# The Influence of Capillary Geometry on the Stiffness of Hydrostatic Bearings in Medium-Sized Cylindrical Grinding Machines: A Simulation Analysis

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**ABSTRACT**

**One of the most crucial parameters in the field of machine tool engineering is the stiffness of the machine tool spindle, as it has a direct impact on the precision and accuracy of machining. This paper presents a research on the effects of geometric parameters of capillary tubes, including diameter ($d_c$) and length ($l_c$), on the stiffness of hydrostatic bearings in machine tools. The oil pump pressure in this study was set at a range from 3 MPa to 5 MPa. The simulation results demonstrate the relationship between the capillary geometric parameters, pressure, and the stiffness of the hydrostatic spindle unit. These findings indicate a direct relationship between stiffness and pump pressure, with increases in pressure also raising lubricant temperature, which may impair cooling. The shape of the capillary directly affects the ratio of oil chamber pressure to pump pressure, which in turn affects hardness. The results of the study indicate that for spindles in machine tools, a $d_c$ ranging from 0.3 mm to 0.6 mm and a ($l_c/d_c$) ratio of between 20 and 100 are feasible based on the given stiffness requirement. It is possible to select an appropriate set of capillary parameters to achieve the best stiffness with specific requirements under the working conditions of hydrostatic bearings based on the results of the simulation analysis.**

***Keywords- hydrostatic bearing; capillary parameters; stiffness; machine tool; simulation analysis***

## I. INTRODUCTION

The spindle unit represents a significant component in the context of general machine tools. The former plays a critical role in determining the productivity and surface quality of workpieces produced in machine tools. In particular, one of the key factors in evaluating the functionality of the spindle hydrostatic unit of a machine tool is its stiffness [1]. The latter is influenced by a number of factors, including the geometric parameters of the bearing, such as the number of oil chambers, radial clearance, length and width of the bearing. However, it is also dependent on hydrostatic lubrication parameters, namely flow rate, pressure and hydrostatic bearing clearance, among which, the flow rate is an important characteristic of the hydrostatic bearing [2, 3]. The pressure and flow rate in the oil chamber are proportional to each other, therefore it is necessary to control the flow rate in order to regulate the oil film pressure in accordance with the acting load. The oil flow into the bearing can be regulated through the use of control mechanisms, including capillary tubes, flow control valves, and nozzles. Accordingly, regulating the oil flow rate will consequently modify the oil film thickness within the hydrostatic bearing or the position of the spindle center.

The control structure of oil film thickness using capillary tubes is a widely used method due to its machining simplicity and relatively low cost. Capillary tubes are manufactured deploying a variety of techniques, involving the use of standard-sized pre-machined capillary tubes with suitable hole diameter/length or the application of Electrical Discharge Machining (EDM) to create deep holes on the bearing wall connected to the oil chamber. The aforementioned control structure ensures that the stiffness of the oil film is more linear within the load range and exhibits high stability. The stiffness of the oil film is not dependent on the viscosity or temperature of the lubricating oil. Authors in [4] investigated the thermal characteristics of a grinding machine with a hydrostatic bearing spindle. An experimental and finite element analysis was carried out, to ascertain the impact of bearing temperature on the machine's thermal deformation. In [5], the replacement of rolling bearings in a cold drawing spindle with liquid hydrostatic bearings was explored. An unloading mechanism was designed with two radial and one thrust bearing, based on the spindle's mechanical characteristics and a mathematical model of the oil pad was developed, taking into consideration the effects of rotation speed on pressure. Authors in [6] presented a study describing the design and development of a specialized test bench for evaluating hydrostatic spindle housings under various experimental conditions. The bench featured a novel design with interchangeable pocket inserts, allowing for the rapid and cost-effective testing of different pocket geometries and dimensions. In [7], a novel design system for hydrostatic spindles was introduced, integrating finite element analysis and hydrostatic principles for optimized dynamic performance. This system, optimized its dynamic performance, ensuring high precision in ultra-precision machining tools.

