Numerical Simulation of Gradually Varying Permanent Flows in a Prismatic Open Channel for Four Geometric Shapes

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ABSTRACT

Standing flows in natural channels often cause phenomena that can be very serious, such as flooding, deformation of channel geometry, and destruction of infrastructure (dams, bridges, and culverts). This study focuses on the computation of gradually varying permanent flows (backwater curves) by two methods: direct integration (Chow) and successive approximation (depth variation). To solve the system of equations governing the problem of gradually varying one-dimensional stationary flows at a free surface, a large amount of data should be taken into account, namely, the flow rate, the water head, the mean flow velocity, the rugosity, and the slope. These parameters are very important, as they cause nonlinear behavior, making the problem and its mathematical solution complex. Digitizing these parameters can help to determine and visualize the longitudinal profile of the water line for known flow rates. This study aimed to: (1) determine the influence of rugosity on gradually varying steady flows and the overclassification of eddy curves in a prismatic channel, (2) study the effect of geometric shape on these flows, and (3) investigate and compare the effects of the calculation methods. The results reveal the great influence of rugosity on gradually varying permanent flows for four selected geometric shapes of the channel, as it has a direct influence on the normal depth and the critical slope. Each time the resistance of the bottom to the flow increases, these results increase. The influence of the geometric shape on these flows is less significant. The comparative study showed a difference between the results obtained.

Keywords: flow; geometry; rugosity; slope; velocity; permanent flow; free-surface flow; permanent regime; eddy curve; flow speed

I. INTRODUCTION

The study of gradually varying one-dimensional free-surface stationary flows requires as much data as possible to be taken into account. Flow rate (Q), water head (h), mean flow velocity (v), rugosity (n), and slope inclination are all important, as they can cause nonlinear behavior, increasing the complexity of the problem and its mathematical solution.
Several studies have focused on different approaches to solving the governing equations of this type of flow. In [1], the differential equation that governs the gradually varying flow was transformed, deriving a generalized theoretical approach to calculate the eddy curves evolving in a triangular channel for a rough turbulent regime. Particular attention was paid to the special cases of critical and horizontal slopes, which are of considerable mathematical importance. In [2], two approaches were compared in branched and looped channel networks: the direct elimination method and the use of branch-segment transformation equations. This study revealed that the technique based on the Newton-Raphson method is at least five times faster than the algorithm based on the branch-segment transformation equations. In [3], a method was developed to model the subcritical flow in channel systems, following the implicit finite-difference method. Backflow effects at junction points were treated based on water level prediction and correction. The results obtained were almost similar to those of the three-phase algorithms and fit well with the observed data. In addition, this algorithm reduced storage requirements and simulation time. Based on Lipschitz's theorem, the study in [4] focused on the transition from subcritical to supercritical flow, finding that the initial value problem for the ordinary differential energy equation has no unique solution. Consequently, the discrete energy equation can have several roots, depending on the conditions. In [5], the finite-volume method was adopted to solve the flow transport equations in the pipes. The results exhibited that inertial forces have an effect on the change in flow in the pipes and that EPANET and EPANET2 may have flaws in calculations.

In [6], two models were developed to simulate water flow in a real fish habitat. The findings, based on one- and two-dimensional assumptions, revealed a clear difference between the results of the two approaches. In [7], Artificial Neural Networks (ANNs) and Genetic Programming (GP) were considered to estimate gradually varying flow lengths when the water depth was higher than the normal and critical depths for various slopes of trapezoidal channels. The results showed that the greater the number of subreaches considered in the direct method is, the better are the results obtained. In [8], an algorithm for a subcritical flow was proposed, which performed correctly and therefore can be considered a valid approach to studying gradually varying flows in open channel networks. In [9], two explicit finite-difference schemes were developed by studying the applicability of the Lax-Wendroff and McCormack diagrams to model unsteady flow with fast and gradual variations in a prismatic open channel. The numerical results of the improved McCormack scheme corresponded more closely to the analytical solution. Furthermore, the McCormack model simulated more accurately the gradually varying flow than the Lax-Wendroff approach.

