A Genetic Programming-Assisted Analytical Formula for Predicting the Permeability of Pervious Concrete

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ABSTRACT

This study proposes a new approach to construct predictive formulas for the permeability of Pervious Concrete (PC), which depends on PC mixture and porosity. To achieve this, a dataset of 195 samples collected from different sources was used. In the dataset the permeability is dependent on porosity, aggregate-to-cement ratio (AC), maximum nominal sizes (MS) of coarse aggregate, and water-to-cement or binder ratios (WC). From the dataset and through applying simple regression techniques, several analytical functions based on the Kozeny-Carman model were constructed and evaluated for their effectiveness in implementing independent datasets and similar analytical functions. Furthermore, for the first time, the Genetic Programming-based Symbolic Regression method was adopted to construct hybrid models combined with the Kozeny-Carman analytical model. The equation of the hybrid model ensures both basic physical conditions and efficiency while being simple enough for engineering-level applications.

Keywords-symbolic regression; genetic programming; permeability; machine learning; pervious concrete

I. INTRODUCTION

Sustainable Urban Drainage Systems (SUDS) have been widely studied in the past few decades to address urban drainage issues caused by urbanization (owning to the increase in the impervious surface area, such as concrete roads, parking lots, and buildings) and the growing threat of global warming (higher intensity rainfall). Based on the concept of natural drainage systems, SUDS directly and quickly drain the surface water from a rainfall to the subbase layer. They either move part of the surface water or all of it into the groundwater through the infiltration process. Various SUDS have been designed for different geographical areas around the world, depending on the type of permeable pavement (porous concrete, self-locking bricks, etc.) and the characteristics of the surface flow [1].

Pervious Concrete (PC) plays an important role in SUDS, serving the dual purpose of being a tool for managing rainfall and a load bearing surface for light/medium exploitation (internal roads, pedestrian and bicycle paths, parking lots, sidewalks, etc.). PC is a special type of concrete characterized by a connected porous structure and high porosity, usually ranging from 15 - 35% by volume, corresponding to an effective permeability of up to 6 mm/s. The surface layers made from PC have many advantages over the conventional concrete such as: reducing the risk of flooding, less traffic noise, lower surface temperature, improving groundwater level, low cost, etc. However, some studies have discussed the main difficulties of using PC in practice, including clogging, long-term durability, and optimal mix design [2].

In order to determine the mix proportion of PC, it is usually necessary to rely on the target value of the desired porosity. Then, the two basic properties of PC, namely strength and permeability, are either determined through experimental work or estimated based on some predictive models. Generally, due to the influence of the random spatial structure of the PC, the prediction of the physical and hydraulic properties of the PC is quite challenging, and it is still a topic of great interest in recent publications. The simplest approach to predict the permeability of PC is based on empirical functions. Due to the natural relationship between permeability and porosity (the higher the porosity is, the higher the permeability is, and vice versa), the empirical functions usually have the form of an exponential, of a power, or of a polynomial function of porosity combined with some other influencing parameters. A list of 34 empirical models is provided in [3]. These empirical analytical functions only represent the available experimental dataset and have little predictive value.

Thus, the PC permeability prediction model remains an unresolved challenge in both cases of the detailed experimental models and general formulas suitable for engineering applications. Inspired by the success of some recent studies in constructing character-based regression models, employing artificial intelligence algorithms and databases, this paper aims to establish a formula for predicting the PC permeability coefficient based on material characteristics. These formulas must ensure physical significance, accuracy, and applicability. To accomplish this task, the research strategy is: first, a suitable dataset is constructed (Section II); then, the basic analytical functions are reconstructed and simple regression functions are proposed (Section III); and finally, the effectiveness of the solutions is enhanced based on modern artificial intelligence algorithms. The outcome is a simple analytical formula to determine permeability based on fundamental material characteristics, providing a basis for constructing more sophisticated models in the future.

II. DATASET

The experimental database was constructed from 195 PC samples with different mix ratios, adopted from reliable open international sources [4-11]. Previous studies have widely recognized that the permeability of PC mainly depends on the characteristics of the voids, such as porosity value, void size, connectivity - continuity of the voids, and the flow features in this void system. Thus, the permeability of PC depends largely on the design mix composition of the material. In this study, a PC dataset was obtained from various sources of different PC samples, which were produced with various aggregates to cement or binder ratios (AC), maximum nominal sizes (MS) of coarse aggregate, water to cement or binder ratios (WC) and two important parameters of this type of material, namely effective porosity (ϕ) and hydraulic permeability coefficient (K). The permeability characteristics of PC are usually investigated in the laboratory by the Falling head method [4], [6] and/or with the constant head method [7-11]. The permeability measured utilizing the constant water column height usually gives higher values than the method employing the variable water column height. Within the scope of this database, the influence of the measurement method was not considered.