It is important to note that the selection of oil used for lubrication can markedly influence the functionality of the hydrostatic bearing. The viscosity of the oil, which can be affected by temperature and additives, plays a crucial role in maintaining the oil film thickness and ensuring the smooth operation of the spindle [8-10]. Moreover, the design and maintenance of the oil supply system, including the pump, filter, and cooling unit, are also of critical importance for the reliable operation of the hydrostatic bearing [11]. The present study examines the influence of capillary tube parameters on the stiffness of the spindle as well as the impact of oil working conditions. The objective is to evaluate the influence of geometric parameters on stiffness, with the aim of identifying a set of suitable parameters for different spindles. The study entails a comprehensive examination of the capillary tube parameters, including the diameter, length, and the manner in which these parameters affect the stiffness of the spindle. By elucidating the relation between capillary tube parameters and spindle stiffness, engineers can develop more efficacious and dependable spindle units, thereby enhancing the overall performance of machine tools.

## II. METERIAL AND METHODS

### *A. Hydrostatic Bearing Configuration*

The spindle unit of a machine with a hydrostatic bearing structure comprises four oil chambers. The hydrostatic spindle unit functions on the basis of an external oil pressure, sufficient to elevate the shaft and guarantee lubrication between the shaft and the bearing. In [2], the hydrostatic bearing parameters are presented, including ratio between the dimensions of the bearing, (width of the edge – $a$, distance between the oil chambers - $b$, length of the bearing – $L$) and the principal structure of the oil supply system used in this study is shown in Figure 1. The oil pump generates a specific oil pressure ($p_s$) and pumps it into the oil chamber with pressure ($p_r$). The oil flow into the oil chamber is regulated by a capillary tube in order to ascertain the stability of the oil film thickness in the bearing. When considering the weight of the shaft ($W$), the external load ($p$), and the effective area of the oil chamber ($F$) with eccentricity ($e$), there is a force balance equation:

$$W = (p_3 - p_1) \times F \qquad (1)$$

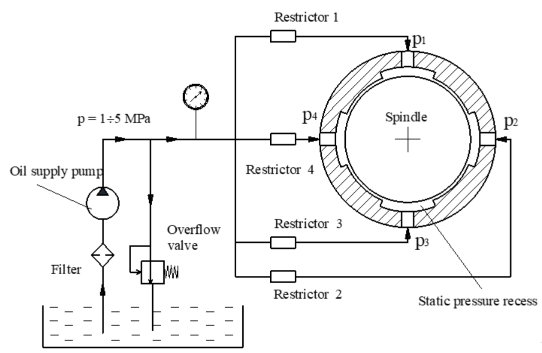


Fig. 1. Principal structure of oil supply system for hydrostatic bearings.

## B. Simulation Model

The capillary flow control structure will facilitate the laminar flow of the lubricant into the oil chamber. The geometric parameters of the capillary tube, capillary length ($l_c$) and capillary diameter ($d_c$), are two essential parameters that directly affect the capillary coefficient ($K_c$) and the ratio of the oil pressure in the oil chamber to the pump pressure. This is expressed through (2):

$$\beta = \frac{p_r}{p_s} = \frac{1}{(1+B_n K_c h^3)} \tag{2}$$

where $B_n$ is the coefficient of the bearing deformation's effect on the oil flow, with bearings that do not have an axial oil drain groove: $B_n = \frac{\pi D}{6a}$, $K_c$ is the conduction coefficient: $K_c = \frac{128 l_c}{\pi d_c^4}$, $a$ is the width of the bearing edge (mm), $D$ is the inner diameter of the bearing (mm), and $h$ is the thickness of the oil film.

A change in the geometric parameter ($l_c/d_c$) of the conduction tube will result in a corresponding alteration in the ratio of the oil pressure in the oil chamber to the pump pressure. This, in turn, will influence the stiffness of the oil film. In the case of the hydrostatic spindle bearing in a cylindrical grinding machine with length ($L$) and a distance between two adjacent oil chambers ($b$), the following selection criteria can be applied: $L = 0.8D$, and $b = a = 0.25L$. In order to ensure the load capacity of the spindle assembly when upgrading to use hydrostatic bearings on the 3K12 cylindrical grinding machine, it is necessary to select an appropriate grinding wheel technology that matches the structure of the machine, the diameter of the spindle, and the diameter of the grinding wheel. This should be done in such a way that the parameters of the hydrodynamic bearings on the machine are maintained. The diameter of the grinding spindle is 70 mm, the rotational speed is 3000 rpm, and the total length of the spindle is 535 mm, certifying complete wet lubrication, thus $L = 56$ mm. The lower limit ($L$) of the clearance is 15 μm, while the upper limit ($U$) is 22.5 μm. Based on experience, the oil supply pressure ($p_s$) must be within the range from 3 MPa to 5 MPa to limit thermal deformations in the bearing.