In [10], a systematic laboratory study was carried out, presenting an empirical equation to distinguish the impact of the vegetation stem distribution pattern on the hydrodynamics of a gradually varied flow through emergent blade-type vegetation. In [11], a semi-analytical solution was proposed to solve the equation of gradually varying flow in circular and parabolic prismatic channels, using the Adomian Decomposition Method (ADM) for any channel size (wide or not) and slope. The results displayed that the water surface profile in circular and parabolic channels using the semi-analytical ADM method was in good agreement with that produced with FDM. In [12], a genetic algorithm-based simulation and optimization method was deployed to solve water surface profiles, and the results were validated using synthetic data generated by FLDWAV. The results disclosed that the accuracy of the proposed method decreased as the rugosity coefficient and the error in the water depth measurements increased. This method provided better results in the subcritical flow regime at high Froude numbers. In [13], the fourth-order Runge-Kutta scheme was applied to numerically solve the differential equation for gradually varying flow and calculate the influence length for rectangular and trapezoidal prismatic channels. The results showed how the influence distance changes with various upstream and downstream flow conditions, as well as with different channel shapes. In [14], the unsteady and gradually varying flow of an open channel in tidal rivers was examined, based on theoretical analysis and two-dimensional numerical simulations. The results demonstrated that the control equations for the total flow of the open channel can be obtained directly from the mathematical model of viscous fluid flow. In [15], theoretical findings were confirmed with numerical simulations performed by solving the complete nonlinear problem implementing the Herschel and Bulkley rheological model. This study concluded that under subcritical conditions, frequently encountered in gradual sludge flows, even in conjunction with the development of rolling waves, capable of inducing deceleration or acceleration of the current, it could prevent or promote the development of instabilities. A good agreement between experimental and numerical results was demonstrated in [16], indicating that both the Reynolds model and the finite volume method are capable of simulating hydraulic flow in open channel transitions (rectangular to trapezoidal). The results demonstrated that as the upstream Froude number increases, the transition efficiency and energy loss coefficient decrease and increase, respectively. In [17, 18], a finite element algorithm was utilized, based on the multiple grid technique, to solve the one-dimensional equations for shallow water in a rectangular prismatic open channel spilling over a weir. The model developed accurately reproduced the behavior of the transient flow, and the multiple grid technique slightly improved the performance of the standard grid in terms of computation time consumed. In [19], the effect of roughness on the transition from laminar to turbulent flow was examined, exhibiting that the transition Reynolds number of a flat plate with a zero pressure gradient is a function of the ratio between the height of the rugosity element and the displacement thickness of the boundary layer at the element. Further findings showed that the effects of roughness are similar in streams with different initial turbulence.

In [20], an overview of the effects of rugosity on surface runoff was presented, where experiments on the hydraulic resistance of concentrated flow channels revealed that the spatial organization of runoff is influenced by rugosity and topography. The location and geometry of large runoff collectors can be predicted deterministically. In [21], three different approaches were adopted to study the influence of rugosity on the laminar flow in microchannels. Three-
dimensional numerical simulations, a one-dimensional rough-layer model, and volumetric averaging experiments disclosed that the Poiseuille number increased with relative rugosity and was independent of the Reynolds number in the laminar regime. The increase in Poiseuille observed in the experiments was well predicted by both the three-dimensional simulations and the rough layer model. In [22], a non-hydrostatic numerical model was used to study the hydraulic effect of different bed geometries. A function was developed for Nikuradse rugosity (\(ks\)), and a new equation was proposed, which directly related \(ks\) to relative bed height, aspect ratio, and side angle. In [23], a three-dimensional thin-wall model was employed, using ANSYS Fluent 14.5 software, to avoid heat transfer effects, demonstrating that the friction factor increased with increasing hydraulic diameter.

In [24], Froude similarity conditions were derived to describe dam failure waves in a smooth rectangular horizontal channel, utilizing a characteristic theory to analytically determine both positive and negative waves. The results provided a better understanding of the mechanism of dam failure waves. In [25], the effect of bed friction on the stability of transverse shear flows in shallow open channels was examined deploying a linear hydrodynamic stability analysis. The results showed that in the limiting case of a shallow transverse shear flow when the velocity change is small, bed rugosity becomes the only parameter governing the stability of transverse shear flows. In [26, 27], a numerical model was developed to understand the dynamic behavior of the flow field, influenced by the shape and rugosity lengths on large bedforms during a tidal cycle. The findings provided a better understanding of the hydrodynamics of natural bedforms in a tidal environment. In [28], the impact of spontaneous reed on flow was evaluated, both in terms of resistance and velocity distribution, employing full-scale field hydraulic experiments on an existing drainage channel. The results revealed that the effect of the plant led to a decrease in flow resistance for both channels, whereas a decrease in rugosity coefficients led to a corresponding increase in water levels, manifested that for both channels, an increase in rugosity coefficients led to a decrease in flow resistances by walls and bottom are homogeneous, and (d) curvature effects are neglected.

\[
dh = \frac{1}{I \times \left(\frac{Q^2}{Q^2 + 2gh} - 1\right)} 
\]

This equation is a first-order differential equation that describes a gradually varied flow. It allows to determine the water depth \(h(x)\) as a function of the distance \(x\) for a given flow rate \(Q\). This is the simplified Barré de Saint-Venant equation and is valid for prismatic and non-prismatic channels. The problem that arises when studying gradually varying flows is to determine the position \(x\) and the shape \(h(x)\) of the free surface for a given flow \(Q\) and geometric shape (section \(S\)). For a given channel, the arguments \(C\) and \(S\) are functions of \(x\) and \(h\), while \(I\) is a function of \(x\).

