The Effect of Construction Joints on the Behavior of Reinforced Concrete Deep Beams

Saba Basim Kadhum
Department of Civil Engineering, Faculty of Engineering, University of Kufa, Najaf, Iraq
sabab.alturk@uokufa.edu.iq (corresponding author)

Alaa Hussein Al-Zuhairi
Department of Civil Engineering, College of Engineering, University of Baghdad, Baghdad, Iraq
alaalwn@coeng.uobaghdad.edu.iq

Received: 20 May 2024 | Revised: 6 June 2024 | Accepted: 8 June 2024

ABSTRACT

The main objective of the present research is to conduct a thorough investigation into the impact of construction joints on the structural performance of reinforced concrete deep beams. This study involves a series of experimental tests and the use of advanced numerical analysis techniques to gain a deeper understanding of the behavior of these beams in the presence of construction joints. The experimental component incorporates analysis findings from both previous and current research. Specifically, six reinforced concrete deep beam specimens featuring horizontal and inclined construction joints were utilized as simply supported with two-point loading. The test findings indicate that the presence of a horizontal construction joint located below, at, or above the mid-height of the beam can lead to reductions in the ultimate load capacity by 9%, 11%, and 1%, respectively. The numerical part of the study focused on creating detailed models of the deep beam specimens with construction joints using the ABAQUS software. The proposed model showed a good agreement with the experimental tests, with estimations not exceeding 7% for the load-carrying capacity. This reduction becomes more significant when the concrete compressive strength is high, necessitating the use of bonding agents and additional reinforcement techniques to mitigate the impact of construction joints on the structural integrity.

Keywords-construction joint; concrete; deep beam; Abaqus

I. INTRODUCTION

Construction joints are necessary to facilitate the staged placement of concrete during construction. They are deliberate separations or discontinuities within concrete structures. Attempting to pour concrete for an entire structure in one continuous operation can be impractical, especially for larger projects. For example, pouring all the concrete in a single day may not be feasible when constructing a floor and its continuing columns. Besides, the amount of concrete that can be mixed and placed on-site is limited by batching capacity, workforce size, and allocated time duration. Accurately positioning and effectively implementing construction joints establishes boundaries for consecutive concrete placements without compromising the structure's integrity. A well-constructed joint ensures a robust, watertight surface, maintaining flexural and shear continuity [1].

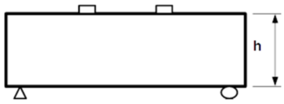
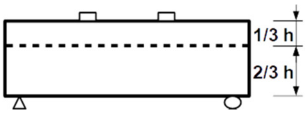
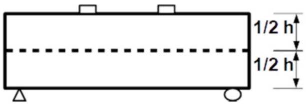
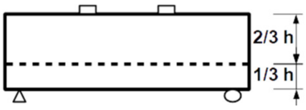
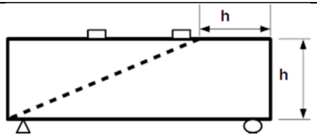
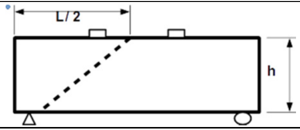
Many studies deal with the effect of construction joints on normal beams, investigating numerous factors on this subject [2-14]. Other concrete members, like slabs, prisms, cubes, and cylinders, were studied under the effect of the existence of construction joints [15-18]. On the other hand, there is a lack of research concerned with examining the construction joints related to deep beams.

Deep beams are defined as beams in which the clear span is equal to or less than four times the overall depth or the concentrated loads are within a distance equal to or less than two times the depth from the face of support [19]. These members are used in structural applications, such as diaphragms, water tanks, foundations, bunkers, offshore structures, shear walls, and girders utilized in multi-story buildings to offset columns and floor slabs subjected to horizontal loads [20-21]. Many researchers investigated experimentally and theoretically the behavior of deep beams under the effect of variable factors, involving prestressing, type of loading, existence of large openings [22, 23]. The only available study concerning construction joints in deep beams was introduced by [24]. In [24], authors studied experimentally the effect of horizontal construction joints on the behavior of reinforced concrete deep beams. According to the study, construction joints located below, at, or above the mid-height of a beam can decrease the failure load by 9%, 11%, and 1%, respectively. Thus, according to the results of this study, the upper section of the beam is the best location for the HCJ in a deep RC beam.