Typically, to establish the oil wedge, the geometric parameters including dc are varied in the range from 0.1 mm to 1 mm and the ratio ($l_c/d_c$) is in the range from 10 to 200, then the pressure ratio ($\beta$) is between 0 and 1, which determines the stiffness of the bearing assembly. In a steady state, the oil film thickness is equal to the limit clearance ($h = h_o$). In order to study the operating characteristics of the hydrostatic bearing according to the geometric parameters of the delivery pipe, the equation for the pressure ratio between the oil chamber pressure and the pump pressure, after substituting the coefficients of influence of the bearing deformation on the oil flow ($B_n$) and the delivery ratio ($K_c$), transforming the variable ($l_c/d_c$), is:

$$\beta = \frac{1}{1+\frac{64 D h_0^3}{3 a d_c^3}\left(\frac{l_c}{d_c}\right)} \tag{3}$$

The stiffness of the spindle unit ($J_0$) is calculated using:

$$J_0 = \frac{p_s L D}{h_0} \frac{3n^2}{2\pi} \frac{\beta\left(1-\frac{a}{L}\right)\sin^2\left(\frac{\pi}{n}\right)}{z+1+2\gamma\sin^2\left(\frac{\pi}{n}\right)} \tag{4}$$

or:

$$J_0 = \frac{p_s L D}{h_0} J_n \tag{5}$$

where $n$ is the number of oil chambers and $\gamma = \frac{N\alpha(L-a)}{\pi D b}$, is the bearing shape factor. For a four-chamber oil bearing with a capillary tube control system for oil film thickness:

$$J_n = 9\frac{\beta(1-\beta)}{\pi+2.4(1-\beta)} \tag{6}$$

so:

$$J_0 = 9LD\frac{p_s}{h_0}\frac{\beta(1-\beta)}{\pi+2.4(1-\beta)} \tag{7}$$

In this study, MATLAB software was utilized for simulation analysis. The purpose of this study is to show the behavior of the stiffness of the oil film under different conditions. The parameters considered are crucial in determining the performance of the hydrostatic bearing. By varying these parameters within the specified ranges, the influence of each of them on the stiffness of the oil film can be observed. This information can then be used to optimize the design and operation of hydrostatic bearings for machine tools.

# III. RESULT AND DISCUSSION

A capillary tube with a diameter of $d_c = 0.1$ mm, exhibits an increasing trend in stiffness as pump pressure increases as observed in Figure 2. This trend is evident in the contour lines for both scenarios, where the bearing is at its upper ($U$) and lower ($L$) limit clearances. The stiffness of a typical machine tool spindle assembly generally falls within the range from 250 N/μm to 500 N/μm. For grinding machine spindles, the stiffness requirement is even higher, ranging from 300 N/μm to 500 N/μm. Based on the stiffness contour lines, it becomes apparent that a minimum pump pressure of approximately 3.8 MPa is necessary. This corresponds to a capillary ratio ($l_c/d_c$) between 10 and 15 for the lower limit ($L$) clearance $h_0 = 15$ μm. In the case of the upper limit ($U$) clearance $h_0 = 22.5$ μm, the maximum achievable stiffness of the hydrostatic film is only 102 N/μm, which is significantly lower than the required stiffness range. Consequently, there are no suitable capillary parameters within the pressure range from 3 MPa to 5 MPa for this scenario.

In the case of a capillary tube with a diameter of 0.2 mm and a lower limit ($L$) clearance of 15 μm, the stiffness of the hydrostatic oil film exhibits a strong dependence on both the capillary ratio ($l_c/d_c$) and the pump pressure ($p_s$). As the pump pressure increases, the stiffness of the oil film also increases, as evidenced by the upward trend in the contour lines in Figure 3. The maximum achievable stiffness in this case is approximately 690 N/μm, which occurs at a pump pressure of $p_s = 5$ MPa, and a capillary ratio of $\frac{l_c}{d_c} = 17$. It is, however, important to note that an increase in pump pressure also leads to an increase in the temperature of the lubricating oil, which can reduce its cooling capacity.