The obtained database contains information including AC, WC, MS, ϕ and K of the PC. It should be noted that the permeability K and porosity ϕ of PC in this dataset vary from 0.1 to 32.7 mm/s, and 10 to 40%, respectively, while the typical permeability of the PC can range from 1.4 to 12.2 mm/s when the porosity is between 15 and 35 % [12]. The statistical parameters of the database are displayed in detail in Table I. In Figure 1, the correlations between the variables of the dataset are presented through the Pearson coefficient, ranging from -1 to 1.

TABLE I. STATISTICAL PARAMETERS OF THE DATASET.

Index	Unit	Mean	Min.	Max.	Recommendations according to [12]
AC	-	4.66	3.03	12.00	4.0-4.5
WC	-	0.3	0.22	0.40	0.27-0.34
MS	mm	11.0	4.5	19.0	9.5-19.0
ϕ	%	28.6	10.0	40.0	15-35
K	mm/s	13.1	0.1	32.7	1.4-12.2

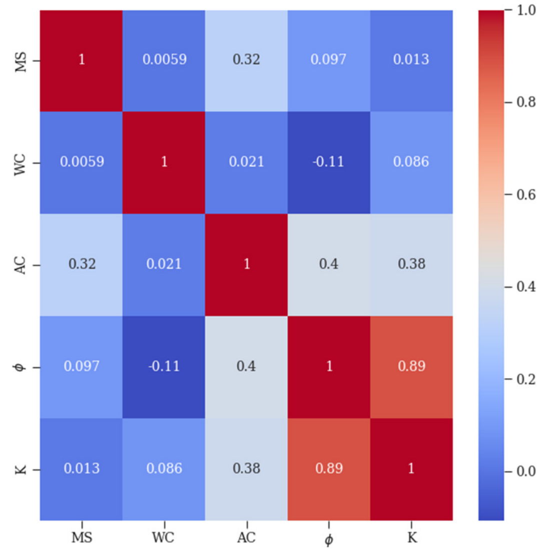

Fig. 1. Correlation heatmap of the dataset.

III. THE THEORY-BASED EVALUATION APPROACH

A. *Kozeny-Carman Equation*

In geotechnical engineering, the hydraulic conductivity an/or the intrinsic permeability can be predicted based on empirical relationships, probabilistic models, or theoretical models of flow. Unlike industrial materials (e.g. composites) that have fairly homogeneous material structures, geotechnical materials (soil, rock, and cement-based materials) often have complex spatial structures, so the theoretical models for predicting the mechanical properties are usually developed from simple theoretical approaches, then improved based on empirical approximations to better reflect the actual results. The Kozeny-Carman equation for predicting the permeability coefficient is also built in this way. Several different forms of the Kozeny-Carman equation have been proposed to determine the hydraulic conductivity/intrinsic permeability of geotechnical materials. For the convenience of the readers, the Kozeny-Carman equation, which has been introduced in previous studies [13], is re-established here. Then, this equation is combined with the current database to propose some new equations suitable for the characteristics of pervious concrete. To establish the theoretical formulas in an attempt to determine the permeability coefficient, two experimental and theoretical equations related to the Darcy flow and the Poiseuille one-dimensional flow are simultaneously considered as illustrated in Figure 2.

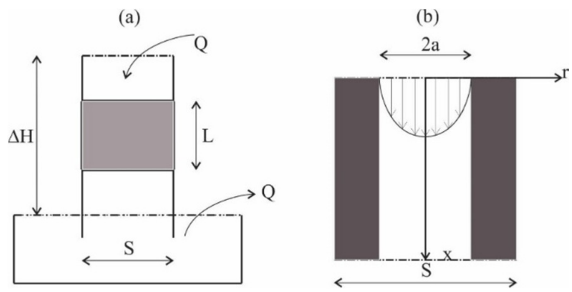

Fig. 2. Theoretical illustration: (a) Darcy permeability experiment, (b) Poiseuille one-dimensional flow.

The basic Darcy equation is expressed as follows:

$$q = \frac{Q}{S} = K\frac{\Delta H}{L} \tag{1}$$

Q is the flow rate, in m^3/s, created by a water pressure ΔH and flowing through the material block of a height L (m) and cross-section S (m^2). As observed in Figure 2(b) and considering the flow through a circular cylindrical channel of radius , the flow velocity will depend on the material viscosity coefficient μ and the hydraulic gradient according to the Poiseuille solution:

$$u(r) = -\frac{1}{4\mu}\frac{dP}{dx}(a^2 - r^2) \tag{2}$$

Balancing the two theoretical and experimental equations of one-dimensional static flow, the formula to determine the hydraulic permeability coefficient K (m/s) is obtained:

$$K = \frac{\rho g}{\mu}\frac{\pi a^4}{8S} \tag{3}$$

In the general case, if the tortuosity coefficient, τ is defined as the ratio between the actual length of the flow and the length of the material sample, and s_v is denoted as the surface-to-volume ratio, then:

$$\phi = \frac{\pi a^2 \tau}{S}, s_v = \frac{2\pi a L \tau}{SL} = \frac{2\phi}{a} \tag{4}$$