II. EXPERIMENTAL PART

Six deep beams were tested experimentally; two were designed in the current study, and four were tested previously [19]. One beam without a construction joint was considered a reference beam. Each one of the other five beams has one construction joint. The shape of the construction joint was chosen based on the most frequent cases that happened when a stop occurred in the casting process. The details of the tested beams are listed in Table I.

TABLE I. DESCRIPTION OF THE TESTED BEAMS

Beam's name	Sketch
DB-R	
DB-H-T	
DB-H-M	
DB-H-B	
DB-I-F	
DB-I-S	

The deep beam specimens were designed according to the ACI318-19 [20] to perform a concrete strength of 23MPa. The beam's cross-section was 400mm in height and 150mm in width. The total length of the beam was 1500mm with a clear span of 1200mm. The beam is reinforced with 3ϕ12mm rebar as a tension main reinforcement, 2ϕ8mm rebars as horizontal reinforcement at 65mm spacing, and ϕ8mm stirrups spaced at 70mm. Table II illustrates the properties of the used reinforcing bars.

TABLE II. MECHANICAL PROPERTIES OF STEEL BARS

Nominal diameter (mm)	*fy* (MPa)	*fu* (MPa)	Elongation (%)
ϕ12	571	701	18.2
ϕ8	452	550	22.8

The details of the dimensions, reinforcement, and loading points are shown in Figure 1.

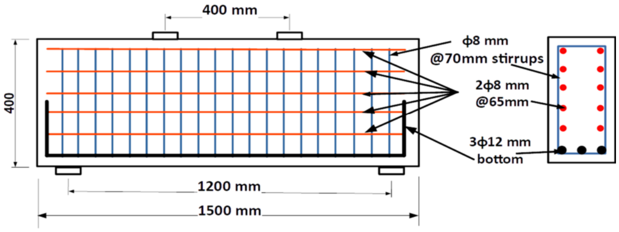


Fig. 1. Schematic diagram of beam dimension and reinforcement details.

The deep beam specimens were tested under two-point loading as a simply supported beam. The load was applied using mechanical jack then divided into two concentrated loads by spreader beam. The beam arrangement in the loading frame and the measured equipment are evidenced in Figure 2.

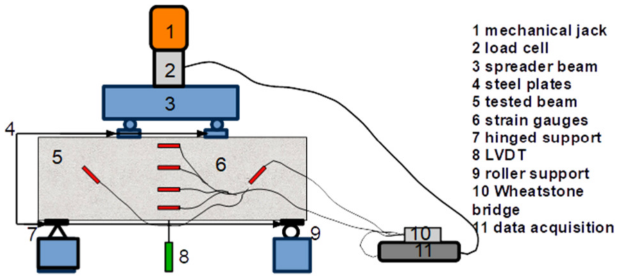


Fig. 2. Schematic diagram of test setup for deep beam specimens.

Figure 3 depicts the cracking modes of the tested deep beam specimens at failure stage. All the deep beam specimens were failed by shear. In the reference beam, the initial flexural crack emerges when a load of 100 kN is applied at the mid-span of the beam, coinciding with the formation of the first shear crack. As the load increases, additional flexural and shear cracks extend toward the mid-depth of the beam, followed by cracks near the support. Subsequently, the diagonal shear cracks proliferate, and cracks are developed near the loading area, in the compression zone at mid-span, and in the vicinity of the support.

The tested beams with horizontal construction joints, DB-H-B, DB-H-M, and SB-H-T exhibited initial cracking at load levels ranging from 80 kN to 120 kN, with the first flexural and shear cracks appearing at each loading stage. Ultimately, all tested beams failed in shear. Notably, the presence of a construction joint at mid-height led to the lowest failure load, accelerating crack propagation towards the upper part of the beam. This resulted in intensified cracking and a reduction in the ultimate load-bearing capacity.

The propagation of cracking in specimen DB-I-F, where the construction joint is located in the middle part of the beam, i.e., at the maximum moment zone, a load of 135 kN, causes the first flexural and shear cracks to appear simultaneously. In areas where the distance between the construction joint and the soffit of the beam is minimal, the crack originating from the bottom of the beam changes its trajectory, causing the construction joint to extend as a shear crack. In the case of specimen DB-I-S, where the inclined construction joint is positioned in the shear zone, the first flexural crack appears at a load of 80 kN, whereas the first shear crack emerges at a load of 110 kN. As the applied load increases, more flexural and shear cracks appear until a crack forms in the joint, leading to beam failure. Table III portrays the first crack and failure load.