(a)

(b)

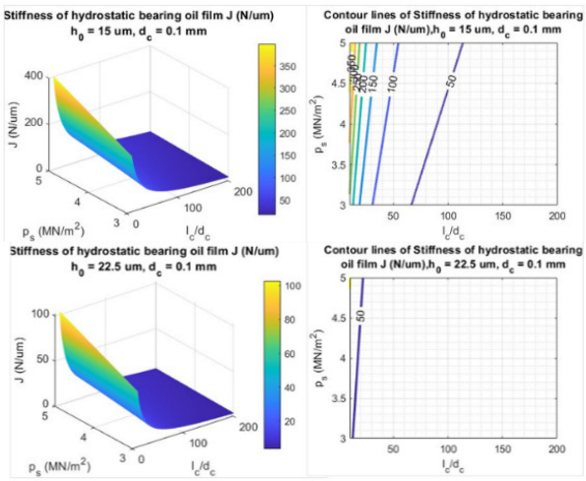


Fig. 2. Hydrostatic oil film stiffness in the case of $d_c$ = 0.1 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

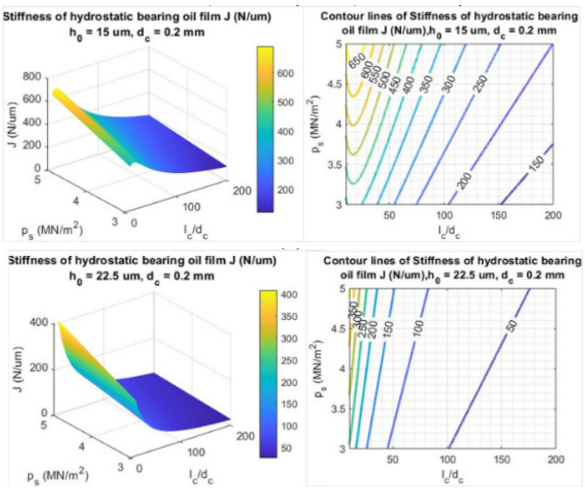


Fig. 3. Hydrostatic oil film stiffness in the case of $d_c$ = 0.2 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

Consequently, in order to achieve a desired stiffness of 500 N/µm, a minimum pump pressure of 3.6 MPa is sufficient at $\frac{l_c}{d_c} = 17$. This ensures a balance between achieving the required stiffness and maintaining optimal operating conditions. For the upper limit ($U$) clearance of 22.5 µm, the maximum achievable stiffness of the hydrostatic oil film is significantly lower than in the previous case. The maximum achievable stiffness in this case is 102 N/µm, which is considerably below the required stiffness range from 250 N/µm to 500 N/µm typically observed in machine tool spindles. Moreover, the analysis indicates that there are no suitable capillary parameters within the pressure range from 3 MPa to 5 MPa that can achieve the desired stiffness for this clearance condition. This suggests that alternative design solutions may be necessary for applications requiring higher stiffness values at larger clearances.

Similarly, this section presents an analysis of the stiffness of the bearing for capillary diameters ranging from 0.3 mm to 1 mm. The analysis exhibits a reduction in the maximum attainable stiffness as the capillary diameter increases. This is evidenced by the stiffness contour plots portrayed in Figures 4-11. This phenomenon can be understood by considering the role of the capillary diameter in determining the oil film stiffness. The capillary acts as a restrictor, regulating the oil flow into the bearing clearance. A smaller capillary diameter results in a restriction of the oil flow, which in turn leads to a higher-pressure buildup within the clearance. This increased pressure results in a higher stiffness of the oil film. Conversely, as the capillary diameter increases, the restriction on the oil flow lessens, leading to a lower pressure buildup and, consequently, a reduced stiffness. This phenomenon can be explained by the observed decrease in the maximum attainable stiffness with an increasing capillary diameter.

(a)

(b)

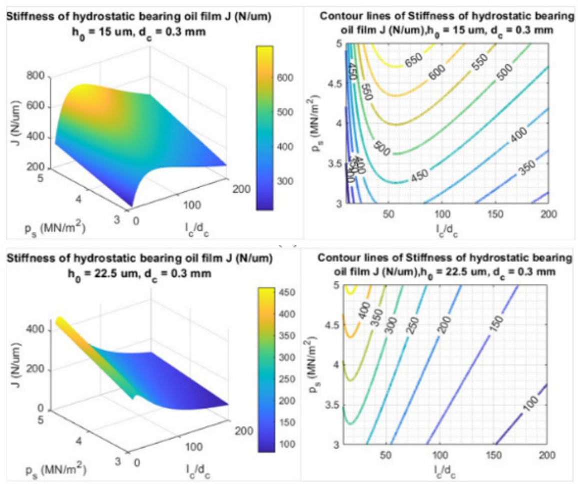


Fig. 4. Hydrostatic oil film stiffness in the case of $d_c$ = 0.3 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

