Overcoming the Limitations of the RAPS Method by identifying Alternative Data Normalization Methods

Nguyen Van Thien
Hanoi University of Industry, Vietnam
nguyenvanthien@haui.edu.vn

Hoang Tien Dung
Hanoi University of Industry, Vietnam
tiendung@haui.edu.vn

Do Duc Trung
Hanoi University of Industry, Vietnam
doductrung@haui.edu.vn (corresponding author)

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ABSTRACT

This study proposes a new approach to improve the performance of the Ranking Alternatives by Perimeter Similarity (RAPS) method in Multi-Criteria Decision-Making (MCDM). RAPS has attracted attention but encounters difficulties when handling zero values in the decision matrix. This study suggests using alternative data normalization methods and assesses their suitability when combined with RAPS in various scenarios. The results identified three additional normalization methods that are appropriate for integration with RAPS. These findings provide a theoretical basis and specific guidelines for selecting data normalization methods when applying RAPS in MCDM.

Keywords-MCDM; RAPS method; data normalization

I. INTRODUCTION

Multi-Criteria Decision-Making (MCDM) techniques are used to identify the best alternative among a set of available options, each characterized by multiple criteria [1]. Although there is no exact statistic on the number of MCDM methods, it is known that they have exceeded 200 and continue to increase [2]. Some MCDM methods do not require data normalization, such as FUCA [3, 4] and SRP [5], while some others do not require weighting criteria, such as PSI [6] and PEG [7]. There are even methods that neither require weighting the criteria nor data normalization, such as CURLI [8]. However, most other MCDM methods require two tasks: weighting the criteria and normalizing the data [9, 10]. Data normalization involves converting criteria with different units into a unitless format to facilitate comparisons among alternatives, and most MCDM methods incorporate at least one data normalization method [11, 12]. However, some normalization methods cannot be used in certain exceptional cases, such as when a criterion's value in an alternative is zero or negative [13]. This requires the use of alternative data normalization methods [13]. However, when using an MCDM method to rank alternatives, the rankings can change significantly depending on the data normalization method used [14, 15]. Thus, before adopting an alternative data normalization method, it is essential to investigate its suitability for integration with the specific MCDM method [13-15]. This approach has been followed in numerous studies by identifying suitable data normalization methods for combination with MCDM, such as MAROCS [13], CODAS [16, 17], PIV [18], Simple WISP [19], ROV [20], CRADIS [21], MACONT [22], TOPSIS [23], and others.

RAPS is a recently proposed MCDM method [24]. This method ranks alternatives using a unique approach, where the score of each alternative is described through perimeter similarity. Specifically, each alternative is considered as the perimeter of a shape, and the score of an alternative is the result of the perimeter of that shape divided by the perimeter of the shape of the optimal alternative. This approach is entirely different from all other MCDM methods [24]. Perhaps due to this difference, this method has been quickly adopted in various fields [25-29]. However, none of the studies applying the RAPS method have considered the case where the decision matrix contains zero elements. When this occurs, the existing data normalization method within RAPS becomes unusable. To use the RAPS method in such cases, it is necessary to find alternative data normalization methods. This necessity is the motivation for conducting this research.

II. MATERIALS AND METHODS

A. RAPS Method

Let m be the number of alternatives to be ranked, n be the number of criteria for each alternative, and x_{ij} be the value of criterion j in alternative i. Criteria where higher values are better are denoted by B, and criteria where lower values are better are denoted by C. To rank the alternatives using the RAPS method, the following steps should be applied sequentially [24]:

- Step 1**:** Normalize the data using (1) and (2). This is a type of linear normalization, denoted as N1.

$$r_{ij} = \frac{x_{ij}}{\max(x_{ij})}, \quad if \quad j \in B \tag{1}$$

$$r_{ij} = \frac{\min(x_{ij})}{x_{ij}}, \quad if \quad j \in C \tag{2}$$

However, it is evident that if there is a zero x_{ij} value, method N1 cannot be used. Therefore, it is necessary to identify suitable data normalization methods that can be combined with RAPS and used when x_{ij} is zero.