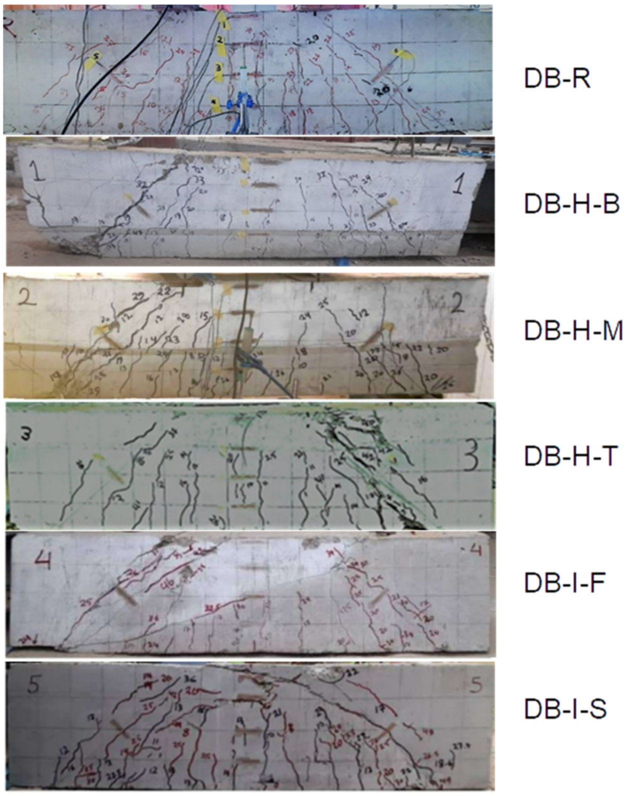


Fig. 3. Cracking patterns at failure stage.

TABLE III. CRACKING AND FAILURE LOAD

Beam specimen	First flexural crack load (kN)	First shear crack load (kN)	Failure load Pu (kN)	Changing in failure load (%)
DB-R	100	100	395	-
DB-H-B	120	120	360	-9
DB-H-M	80	80	350	-11
DB-H-T	110	120	380	-4
DB-I-F	135	135	390	-1
DB-I-S	80	110	400	+1

Table III demonstrates that the cracks appeared earlier in the case of the deep beam with a horizontal construction joint in the mid-height or inclined construction joint at the shear zone. Figure 4 showcases the deflection in each case.

III. NUMERICAL ANALYSIS

Numerical analysis with ABAQUS software is highly significant in engineering and scientific research. It provides accurate solutions to complex problems, is applied to versatile and wide problems, offers deep insights into structural behavior, and supports parametric studies and optimization while saving time and cost. Additionally, it provides high-performance computing. Recently, researchers have used numerical analysis to study and investigate various cases [26-34].

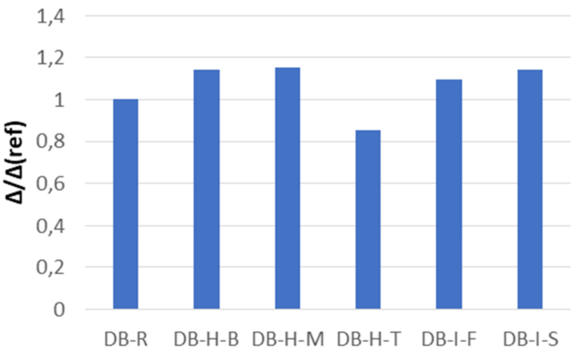


Fig. 4. Deflection.

A. *Modeling of Tested Deep Beams*

1) *Inelasticity*

The Inelasticity of concrete is represented in the FE model using the concrete damaged plasticity (CDP) model presented by authors in [35]. Information about concrete behavior under compression and tension is necessary to establish the CDP model in ABAQUS. In [36], the proposed stress-strain relationship for compression was utilized. The modified model by [37] represented the stress-strain relationship for concrete in tension. Five parameters are related to CDP in the FE model. The parameters are listed in Table IV. A trial was made to choose the dilation angle and viscosity parameter values, while the other values were taken from the literature.