III. ITERATIVE METHOD [34]

The iterative method generally takes the longest to apply but is often the most accurate. Its basic equation is (1), which gives:

\[
dx = \frac{(1 - \frac{Q^2}{gh}) dh}{\frac{Q}{gh} \frac{dh}{dx}} = \frac{\Delta h}{I - J_m} = \frac{\Delta h}{I - J_m} = \frac{\Delta h}{I - J_m} (2)
\]

Passing from differential equations to finite differences can give:

\[
\Delta x = x_1 - x_2 = \frac{\Delta h}{I - J_m} = \frac{\Delta h}{I - J_m} = \frac{\Delta h}{I - J_m} (3)
\]

where \(J_m\) represents the average pressure drop, defined in the middle of the \(\Delta x\) interval by:

\[
J_m = \frac{(h_2 + h_1)}{2} (4)
\]

\[
\Delta x = x_2 - x_1 = \frac{(h_2 + h_1)}{2} = \frac{(h_2 + h_1)}{2} (5)
\]

Therefore:

\[
h_1 - h_2 = (x_2 - x_1) \left(1 - \frac{(h_2 + h_1)}{2}\right) - \frac{v^2}{2g} - \frac{v^2}{2g} (6)
\]
Finally, the equation for gradually varying flows can be written simply as:

\[ h_1 - h_2 = (x_2 - x_1) \left( I - \frac{(h_1^{3/2}-h_2^{3/2})}{2} \right) \]  

This can be solved in two ways: by the section method \((\Delta x\) fixed) or by the depth variation method \((\Delta h\) fixed).

A. Depth Variation Method \((\Delta h\) is fixed)

The depth variation or direct step method applies to (3) for gradually varied motion. This method consists of finding the value of the abscissa \(x_2\) for a depth \(h_2\) very close to \(h_1\) using (5), then calculating the abscissa \(x_2\) and moving on to the next section, etc.

B. Chow Method [35]

Chow (1959) considered that the parameter \(\beta\) is not constant. He generalized \(\frac{dh}{dx} = I \frac{h^N - h^M}{h^M - h^0}\) and wrote it as:

\[ \frac{dh}{dx} = I \frac{h^N - h^0}{h^M - h^0} \]  

where \(N\) and \(M\) are always hydraulic exponents that depend on the shape of the cross-section and the roughness coefficient chosen. For a section of any shape, the Chow method gives:

\[ N(h) = \frac{2h}{3} (5B - 2R h \frac{dP}{dh}) \]  

\[ M(h) = \frac{h}{2} (3S - \frac{d}{dh}) \]  

The two hydraulic exponents are functions of the water depth \(h\). Their usual variations are:

\[ 2.0 < N < 5.3 \quad \text{and} \quad 3 < M < 4.8 \]  

Using the same Bresse variables, that is:

\[ \frac{h}{h_n} = \eta \quad \text{and} \quad dh = h_n \times d\eta \]  

Equation (8) becomes:

\[ dx = \frac{h_n}{I} \left( 1 - \left( \frac{1}{1-\eta^s} \right) \right) + \left( \frac{h_n}{I} \frac{M}{h^M - h^0} \right) \left( \frac{\eta^{M-M}}{1-\eta^s} \right) \]  

By integrating between two sections with abscissas \(x_0\) and \(x_1\), it is obtained:

\[ x_1 - x_0 = \frac{h_n}{I} \left( (\eta_1 - \eta_0) - \int \frac{1}{1-\eta^s} d\eta \right) + \left( \frac{h_n}{I} \right) \left( \frac{M}{h^M - h^0} \right) \left( \int \frac{\eta^{M-M}}{1-\eta^s} d\eta \right) \]  

Equation (12) then becomes:

\[ x_1 - x_0 = \frac{h_n}{I} \left( (\eta_1 - \eta_0) - (\Phi(\eta_1, N) - \Phi(\eta_0, N)) \right) + \]  

\[ \left( \frac{h_n}{I} \right)^M \left( \Phi(\xi, \chi) - \frac{\xi}{h_n} \Phi(\xi, \chi) \right) \]  

\[ (13) \]

IV. NUMERICAL SIMULATION

The complexity and large number of parameters involved in the study of gradually varying permanent flows make this problem difficult to solve using conventional analytical methods. The solution to this type of problem is based on numerical simulation deploying a calculation code (FBA) designed in-house, programmed utilizing the Delphi programming language, and based on Pascal.