- Step 2: Let w_j be the weight of criterion j. Calculate the normalized values considering the weights of the criteria using:

$$u_{ij} = w_j \cdot r_{ij} \tag{3}$$

- Step 3: Determine the optimal alternative using (4), where each element of the optimal alternative is determined according to (5).

$$\begin{cases} Q = \{q_1, q_2, \dots, q_j\} \\ \quad j = 1,2, \dots, n \end{cases} \tag{4}$$

$$\begin{cases} q_j = max\left(u_{ij} \,/\, 1 \le j \le n\right) \\ \quad \forall i \in [1,2, \dots, m] \end{cases} \tag{5}$$

- Step 4: Decompose the optimal alternative into two subsets according to:

$$Q = Q^{max} \cup Q^{min} \tag{6}$$

- If the number of criteria of type B is k, then h=n-k is the number of criteria of type C. The optimal alternative is then decomposed according to:

$$\begin{cases} Q = \{q_1, q_2, \dots, q_k\} \cup \{q_1, q_2, \dots, q_h\} \\ \qquad k + h = n \end{cases} \tag{7}$$

- Step 5: Decompose the alternative solutions according to (8) and (9).

$$\begin{cases} U_i = U_i^{max} \cup U_i^{min} \\ \forall i \in [1,2, \dots, m] \end{cases} \tag{8}$$

$$\begin{cases} U_i = \{u_{i1}, u_{i2}, \dots, u_{ik}\} \cup \{u_{i1}, u_{i2}, \dots, u_{ih}\} \\ \qquad \forall i \in [1,2, \dots, m] \end{cases} \tag{9}$$

- Step 6: Calculate the magnitude of the components.
  - For the optimal alternative, apply (10) and (11):

$$Q_k = \sqrt{q_1^2 + q_2^2 + \dots + q_k^2} \tag{10}$$

$$Q_h = \sqrt{q_1^2 + q_2^2 + \dots + q_h^2} \tag{11}$$

  - For the alternative solutions, apply (12) and (13):

$$\begin{cases} U_{ik} = \sqrt{u_{i1}^2 + u_{i2}^2 + \dots + u_{ik}^2} \\ \quad \forall i \in [1,2, \dots, m] \end{cases} \tag{12}$$

$$\begin{cases} U_{ih} = \sqrt{u_{i1}^2 + u_{i2}^2 + \dots + u_{ih}^2} \\ \quad \forall i \in [1,2, \dots, m] \end{cases} \tag{13}$$

- Step 7: Calculate the perimeter of the optimal alternative using:

$$P = Q_k + Q_h + \sqrt{Q_k^2 + Q_h^2} \tag{14}$$

- Step 8: Calculate the perimeter of alternative i using:

$$P_i = U_{ik} + U_{\text{!}} + \sqrt{U_{ik}^2 + U_{ih}^2} \tag{15}$$

- Step 9: The score of alternative i is calculated using (16). The alternative with the highest score is the best, and vice versa.

$$PS_i = \frac{P_i}{P}, \quad \forall i \in [1,2, \dots, m] \tag{16}$$

B. Data Normalization Methods

As mentioned above, method N1 cannot be used if there is a value x_{ij} that is zero. Therefore, identifying alternative data normalization methods to replace N1 is necessary. According to [14], there are more than 10 commonly used data normalization methods for normalizing data when applying MCDM methods, but only four of them can be used in cases where the value of x_{ij} is zero. These include the Weitendorf normalization, vector normalization, z-score normalization, and the enhanced accuracy normalization method. These methods are denoted as N2, N3, N4, and N5, respectively.

- The N2 method uses (17) for criteria of type B and (18) for criteria of type C.

$$r_{ij} = \frac{x_{ij} - min(x_{ij})}{max(x_{ij}) - min(x_{ij})}, \quad if \quad j \in B \tag{17}$$

$$r_{ij} = \frac{max(x_{ij}) - x_{ij}}{max(x_{ij}) - min(x_{ij})}, \quad if \quad j \in C \tag{18}$$

- The N3 method uses (19) for criteria of type B and (20) for criteria of type C.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij})^2}} \tag{19}$$

$$r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} (x_{ij})^2}} \tag{20}$$

- The N4 method uses (21) for criteria of type B and (22) for criteria of type C.