Several semi-empirical expressions have been developed for different types of materials. For PC, it is first assumed that the granular material consists of spheres of diameter d, then the surface-to-volume ratio can be calculated by:

$$s_v = \frac{6(1-\phi)}{d} \tag{5}$$

Substituting (4) and (5) into the basic equation (3), a semi-empirical equation is obtained to determine the hydraulic permeability coefficient K of PC:

$$K = \frac{\rho g}{\mu}\frac{\phi^3 d^2}{72(1-\phi)^2\tau^2} = \frac{(\phi-\phi_p)^3}{(1-\phi+\phi_p)^2}A \tag{6}$$

ϕ_p is the value of porosity at which permeability begins to occur. The coefficient A is characteristic of the fluid properties as well as the spatial structure of the porous material.

B. Application to Pervious Concrete

The task of this section is to evaluate the applicability of the theoretical equation developed above and propose some specific applications. To ensure the practicality of the research solution, this study sets hydraulic conductivity in units of mm/s, and the particle size of the aggregate, d, in mm. It is obvious that, in terms of physical dimension, equation (6) is consistent with the intrinsic permeability of the material when A is a dimensionless parameter. In the case of hydraulic conductivity, A can have units of 1/mm.s, owing to the $\rho g/\mu$ coefficient. However, this conversion does not affect the process of estimating the coefficient of the equation. Based on the data constructed in Section II, two equations using simple regression analysis are proposed in Table II, along with the corresponding R^2 coefficients.

TABLE II. THE THEORY-BASED AND EMPIRICAL PREDICTING EQUATIONS

	Ref.	Equation	R^2 coefficient		
			Dataset in [3]	This work dataset	Dataset in [14]
(1)	This work	$K = \frac{(\phi)^3}{(100-\phi)^2}1.92$	-	0.73	0.89
(2)	This work	$K = \frac{(\phi)^3}{(100-\phi)^2}(-0.3WC + 0.093MS + 2.4CA + 43.5CA^2 - 126.18CA^3)$	-	0.86	0.89
(3)	[3]	$K = 0.008\phi^{2.1621}$	0.7	0.75	0.85
(4)	[3]	$K = 0.603\phi + 0.1498AC + 2.051WC - 6.545$	0.66	0.85	0.8

(note: CA= l/AC)

In Figure 3, the correlations between the experimental and the predicted values of the proposed equations are presented. It should be emphasized that these two models are selected from many different options through the least squares rule. The results exhibit that parameter A depends on the input parameters WC, AC, MS based on a polynomial function, corresponding to (2), which provides better results than the complex nonlinear power functions that are often used. Equation (1), on the other hand, represents the simplest form possible, but still achieves an impressive R^2 of 0.73.

Next, the effectiveness of the constructed predicting equations is examined. Authors in [3] conducted a rather detailed study on creating a simple analytical function to predict the permeability coefficient of PC. In this study, 34 equations determining the permeability coefficient of the porous material in the form of linear trend, two-degree polynomial, exponential, power, and Carman–Kozeny relationships were synthesized from various sources. Subsequently, the authors built a database consisting of 695 samples for evaluation. From there, they selected and constructed the 2 best equations ((3) and (4) in Table II), where (3) is a power function only dependent on porosity and (4) is a polynomial function taking into account both AC and WC indices. The results for the dataset of [3] are not exceptional, exhibiting $R^2 = 0.7$, and 0.66 for (3) and (4), respectively. This indicates that the dataset is quite scattered. Unfortunately, the dataset is not available, making it difficult to be assessed in detail.

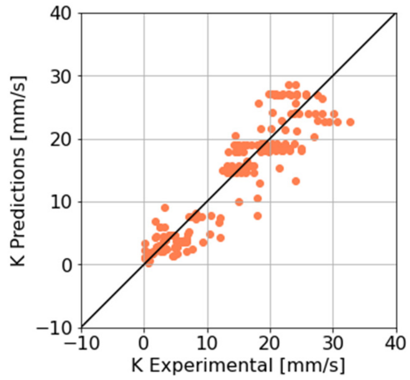

Fig. 3. Compare the predictions results and experimental results of the proposed theory-based equations.