TABLE IV. PARAMETERS OF THE CDP MODEL

Parameter	Value
dilation angle (φ)	32°
eccentricity (ε)	0.1
f_{b0} / f_{c0}	1.16
coefficient K	2/3
viscosity parameters (μ)	0.00001

2) *Steel Reinforcing*

The reinforcement of steel is simulated as a material that behaves like an elastic substance until it reaches a certain point, after which it behaves like a plastic substance. This behavior is observed in both cases when the steel is subjected to tension or compression.

3) *Concrete*

Two types of elements were used to represent the concrete. The aspect employed was C3D8, which can experience cracking when subjected to tension and crushing when subjected to compression to idealize the concrete in the reference beam and the beam with a horizontal construction joint. A 4-node linear tetrahedron element, C3D4, was deployed to represent the concrete in the rest of the beams, where the shape of the sections was not rectangular. Simulating steel rebars involves utilizing a truss element (T3D2) in three dimensions. ABAQUS employed a solid finite element to model the plates at the loading points and supports. The types of used elements are observed in Figure 5.

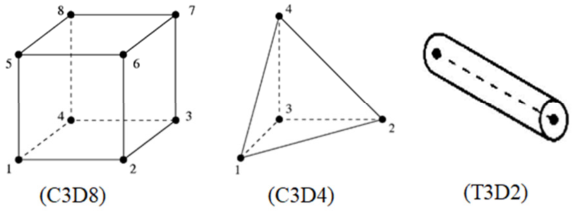
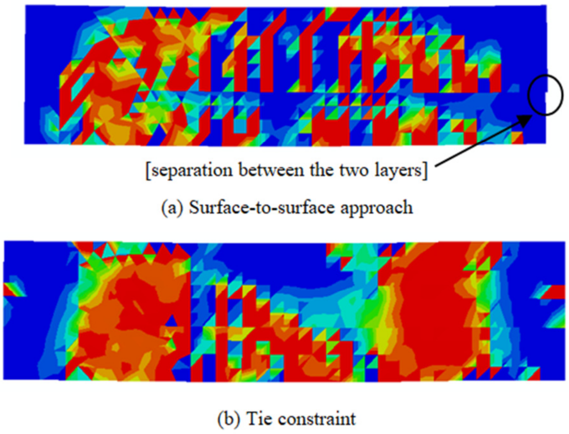


Fig. 5. ABAQUS elements for concrete.

4) *Joints*

Various methods utilize ABAQUS software to model the interface between two layers of concrete in a reinforced concrete beam. Some common approaches include:

- Tie Constraints: This method assumes perfect bonding between the two concrete layers. The tie constraint connects the nodes on the interface of the two layers, ensuring they move together without any separation.
- Contact Modeling: ABAQUS provides contact pairs to model the interaction between the two concrete layers. Contact pairs allow for sliding and separation between the surfaces, simulating a more realistic interface behavior.
- Cohesive Zone Modeling: This method utilizes cohesive elements to model the interface between the two layers. Cohesive elements simulate the bond behavior between the concrete layers, including separation and sliding. Cohesive elements are defined by their stiffness, strength, and failure criteria.
- Embedded Element Technique: A thin layer of elements is embedded between the two concrete layers to model the interface. These elements have different material properties to represent the bond between the layers.
- Fracture Mechanics Approach: This approach models the interface as a crack propagation problem, where cracks are initiated and propagated along the interface based on the applied loading conditions and material properties.
- The current study experimented with the first and second approaches. The second approach separates the two layers of the beams, which is not noticed in the experimental work. The method adopted to simulate the connection between the two concrete segments was through a tie interaction model, mainly because the bond effect was considered in modeling concrete tension behavior. Figures 6a and 6b show the difference between surface-to-surface contact and tie constraint.



(a) Surface-to-surface approach

(b) Tie constraint

Fig. 6. The connection between the two segments.

5) *Steel and Concrete Bond*

For this analysis, the concrete and the reinforcing rebars are supposed to be in complete bonding, and this is achieved by using a constraint called Embedded Element, as illustrated in Figure 7. The reinforcing rebar is embedded within the concrete host element, which means that all nodes belonging to the embedded element possess identical translational degrees of freedom with the nodes belonging to the concrete host element.

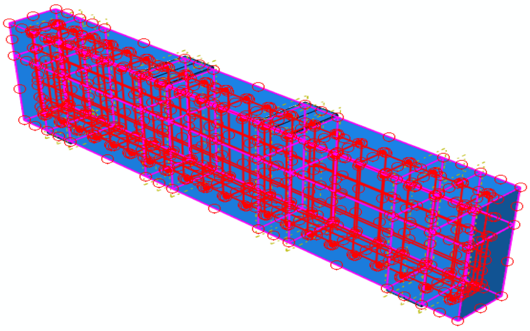


Fig. 7. Embedded reinforcement in host concrete.