$$r_{ij} = \frac{x_{ij} - \frac{\sum_{i=1}^{m} x_{ij}}{m}}{\sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij} - \frac{\sum_{i=1}^{m} x_{ij}}{m}\right)^2}{m}}} \quad (21)$$

$$r_{ij} = -\frac{x_{ij} - \frac{\sum_{i=1}^{m} x_{ij}}{m}}{\sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij} - \frac{\sum_{i=1}^{m} x_{ij}}{m}\right)^2}{m}}} \quad (22)$$

- The N5 method uses (23) for criteria of type *B* and (24) for criteria of type *C*.

$$r_{ij} = 1 - \frac{max(x_{ij}) - x_{ij}}{\sum_{i=1}^{m}(max(x_{ij}) - x_{ij})} \quad (23)$$

$$r_{ij} = 1 - \frac{x_{ij} - min(x_{ij})}{\sum_{i=1}^{m}(x_{ij} - min(x_{ij}))} \quad (24)$$

III. IDENTIFYING SUITABLE DATA NORMALIZATION METHODS FOR THE RAPS METHOD

This section investigates the suitability of data normalization methods for RAPS in various scenarios. Differences between these scenarios include variations in the application fields, the number of alternatives to be ranked in each case, the number of criteria in each case, and the form of criteria in each case. Selecting cases with such variations is intentional to ensure the objectivity of the results obtained.

A. Case 1

This case involves a ranking of ten types of fertilizers for mushroom cultivation. Three criteria are used to describe each fertilizer, including the percentage of nitrogen, phosphorus, and potassium contents. In mushroom cultivation, nitrogen, phosphate, and potassium play crucial roles in their development. Nitrogen is vital for cell growth, promoting rapid growth and protein synthesis. Phosphate is necessary for energy metabolism and cellular structure development, supporting root growth and cell division. Potassium regulates water balance and enzyme activities, enhancing stress and disease resistance, thus improving mushroom quality and yield. Higher levels of these nutrients are beneficial for optimal mushroom growth. All three criteria are type *B*. Table I shows the values and weights of the criteria for each alternative. Fertilizer ranking has also been previously performed using the RAM method [30]. First, data normalization is performed using the N1 method. The ranking sequence of the options using the RAPS method is as follows:

- Apply (1) and (2) to calculate the normalized values as shown in Table II.
- Apply (3) to calculate the normalized values considering the weights of the criteria, as shown in Table III.
- Applying (4) to (11), the values of Q_k and Q_h are calculated as 0.75735 and 0, respectively. The values of U_{ik}, U_{ih}, P, P_i, and PS_i are determined by applying the corresponding formulas (12)-(16), with the results shown in Table IV. The ranking of the options according to the values of PS_i is also performed in the last column of this table.

TABLE I. FERTILIZER TYPES FOR MUSHROOM CULTIVATION [30]

Manure	N (%)	P (%)	K (%)
Fresh cow manure	0.6	0.4	0.5
Dried cow manure	1.2	2	2.1
Fresh chicken manure	0.9	0.5	0.5
Dried chicken manure	1.6	1.8	2
Fresh pig manure	0.6	0.3	0.4
Dried pig manure	2.2	2.1	1
Fresh horse manure	0.6	0.3	0.5
Fresh rabbit manure	2.4	1.4	0.6
Fresh turkey manure	1.3	0.7	0.5
Fresh earthworm castings	0.91	1.14	0.21
Weight	1/3	1/3	1/3

TABLE II. TABLE II. NORMALIZED VALUES

Manure	N	P	K
Fresh cow manure	0.2500	0.1905	0.2381
Dried cow manure	0.5000	0.9524	1.0000
Fresh chicken manure	0.3750	0.2381	0.2381
Dried chicken manure	0.6667	0.8571	0.9524
Fresh pig manure	0.2500	0.1429	0.1905
Dried pig manure	0.9167	1.0000	0.4762
Fresh horse manure	0.2500	0.1429	0.2381
Fresh rabbit manure	1.0000	0.6667	0.2857
Fresh turkey manure	0.5417	0.3333	0.2381
Fresh earthworm castings	0.3792	0.5429	0.1000