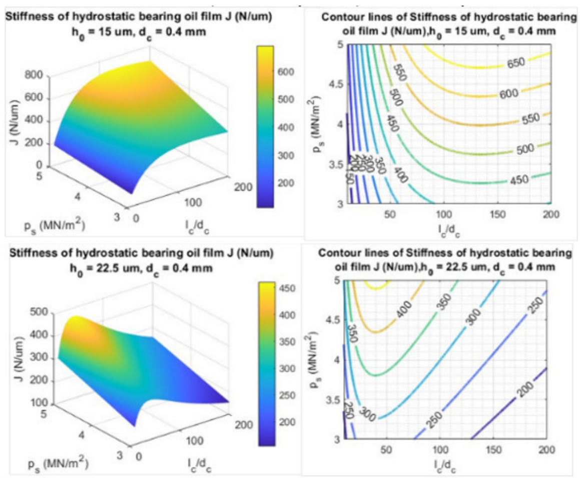


Fig. 5. Hydrostatic oil film stiffness in the case of $d_c$ = 0.4 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

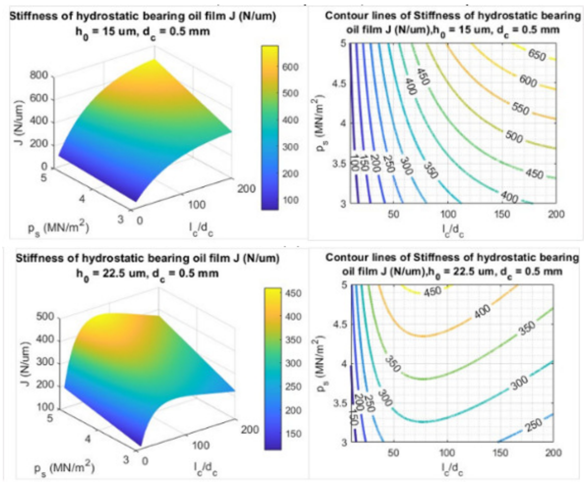


Fig. 6. Hydrostatic oil film stiffness in the case of $d_c$ = 0.5 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

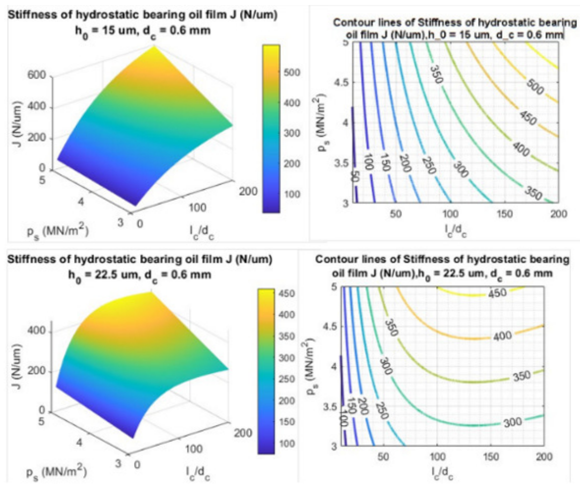


Fig. 7. Hydrostatic oil film stiffness in the case of $d_c$ = 0.6 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

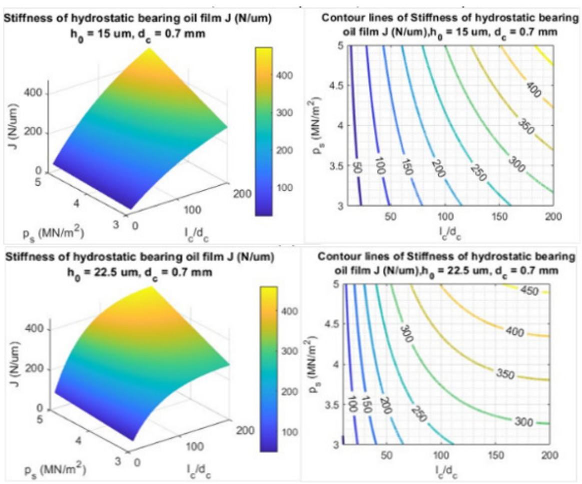


Fig. 8. Hydrostatic oil film stiffness in the case of $d_c$ = 0.7 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