6) *Boundary Conditions*

The boundary conditions refer to the beam's support and loading type. The specimens are modeled as supported beams. The supports were modeled as displacement, with one support constrained in the X and Y directions representing hinged support and the other constrained in the Y direction only as roller support. The load was applied as a displacement on a reference point on the loading plate in the Y direction. The displacement given was more significant than the experimental one to ensure the capture of the complete behavior of the beam until failure. Figure 8 manifests the loading and boundary conditions.

B. *Numerical Results*

This section compares the output of the current numerical model with experimental results. Adequate compatibility was obtained from the comparison of analytical outcomes, and the results acquired from the experimental work of the tested specimen can be noticed through the comparison displayed in the following table and figures.

The tested deep beam specimens were analyzed using the proposed model and the result is that the ultimate load fails. A comparison between the numerical and experimental results is listed in Table V and Figure 9. The percentage of the difference between the numerical and experimental results changes between 0.75% and 6.8%.

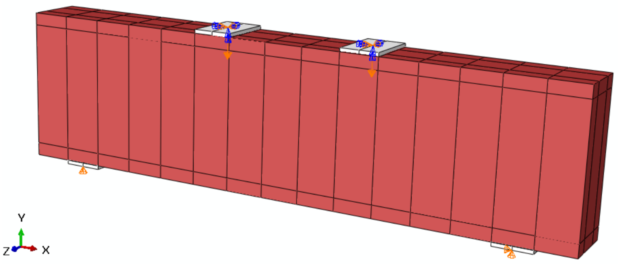


Fig. 8. Loading and boundary conditions.

TABLE V. COMPARISON BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS.

Beam	f'_c=30		f'_c=40		f'_c=50	
	Decreasing in Pu%	**Changing in Δ**	**Decreasing in Pu%**	**Changing in Δ**	**Decreasing in Pu%**	**Changing in Δ**
DB-R	-	-	-	-	-	-
DB-H-B	3.80	-11.63	8.24	-3.77	12.53	-1.24
DB-H-M	7.22	-10.85	7.95	-17.97	9.97	13.81
DB-H-T	5.90	-8.30	9.57	-36.29	15.13	-24.35
DB-I-F	4.80	-5.76	2.30	-8.22	3.54	16.37
DB-I-S	5.38	24.65	6.21	-14.83	9.16	7.93

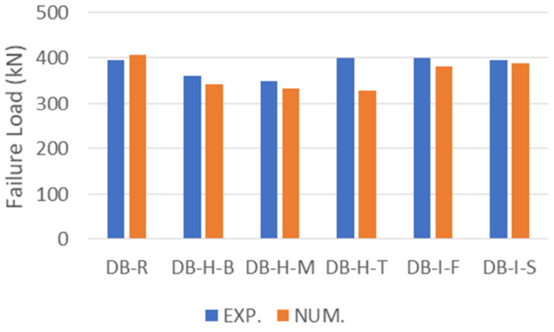


Fig. 9. Comparison between experimental and numerical failure load.

C. *Case Study*

In this section, the effect of increasing concrete compressive strength (f'_c) was investigated using the proposed theoretical model. Three values of concrete compressive strength, 30, 40, and 50 MPa, were analyzed.

The results show that the effect of the construction joint becomes more apparent as the compressive strength increases, in which the decrease in the ultimate load failure in the tested beam, where f'_c =23, ranged between 1 and 11%. In comparison, the decrease reached about 15% when the value of the construction joint became 40 and 50. Proper load transfer across construction joints is crucial in high-strength concrete. Ineffective joint design or execution may lead to heightened stress and compromised performance. The increasing concrete compressive strength results are presented in Table VI as changes in both the ultimate load and the deflection.

TABLE VI. EFFECT OF INCREASING CONCRETE COMPRESSIVE STRENGTH ON ULTIMATE LOAD AND DEFLECTION.

Beam specimen	EXP.	NUM.	*Pu* (Num)/ *Pu* (Exp) *100%	Chaining in predicting *Pu* (%)
DB-R	395	406	102.78	2.78
DB-H-B	360	376	104.44	4.44
DB-H-M	350	374	106.85	6.85
DB-H-T	400	385	96.25	-3.75
DB-I-F	400	382	95.5	-4.5
DB-I-S	395	388	98.22	-1.77

Figures 10 and 11 represent the difference in maximum load and deflection in each case respectively.