TABLE III. VALUES OF U_{IJ}

Manure	N	P	K
Fresh cow manure	0.0833	0.0635	0.0794
Dried cow manure	0.1667	0.3175	0.3333
Fresh chicken manure	0.1250	0.0794	0.0794
Dried chicken manure	0.2222	0.2857	0.3175
Fresh pig manure	0.0833	0.0476	0.0635
Dried pig manure	0.3056	0.3333	0.1587
Fresh horse manure	0.0833	0.0476	0.0794
Fresh rabbit manure	0.3333	0.2222	0.0952
Fresh turkey manure	0.1806	0.1111	0.0794
Fresh earthworm castings	0.1264	0.1810	0.0333

Thus, the ranking of the options using the RAPS method combined with the N1 normalization method has been completed. Figure 1 shows the ranking results of the options using the N2, N3, N4, and N5 methods for data normalization. This figure also includes the ranking results of the options using the RAM method [30].

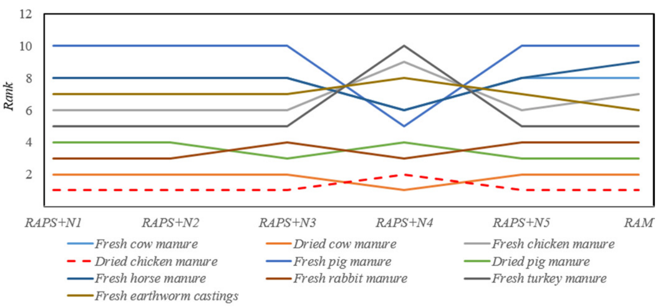

Fig. 1. Ranking of fertilizer types for mushroom cultivation.

TABLE IV. CASE 1: PARAMETERS IN RAPS AND RANKING OF THE OPTIONS

Manure	U_{ik}	U_{ih}	P	P_i	PS_i	Rank
Fresh cow manure	0.1151	0	1.1547	0.2302	0.1993	8
Dried cow manure	0.3727			0.7454	0.6455	2
Fresh chicken manure	0.1481			0.2961	0.2565	6
Dried chicken manure	0.3875			0.7750	0.6712	1
Fresh pig manure	0.1048			0.2095	0.1815	10
Dried pig manure	0.3443			0.6886	0.5964	4
Fresh horse manure	0.1151			0.2302	0.1993	8
Fresh rabbit manure	0.3467			0.6933	0.6005	3
Fresh turkey manure	0.1972			0.3945	0.3416	5
Fresh earthworm castings	0.1307			0.2614	0.2264	7

Observing Figure 1, when using the RAPS method to rank the fertilizers, the rankings are quite stable when using data normalization methods N1, N2, N3, and N5. The plus sign is used to describe the combination of the RAPS method with data normalization methods, for example, RAPS+N1 means the combination of the RAPS method with N1 normalization. There is no difference in rankings when using RAPS+N1 and RAPS+N2. Similarly, the rankings are identical when using RAPS+N3 and RAPS+N5. Fertilizer types ranked first (dried chicken manure), second (dried cow manure), fifth (fresh turkey manure), eighth (fresh cow manure), and tenth (fresh pig manure) are completely identical when using RAPS+N1, RAPS+N2, RAPS+N3, RAPS+N5, and the RAM method [30]. This indicates that in this case, the data normalization methods N1, N2, N3, and N5 are identified as suitable for the RAPS method. Conversely, using RAPS+N4 produces significantly different fertilizer rankings compared to the other combinations and the RAM method. This confirms that N4 is not suitable for combination with the RAPS method.

B. *Case 2*

This case ranks seven different robot types. Each robot type is characterized by five criteria, including load capacity, maximum speed of the actuator, repeatability, memory capacity, and range of the actuator. These criteria are denoted as *C1*, *C2*, *C3*, *C4*, and *C5*, where only *C3* is of type *C*, and the remaining four criteria are of type *B*. *C1* refers to the maximum weight the robot can handle, *C2* denotes the speed of movement of the robot's actuators, *C3* measures the precision of the robot in returning to a position, *C4* relates to the amount of data the robot can store, and *C5* determines the extent of movement of the robot's actuators. Table V shows the values and weights of the criteria for each robot. The robots were also previously ranked using the CODAS method [31]. Figure 2 shows the ranking results of the robots using the RAPS method with the five data normalization methods and the CODAS method.