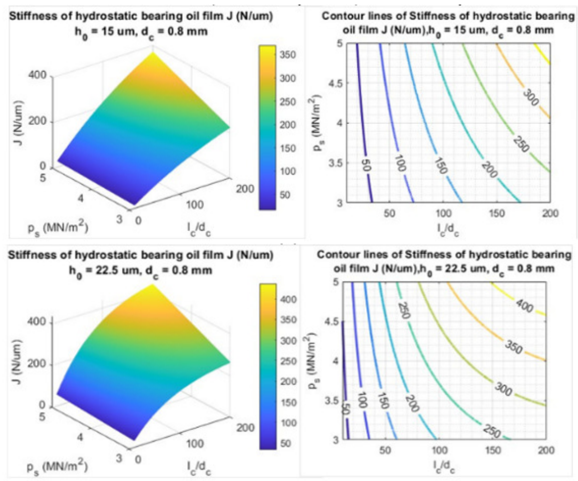


Fig. 9. Hydrostatic oil film stiffness in the case of $d_c$ = 0.8 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

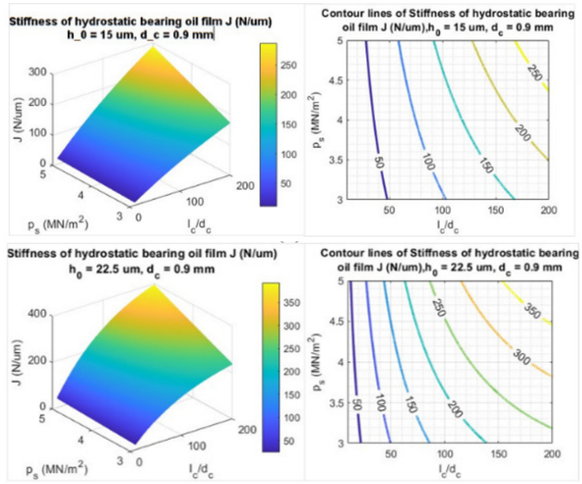


Fig. 10. Hydrostatic oil film stiffness in the case of $d_c$ = 0.9 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

(a)

(b)

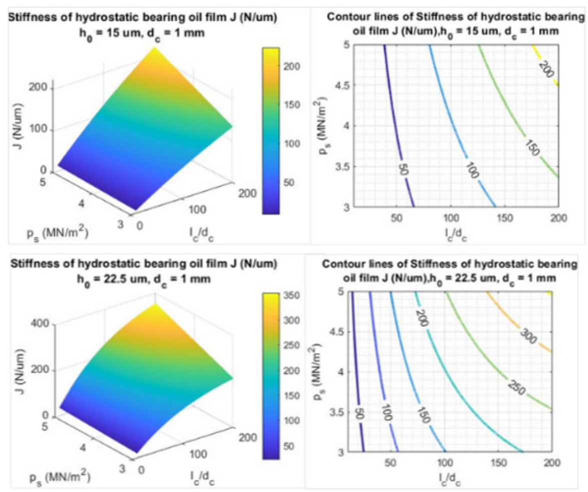


Fig. 11. Hydrostatic oil film stiffness in the case of $d_c$ = 1 mm, limiting clearance: (a) $h_o$ = 15 µm, (b) $h_o$ = 22.5 µm.