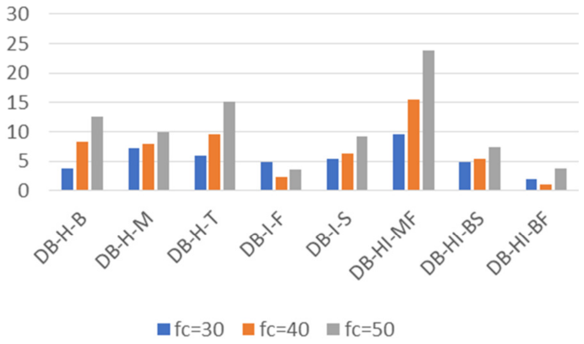


Fig. 10. Changing in maximum load.

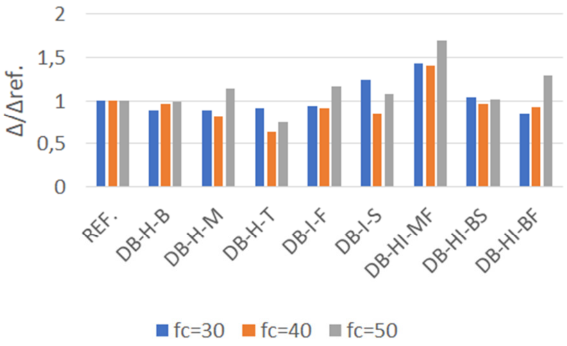


Fig. 11. Changing in deflection.

IV. CONCLUSIONS

- The existence of the horizontal construction joint below, at, or above the beam mid-height decreases the ultimate load by 9%, 11%, and 1%, respectively.
- Horizontal construction joints in the lower part of the beam should be avoided, as it was noticed from the study that when the construction joint is located in a region where

cracks are formed, it behaves as a control joint and accelerates the appearance of cracks.

- Placing the ends of the joints in the region where forces are applied should be avoided, as failure may be severe.
- The existence of a construction joint in the shear zone led to a brittle and sudden failure.
- Although construction joints at the top part of the deep beam do not affect the ultimate load, they decrease the deflection.
- The numerical analysis showed that the increase of concrete compressive strength led to a decrease in the ultimate load capacity. The decrease reached 15% when the compressive concrete strength increased to 50 MP.
- Since the numerical analysis revealed that the effect of construction joints becomes more effective when the concrete compressive strength is high, then bonding agents and strengthening methods are needed in the deep beams of highly compressive concrete.

REFERENCES

[1] G. Halvorsen et al., *ACI 224.3R-95 Joints in Concrete Construction*, American Concrete Institute, 1995.

[2] A. K. Ismail, "Flexural behavior of reinforced concrete beams having horizontal construction joints," M.S. thesis, University of Technology, Baghdad, Iraq, 2005.

[3] H. J. Mehrath, "Flexural behavior of reinforced concrete beams having transverse construction joints," M.S. thesis, University of Technology, Baghdad, Iraq, 2008.

[4] Q. Abdul-Majeed, "Evaluation of transverse construction joints of reinforced concrete beams," *Engineering and Technology Journal*, vol. 28, no. 14, pp. 4750–4773, 2010.

[5] M. Ghalib Ghaddar, L. Ali Ghaleb, and Q. Abdul-Majeed, "Effect of the Number of Horizontal Construction Joints In Reinforced Concrete Beams," *Engineering and Technology Journal*, vol. 28, no. 19, pp. 5803–5821, Sep. 2010, https://doi.org/10.30684/etj.28.19.1.

[6] Z. W. Abass, "Effect of Construction Joints on Performance of Reinforced Concrete Beams," *Al-Khwarizmi Engineering Journal*, vol. 8, no. 1, pp. 48–64, Feb. 2012.

[7] C. A. Issa, N. N. Gerges, and S. Fawaz, "The effect of concrete vertical construction joints on the modulus of rupture," *Case Studies in Construction Materials*, vol. 1, pp. 25–32, Jan. 2014, https://doi.org/10.1016/j.cscm.2013.12.001.