TABLE V. ROBOT TYPES [31]

Robots	*C1* (kg)	*C2* (mm/s)	*C3* (mm)	*C4* (steps)	*C5* (mm)
ASEA-IRB 60/2 Cincinnati	60	0.4	2540	500	990
Milacrone T3-726 Cybotech V15	6.35	0.15	1016	3000	1041
Electric Robot Hitachi America	6.8	0.1	1727.2	1500	1676
Process Robot Unimation PUMA	10	0.2	1000	2000	965
Unimation PUMA 500/600	2.5	0.1	560	500	915
United States Robots Maker 110	4.5	0.08	1016	350	508
Yaskawa Electric Motoman L3C	3	0.1	1778	1000	920
Weight	0.036	0.326	0.192	0.326	0.12

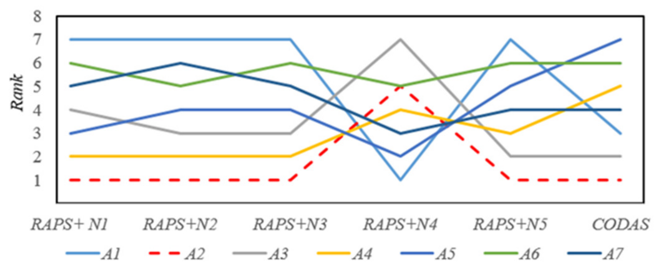

Fig. 2. Ranking of robots.

In this case, when using RAPS+N1, RAPS+N2, RAPS+N3, and RAPS+N5, all show that the ASEA-IRB 60/2 Cincinnati is ranked seventh (the lowest ranking) and the Milacrone T3-726 Cybotech V15 is ranked first. It should be noted that using the CODAS method also ranked the Milacrone T3-726 Cybotech V15 as the best robot [31]. This result indicates that, in this case, data normalization methods N1, N2, N3, and N5 are suitable for combination with RAPS. Conversely, the rankings of the robots when using RAPS+N4 are very different from when using other combinations and the CODAS method. This means that N4 is determined to be unsuitable for combination with RAPS in this case.

C. *Case 3*

This case ranks nine different machining options for metal cutting. Cutting force components along three axes F_x, F_y, F_z, and Material Removal Rate (*MRR*) are the four criteria used to characterize each option. Lower F_x, F_y, and F_z components help reduce tool wear and improve the accuracy and surface quality of the machining process. Conversely, *MRR* is the rate at which material is removed from the workpiece during machining, and a higher value is better since it increases efficiency and reduces machining time. Table VI shows the values and the weights of the criteria for each machining option. Ranking the machining options has also been previously conducted using seven methods including SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV [32]. Figure 3 shows the ranking results of the options using various methods.

TABLE VI. MACHINING OPTIONS [32]

Alternative	F_x (N)	F_y (N)	F_z (N)	*MRR* (mm^3/s)
A1	59.844	187.437	44.165	11.561
A2	87.943	199.762	99.125	49.062
A3	78.913	127.456	69.874	109.108
A4	54.816	172.714	60.19	28.588
A5	63.117	180.361	68.869	99.039
A6	68.79	113.951	70.694	61.669
A7	46.654	116.88	92.222	57.177
A8	44.989	162.337	63.25	55.462
A9	54.846	167.837	74.165	151.09
Weight	0.2427	0.25976	0.24616	0.25138

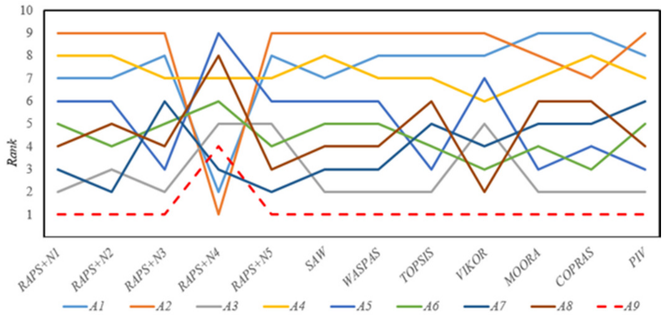

Fig. 3. Ranking of machining options.