For a diameter of 0.3 mm as displayed in Figure 4, the maximum stiffness can be estimated to reach approximately 690 N/μm at a pressure of 5 MPa and $\frac{l_c}{d_c} = 54$. In this case, any pump pressure value ($p_s$) can be selected in conjunction with an appropriate ($l_c/d_c$). For the upper limit clearance, the maximum stiffness can reach approximately 460 N/μm at a pressure of 5 MPa, $\frac{l_c}{d_c} = 17$, and a range of $\frac{l_c}{d_c} < 65$ with a minimum pump pressure of about 3.2 MPa. This is necessary to achieve the required stiffness. With a capillary tube diameter of 0.4 mm (Figure 5), it is apparent that to achieve the required stiffness, the range of pump pressure and the guide ratio is relatively broad. In order to attain the requisite stiffness, it is necessary to ensure that the minimum ($l_c/d_c$) ratio is greater than 20. The maximum stiffness can be obtained at a pressure of 5 MPa and a ($l_c/d_c$) ratio of 134, with a stiffness of approximately 690 N/μm. For the upper limit ($U$) clearance, the ($l_c/d_c$) ratio should be less than 155, and the maximum stiffness can reach approximately 460 N/μm at a pressure of 5 MPa, $\frac{l_c}{d_c} = 40$. In the case of a capillary tube with a diameter of 0.5 mm (Figure 6), the maximum stiffness value ($J$) is 679 N/μm at $h_0 = 15\ \mu\text{m}$, $p_s = 5$ MPa, $\frac{l_c}{d_c} = 200$, and ($J$) is 460 N/μm at $h_0 = 22.5\ \mu\text{m}$, $p_s = 5$ MPa, $\frac{l_c}{d_c} = 77$. In order to achieve the requisite stiffness, a minimum ($l_c/d_c$) ratio of greater than 35 is necessary at $h_0 = 15\ \mu\text{m}$, while a minimum ($l_c/d_c$) ratio of greater than 20 is required at $h_0 = 22.5\ \mu\text{m}$. Furthermore, the maximum stiffness value ($J$) is 589 N/μm at $h_0 = 15\ \mu\text{m}$, $p_s = 5$ MPa, $\frac{l_c}{d_c} = 200$, and 460 N/μm at $h_0 = 22.5\ \mu\text{m}$, $p_s = 5$ MPa, $\frac{l_c}{d_c} = 134$, respectively, for a capillary tube diameter of 0.6 mm (Figure 7). In order to attain the required stiffness, a minimum ($l_c/d_c$) ratio of greater than 65 is needed in the case of the lower limit ($L$) clearance, while a minimum ($l_c/d_c$) ratio of greater than 35 is required in the case of the upper limit ($U$) clearance.

In Figures 8-11, it is obvious that as the capillary tube diameter increases, the maximum stiffness decreases. This is particularly noticeable for larger capillary tube diameters, such as 0.7 mm, 0.8 mm, 0.9 mm, and 1 mm. The contour plots of the oil film stiffness values indicate that to achieve the range of oil film stiffness from 300 μm to 500 μm, the guide ratio ($l_c/d_c$) must have relatively large values, typically exceeding 100. Nevertheless, when the ($l_c/d_c$) ratio is smaller than 0.3 mm or larger than 0.6 mm, the oil film stiffness fails to meet the requisite specifications throughout the surveyed parameter range. With ($l_c/d_c$) values spanning from 0.3 to 0.6, it is possible to select the geometric parameters of the delivery pipe and the corresponding pump pressure in order to obtain the desired stiffness, which is optimal for the design requirements.

Nevertheless, an increase in pump pressure will result in an elevated lubricating oil temperature and a concomitant reduction in the cooling capacity of the bearing. An increase in the ($l_c/d_c$) ratio will lead to the necessity of a longer delivery pipe, which in turn will present difficulties in the manufacturing of small-diameter, deep holes for oil delivery in the hydrostatic bearing. Therefore, the selection of appropriate parameters and pump pressure should be based on the practical manufacturing conditions. The stiffness analysis of the hydrostatic bearing with a capillary-compensated oil film demonstrates the impact of capillary diameter and pressure conditions on stiffness.

## IV. CONCLUSION

This study presents a simulation analysis to investigate the influence of characteristic parameters, specifically the diameter ($d_c$) and length ($l_c$) of the capillary tube, as well as their ratio, on the stiffness of the hydrostatic bearing within the pressure range from 3 MPa to 5 MPa. The formation of film thickness using the capillary tube is dependent on the geometric parameters of the capillary tube, which consequently affects the pressure ratio between the oil chamber and the pump pressure. This, in turn, influences the stiffness of the hydrostatic bearing. Specifically, machine tool spindles must comply with strict stiffness standards to ensure the precision of machining processes. Based on the calculated simulation and the contour plot, the optimal range of the capillary tube diameter ($d_c$) can be selected between 0.3 mm and 0.6 mm, and the ($l_c/d_c$) ratio should be within the range from 20 to 100 to achieve the optimal stiffness of the oil film. This range is feasible for spindles of machine tools.

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