[8] N. N. Gerges, C. A. Issa, and S. Fawaz, "The effect of construction joints on the flexural bending capacity of singly reinforced beams," *Case Studies in Construction Materials*, vol. 5, pp. 112–123, Dec. 2016, https://doi.org/10.1016/j.cscm.2016.09.004.

[9] H. Jabir, S. Salman, and J. Mhalhal, "Effect of Construction Joints on the Behavior of Reinforced Concrete Beams," *Engineering Journal*, vol. 23, pp. 47–60, May 2017, https://doi.org/10.31026/j.eng.2017.05.04.

[10] A. N. Abbas, H. K. Al-Naely, H. H. Abdulzahra, and Z. S. Al-Khafaji, "Structural behavior of reinforced concrete beams having construction joint at different elevation," *International journal of civil engineering and technology (ijciet)*, vol. 9, no. 13, pp. 712–720, Dec. 2018.

[11] S. B. Kadhum and S. R. Al-Zaidee, "A Review of Previous Studies on the Construction Joints in Reinforced Concrete Beams," *Design Engineering*, no. 8, pp. 16464–16472, Nov. 2021.

[12] M. A. Ismael, Y. M. Hameed, and H. J. Abd, "Effect of construction joint on structural performance of reinforced self-compacting concrete beams," *International journal of civil engineering and technology*, vol. 10, no. 1, pp. 297–306, Jan. 2019.

[13] A. A. Abbood and M. K. Kharnoob, "Influence of Fire-Flame Temperature and Duration on the Behavior of Reinforced Concrete Beams with Construction Joints," *Journal of Engineering*, vol. 30, no. 5, pp. 132–150, May 2024, https://doi.org/10.31026/j.eng.2024.05.09.

[14] A. Al-Rifaie, H. Al-Hassani, and A. Shubbar, "Flexural behaviour of reinforced concrete beams with horizontal construction joints," *IOP Conference Series: Materials Science and Engineering*, vol. 1090, Mar. 2021, Art. no. 012003, https://doi.org/10.1088/1757-899X/1090/1/012003.

[15] R. F. Abbas and W. H. Sultan, "Effect of type and position of construction joint on behavior and capacity of reinforced concrete one way slabs," *Journal of Engineering and Sustainable Development*, vol. 23, no. 2, pp. 22-41, Mar. 2019.

[16] I. Yakovleva and V. Kurochkina, "Arrangement of construction joints in cast-in-place slabs," in *E3S Web of Conferences*, Jan. 2019, vol. 97, Art. no. 04023, https://doi.org/10.1051/e3sconf/20199704023.

[17] O. Qasim and H. Sultan, "Experimental Investigation of Effect of Steel Fiber on Concrete Construction Joints of Prism," *IOP Conference Series Materials Science and Engineering*, vol. 745, Mar. 2020, Art. no. 13, https://doi.org/10.1088/1757-899X/745/1/012170.

[18] H. B. Osman, H. B. Tami, and N. A. A. Rahman, "A comparison of construction joint ability on concrete slab applied at construction site," *ARPN Journal of Engineering and Applied Sciences*, vol. 11, no. 4, pp. 2576–2580, 2016.

[19] *ACI CODE-318-19(22): Building Code Requirements for Structural Concrete and Commentary (Reapproved 2022)*, ACI: Farmington Hills, MI, USA, 2019.

[20] A. Nilson, D. Darwin, and C. Dolan, *Design of Concrete Structures*, 14th ed. Boston, MA, USA: McGraw-Hill Education, 2009.

[21] G. Russo, R. Venir, and M. Pauletta, "Reinforced Concrete Deep Beams-Shear Strength Model and Design Formula," *ACI Structural Journal*, vol. 102, no. 03, May 2005, Art. no. 429.

[22] R. M. Abbas and L. T. Hussein, "Transient response and performance of prestressed concrete deep T-beams with large web openings under impact loading," *Journal of the Mechanical Behavior of Materials*, vol. 32, no. 1, Jan. 2023, https://doi.org/10.1515/jmbm-2022-0268.

[23] H. K. Kadhim and M. D. Abdullah, "Effect of loading type in concrete deep beam with strut reinforcement," *Open Engineering*, vol. 14, no. 1, Jan. 2024, https://doi.org/10.1515/eng-2022-0488.

[24] S. B. Kadhum, A. H. Al-Zuhairi, and S. R. Al-Zaidee, "Experimental investigation of the effect of horizontal construction joints on the behavior of deep beams," *Open Engineering*, vol. 14, no. 1, Jan. 2024, https://doi.org/10.1515/eng-2022-0554.