When using RAPS+N1, RAPS+N2, RAPS+N3, and RAPS+N5, option A2 is ranked ninth (the worst option), which is also consistent with the rankings obtained using the SAW, WASPAS, TOPSIS, VIKOR, and PIV methods [32]. Remarkably, option A9 is identified as the best option when using RAPS+N1, RAPS+N2, RAPS+N3, and RAPS+N5, as well as when using the remaining seven MCDM methods. This indicates that, in this case, data normalization methods N1, N2, N3, and N5 are suitable for combination with RAPS. The rankings of the options when using RAPS+N4 are significantly different from when using other methods, which also indicates that N4 is not suitable for combination with RAPS in this case.

One noticeable observation across all three cases is that the four normalization methods N1, N2, N3, and N5 are consistently evaluated as suitable for combination with RAPS. It is worth noting that N1 is the data normalization method available within the RAPS method, but this method cannot be used if there are elements equal to 0 in the decision matrix. The three methods N2, N3, and N5 can be used in this case. This means that if N1 cannot be used, users can replace it with N2, N3, or N5. This discovery has expanded the application scope of the RAPS method compared to its original version. Another example case was examined to verify this result.

D. Case 4

Table VII shows a generated example dataset. This table presents four options to rank, labeled A1, A2, A3, and A4. Four criteria are considered: C1, C2, C3, and C4. The first two criteria are of type *C*, while the remaining two are of type *B*. For simplicity, the weights of all criteria are assumed to be equal.

TABLE VII. EXAMPLE OF CASE 4

No.	C1	C2	C3	C4
A1	0	2	5	4
A2	4.5	0	0	3
A3	3	2.5	6	3
A4	3	2	6	4
Weight	1/4	1/4	1/4	1/4

In this scenario, a deliberate situation is created, where the N1 method cannot be used, by assigning a value of zero to criterion C1 at A1, criterion C2 at A4, and criterion C3 at A4. In this case, combinations of RAPS+N2, RAPS+N3, and RAPS+N5 are used to rank the options. Additionally, methods such as ROV, FUCA, TOPSIS, MOORA, and PIV were employed for ranking. The ROV, TOPSIS, MOORA, and PIV methods were chosen because they have built-in normalization procedures that can handle zero values of x_{ij} [32]. The FUCA method is selected because it does not require data normalization [3, 4]. Figure 4 shows the ranking results using various methods.

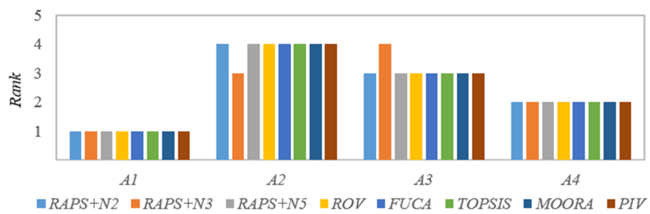

Fig. 4. Ranking of options in case 4.

In this case, the ranking of options is consistently determined when using RAPS+N2, RAPS+N5, and the ROV, FUCA, TOPSIS, MOORA, and PIV methods. All methods, including RAPS+N3, indicate that A1 ranks first and A4 ranks second. This demonstrates that the use of N2, N3, and N5 for data normalization ensures the accuracy of selecting the best option. Therefore, N2, N3, and N5 are reaffirmed once again as suitable for integration with RAPS.

IV. CONCLUSION

Four normalization methods can be used when there are x_{ij} values of zero in the decision matrix, including Weitendorf normalization (N2), vector normalization (N3), z-score normalization (N4), and enhanced accuracy normalization (N5). If there are x_{ij} values of zero in the decision matrix, the linear normalization method (N1) available in RAPS cannot be used. In this case, users can replace N1 with N2, N3, or N5. Conversely, N4 cannot replace N1 in combination with RAPS. Identifying that N2, N3, and N5 are suitable for combination with RAPS improves the applicability of this method when N1 cannot be used. This study has only confirmed the suitability of the N2, N3, and N5 data normalization methods when combined with RAPS in the case of using a specific set of weights. The question remains whether this suitability is maintained when the weights of the criteria change. To answer this question, further studies are needed.

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