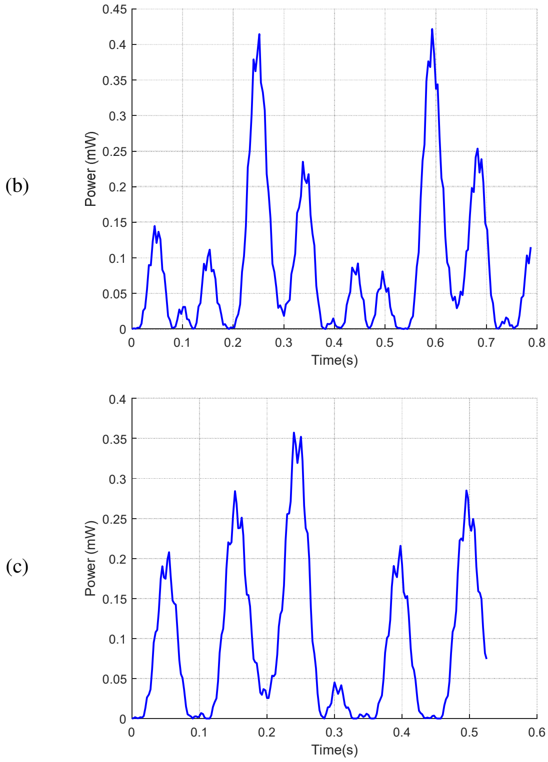


Fig. 5. Piezoelectric output power for a moving load at speed: (a) 10 m/s, (b) 20 m/s, and (c) 30 m/s.

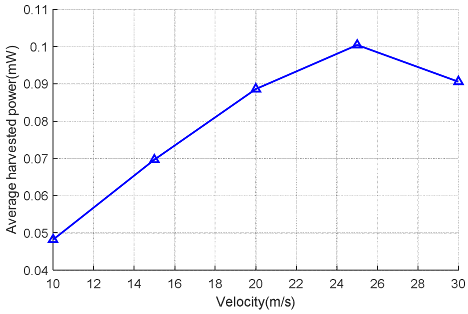


Fig. 6. Dependence of average harvested power on speed.

V. CONCLUSIONS

This study utilized analytical methods to find a closed form for the displacement of a steel–concrete composite beam subjected to a moving harmonic load. The displacement was expanded according to the mode shapes to find the exact solution of the equation of motion. The displacement expression of the beam was substituted into the governing circuit equation of the piezoelectric material layer. The governing circuit equation was solved to find the exact solution for the voltage and power of the piezoelectric patch. Numerical simulations were performed with steel–concrete composite beams subjected to a moving harmonic load at various speeds. The results show that the velocity of the moving load substantially affects the vibration of the beam and the energy harvested from the piezoelectric material layer. As the velocity of the load increases, the fluctuations in voltage reduce in frequency but increase in amplitude, leading to an increase in average harvested energy output.

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