[25] *Abaqus Analysis user's manual version 6.14*. Simulia, 2014.

[26] S. Q. Abdualrahman and A. H. Al-Zuhairi, "A Comparative Study of the Performance of Slender Reinforced Concrete Columns with Different Cross-Sectional Shapes," *Fibers*, vol. 8, no. 6, Jun. 2020, Art. no. 35, https://doi.org/10.3390/fib8060035.

[27] A. H. A. Al-Ahmed, A. H. Al-Zuhairi, and A. M. Hasan, "Behavior of reinforced concrete tapered beams," *Structures*, vol. 37, pp. 1098–1118, Mar. 2022, https://doi.org/10.1016/j.istruc.2022.01.080.

[28] A. H. Al-Zuhairi, A. H. A. Al-Ahmed, A. N. Hanoon, and A. A. Abdulhameed, "Structural Behavior of Reinforced Hybrid Concrete Columns under Biaxial Loading," *Latin American Journal of Solids and Structures*, vol. 18, no. 6, Sep. 2021, Art. no. e390, https://doi.org/10.1590/1679-78256640.

[29] H. Hernoune, B. Benabed, A. Kanellopoulos, A. H. Al-Zuhairi, and A. Guettala, "Experimental and Numerical Study of Behaviour of Reinforced Masonry Walls with NSM CFRP Strips Subjected to Combined Loads," *Buildings*, vol. 10, no. 6, Jun. 2020, Art. no. 103, https://doi.org/10.3390/buildings10060103.

[30] S. Q. Abdualrahman and A. H. Al-Zuhairi, "Numerical Study of Specially Shaped Slender RC Columns under Compressive Load," *IOP Conference Series: Materials Science and Engineering*, vol. 881, no. 1, Apr. 2020, Art. no. 012042, https://doi.org/10.1088/1757-899X/881/1/012042.

[31] A. J. Daraj and A. H. Al-Zuhairi, "The Combined Strengthening Effect of CFRP Wrapping and NSM CFRP Laminates on the Flexural Behavior

of Post-Tensioning Concrete Girders Subjected to Partially Strand Damage," *Engineering, Technology & Applied Science Research*, vol. 12, no. 4, pp. 8856–8863, Aug. 2022, https://doi.org/10.48084/etasr.5008.

[32] M. M. Hason, A. H. Al-Zuhairi, A. N. Hanoon, A. A. Abdulhameed, A. W. Al Zand, and I. S. Abbood, "Peak Ground Acceleration Models Predictions Utilizing Two Metaheuristic Optimization Techniques," *Latin American Journal of Solids and Structures*, vol. 19, no. 3, Jun. 2022, Art. no. e447, https://doi.org/10.1590/1679-78256940.

[33] A. Jalil and A. H. Al-Zuhairi, "Behavior of Post-Tensioned Concrete Girders Subject to Partially Strand Damage and Strengthened by NSM-CFRP Composites," *Civil Engineering Journal*, vol. 8, no. 7, pp. 1507–1521, Jul. 2022, https://doi.org/10.28991/CEJ-2022-08-07-013.

[34] H. Q. Abbas and A. H. Al-Zuhairi, "Use of EB-CFRP to Improve Flexural Capacity of Unbonded Post-Tensioned Concrete Members Exposed to Partially Damaged Strands," *Civil Engineering Journal*, vol. 8, no. 6, pp. 1288–1303, Jun. 2022, https://doi.org/10.28991/CEJ-2022-08-06-014.

[35] J. Lubliner, J. Oliver, S. Oller, and E. Oñate, "A plastic-damage model for concrete," *International Journal of Solids and Structures*, vol. 25, no. 3, pp. 299–326, Jan. 1989, https://doi.org/10.1016/0020-7683(89)90050-4.

[36] L. P. Sanez, "Discussion of Equation for the Stress - Strain Curve of Concrete' by Desayi and Krishnan," vol. 61, pp. 1229–1235, Jan. 1964.

[37] B. Wahalathantri, D. Thambiratnam, T. Chan, and S. Fawzia, "A material model for flexural crack simulation in reinforced concrete elements using ABAQUS," in *Proceedings of the First International Conference on Engineering, Designing and Developing the Built Environment for Sustainable Wellbeing*, C. J. L. Cowled, Ed., 2011, pp. 260–264.