A. Example of Calculation 1

A trapezoidal channel with fixed parameters (positive slope \(I = 0.001\), width \(b = 10\) m, flow \(Q = \) m³/s, slope coefficient \(m = 1.5\) and water depth \(h = 1.5\) m), and varied rugosity coefficients \((n = 0.012\) m⁰/s, \(n = 0.08\) m⁰/s, \(n = 0.13\) m⁰/s).

\[ Q = V 	imes S = \frac{1}{n} \times R^{2/3} \times I^{1/2} \times S \]  

and given the above conditions:

1) 1st Case

\[ h_1 = 0.723\ m, h_2 = 0.842\ m, I = 0.0017. \quad \text{Since} \quad h > h_n > h_c \quad \text{and} \quad I > I_c, \quad \text{the Froude number} \quad F_c < 1, \quad \text{the backwash curve obtained is of the type M branch M1.} \]

2) 2nd Case

\[ h_1 = 0.723\ m, h_2 = 2.552\ m, I = 0.0739. \quad \text{Since} \quad h_n > h > h_c \quad \text{and} \quad I > I_c, \quad \text{the Froude number} \quad F_c < 1, \quad \text{the backwash curve obtained is of the type M branch M2.} \]

3) 3rd Case

\[ h_1 = 0.723\ m, h_2 = 3.358\ m, I = 0.1952. \quad \text{Since} \quad h_n > h > h_c \quad \text{and} \quad I > I_c, \quad F_c < 1, \quad \text{the backwash curve obtained is of the type M branch M2.} \]

B. Example of Calculation 2

A rectangular channel with fixed parameters (positive slope \(I = 0.001\), width \(b = 10\) m, flow \(Q = \) m³/s, slope coefficient \(m = 1.5\) and water depth \(h = 1.5\) m), and varied rugosity coefficients \((n = 0.012\) m⁰/s, \(n = 0.08\) m⁰/s, \(n = 0.13\) m⁰/s).

Given the above conditions:
1) 1st Case

$h_c = 0.742 \text{ m}, h_a = 0.866 \text{ m}, I_c = 0.0018$. Since $h > h_a > h_c$ and $I_c > I$, and the Froude number $F < 1$, the backwash curve obtained is of the type M branch M1.

2) 2nd Case

$h_c = 0.742 \text{ m}, h_a = 3.03 \text{ m}, I_c = 0.0786$. Since $h_c > h > h_a$ and $I_c > I$, and the Froude number $F < 1$, the backwash curve obtained is of the type M branch M2.

3) 3rd Case

$h_c = 0.742 \text{ m}, h_a = 4.233 \text{ m}, I_c = 0.2076$. Since $h_a > h > h_c$ and $I_c > I$, and the Froude number $F < 1$, the backwash curve obtained is of the type M branch M3.

C. Example of Calculation 3

A triangular channel with fixed parameters (positive slope $I = 0.001$, width $b = 10 \text{ m}$, flow $Q = 30 \text{ m}^3/\text{s}$, slope coefficient $m = 1.5$ and water depth $h = 1.5 \text{ m}$), and varied rugosity coefficients ($n = 0.012 \text{ m}^{-1/3}, n = 0.08 \text{ m}^{-1/3}, n = 0.13 \text{ m}^{-1/3}$). Given the above conditions:

1) 1st Case

$h_a = 2.411 \text{ m}, h_a = 2.662 \text{ m}, I_c = 0.0017$. Since $h_a > h > h_c$ and $I_c > I$, and the Froude number $F < 1$, the backwash curve obtained is of the type M branch M3.

2) 2nd Case

$h_a = 2.411 \text{ m}, h_a = 5.423 \text{ m}, I_c = 0.0754$. Since $h_a > h > h_c$ and $I_c > I$, and $F < 1$, the backwash curve obtained is of the type M branch M3.

3) 3rd Case

$h_a = 2.411 \text{ m}, h_a = 6.506 \text{ m}, I_c = 0.1991$. Since $h_a > h > h_c$ and $I_c > I$, and $F < 1$, the backwash curve obtained is of the type M branch M3.

D. Example of Calculation 4

A parabolic channel with fixed parameters (positive slope $I = 0.001$, width $b = 10 \text{ m}$, flow $Q = 30 \text{ m}^3/\text{s}$, slope coefficient $m = 1.5$ and water depth $h = 1.5 \text{ m}$), and varied rugosity coefficients ($n = 0.012 \text{ m}^{-1/3}, n = 0.08 \text{ m}^{-1/3}, n = 0.13 \text{ m}^{-1/3}$). Given the above conditions:

1) 1st Case

$h_a = 0.985 \text{ m}, h_a = 1.144 \text{ m}, I_c = 0.0016$. Since $h_a > h > h_c$ and $I_c > I$, and the Froude number $F < 1$, the backwash curve obtained is of the type M branch M1.

2) 2nd Case

$h_a = 0.985 \text{ m}, h_a = 3.713 \text{ m}, I_c = 0.073$. Since $h_a > h > h_c$ and $I_c > I$, and $F < 1$, the backwash curve obtained is of the type M branch M1.

3) 3rd Case

$h_a = 0.985 \text{ m}, h_a = 5.15 \text{ m}, I_c = 0.2076$. Since $h_a > h > h_c$ and $I_c > I$, $F < 1$, the backwash curve obtained is of the type M branch M2.

V. RESULTS AND DISCUSSION

The influence of rugosity on gradually varying permanent flows is decisive, as it has a direct influence on normal depth and critical slope. Each time the bottom resistance to flow increases, these parameters increase, and the desired distance on which the second boundary condition depends also increases, so these parameters are said to be directly proportional to rugosity. When these parameters are size parameters in the determination and classification of eddy curves and are influenced by rugosity, rugosity can be said to be one of the parameters for classifying eddy curves.

The influence of channel geometry on gradually varying permanent flows is less significant. This slight influence on the normal and critical depths and the critical slope is clearly shown in Figures 2 to 6, except for the triangular shape, where the influence is more significant and the eddy curve is always of branch 3 regardless of the class of the curve. The cross-section of the channel should be as close as possible to the actual cross-section, for both economic reasons and to ensure that the channel is correctly dimensioned, so that the developments to be planned do not influence the flow regime and do not alter the natural flow. When calculating the water line using both the iterative and direct integration methods, it can be seen that rugosity has a major influence on the results.
For this purpose, the method of depth variations for low rugosity is the most suitable for all geometric shapes except the parabolic, where Chow's method is the most suitable for medium rugosity. For fairly pronounced rugosity, Chow's method is the most suitable for all geometric shapes, except for the rectangular, where the first method is the most suitable.

Chow's method is best suited for trapezoidal and parabolic geometries, while for rectangular and triangular geometries, the depth variation method is the most appropriate.
VI. CONCLUSION

This study investigated the influence of rugosity, geometric form, and calculation methods on gradually varying permanent flows. The eddy curves were determined using two methods, namely the successive approximation method and the Chow method. These methods were coded, allowing visualization of the longitudinal profile of the water line for flows of known volume. When the eddy curve was calculated deploying these methods, rugosity was found to have a major influence on the calculation results. For low rugosity, the method of depth variations is the most suitable for all geometric shapes, except for the parabolic shape, where Chow's method is the most suitable. For medium rugosity, Chow's method is the most suitable for all geometric shapes, except for the rectangular shape where the first method is the most suitable. For high rugosity, Chow's method is the most suitable for trapezoidal and parabolic geometries, whereas for rectangular and triangular geometries, the depth variation method is the most suitable.

Determining the backwater curves (lowering/raising) enables us to make a judicious choice regarding the development measures to be taken along the non-prismatic
channel under consideration. Ultimately, the accuracy of the calculations must be considered about all the physical and hydrodynamic parameters that can influence the results. As these parameters are generally difficult to estimate, we were able to only estimate an average. Thus, knowing the flow parameters (channel size, flow rate, slope, rugosity, etc.), they could determine the longitudinal profile of the water line in the channel at the level of the various sections, and thus localize the phenomena generating surface waves, and perhaps even reproduce the waves and disturbances themselves. The direct consequences of flooding are often considered to be limited to the maximum flood zone generated at a particular point on the river. However, given the magnitude of the exceptional flood flows, the river bed could undergo profound upheavals, which can be summed up in very significant changes to the channel geometry. There is still a lot of work to be done on this subject. That said, superimposing the various specific treatments on the scale of a given channel seems likely to produce interesting visual results, and, above all, at a reasonable computational cost.

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