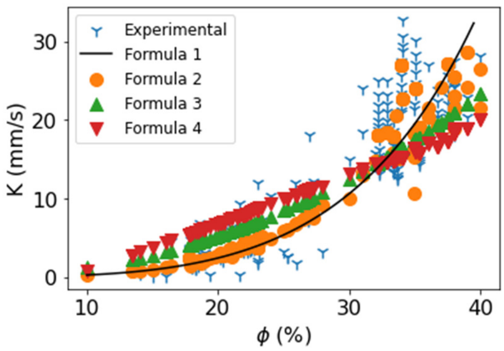

Fig. 4. Comparison between the prediction results of the analytical equations.

Returning to the evaluation of the effectiveness of the constructed models, first, 2 models proposed from [3] are applied in the proposed dataset. The results of (3), and (4) exhibit correlation coefficients R^2 of 0.75 and 0.85, respectively, indicating that this dataset considers the role of other material parameters, such as WC and AC, and is more suitable. The corresponding results for (1) and (2) are also exhibited in Table II. To provide an objective evaluation, this study uses independent data proposed by [14] to assess the effectiveness of the 4 predicting formulas mentioned above. The results are depicted in Table II. The models proposed in this study perform better with a correlation coefficient that reaches $R^2 = 0.89$ for the simple and complex equations with the independent test dataset. From Figure 4, it is evident that predicting the permeability coefficient solely based on porosity (1) does not offer any advantage. Conversely, to account for the influence of other material parameters such as WC and AC, more complex nonlinear equations need to be constructed. This topic will be discussed in the following section.

IV. HYBRID MODEL

In recent years, several analytical models [15] and artificial intelligence approaches [16] have been developed to predict the fundamental properties of porous concrete. In [17] a symbolic regression approach based on the genetic programming framework was utilized to assess the compressive strength of the porous concrete, achieving favorable results compared to purely analytical solutions or black-box machine learning models such as ANN and XGB [16]. However, symbolic regression models often suffer from drawbacks such as lack of physical significance and violations of fundamental physical laws. To enhance efficiency and ensure accuracy and applicability, a hybrid equation will be developed based on both analytical development and the dataset from the previous section, along with artificial intelligence tools focusing on symbolic regression.

A. Genetic Programming-based Symbolic Regression Method

Within the field of artificial intelligence, Genetic Programming (GP) is an automated methodology that emulates natural selection to seek an optimal outcome, initially pioneered in 1992 [18]. However, one of the major drawbacks of the GP method compared to other machine learning approaches is the computation time. To accelerate the speed of computation, this study applies a new algorithm proposed in [19], called Operon, a modern C++/Python framework for symbolic regression based on the genetic programming method to explore mathematical expressions and to find the best fit model for a given problem. Unlike other standard tree-based GP methods, Operon utilizes parallelism in the evolutionary process, generating each new individual independently in its own logic stream. The main algorithm and the C++/Python script are introduced in [19].

Once the computational framework is established to search for solutions, selecting the best function remains a critical question to address. For each function that is the result of symbolic regression, there are two most important criteria for evaluating the accuracy and complexity of the function. A function with high accuracy sometimes has too high complexity, leading to difficulties in explicit representation and practical application. Conversely, with concise functions, it is often difficult to achieve high efficiency compared to other functions. Choosing the best solution that balances the efficiency and accuracy of the model in the obtained result set is not a simple task. To solve this problem, this study performed many GP calculations and saved the resulting functions along with their parameters, such as statistical measures, complexity, and running time. Then, the Pareto front was constructed as the optimal result in terms of accuracy and complexity. Finally, the optimal option from the Pareto front was chosen.

B. Application to Pervious Concrete

According to [17], there are four main parameters that affect the accuracy of the model: initial population depth, tournament size, population size, and number of generations. In this study, simulations were conducted using the following parameter values: initial population depth ranging from 2 to 6, tournament sizes of 5, 10, 15, 20, 25, and 30, population sizes varying from 1000 to 2000, and number of generations set at 20, 40, 60, 80, and 100. Each parameter set for the model was calculated 10 times, totaling 15,000 calculations. In each calculation, the functions were randomly generated; then the function with the highest accuracy was saved and evaluated with the validation dataset. Then, two scenarios were examined. The first scenario, denoted as the GP formula, corresponds to applying the model to the entire constructed dataset. The second scenario, based on the database, only predicts the *A* coefficient in the Kozeny-Carman equation, whereas the first coefficient related to porosity remains unchanged. This case is denoted as the hybrid formula, representing the combination of an analytical equation and a character regression algorithm. Figure 5 introduces the trade-off between accuracy and complexity of the proposed formulas. It should be noted that the definition of a reasonable complexity of a mathematical expression is a topic of debate. Here, a simple definition that complexity is the number of mathematical operators, features, and constants in the model, also known as the node length in GP, is accepted. Next, the corresponding Pareto fronts are constructed (the blue line shown in Figure 5). It is worth mentioning again that the Pareto front consists of optimal points that are not dominated by any other solution in both accuracy and complexity.

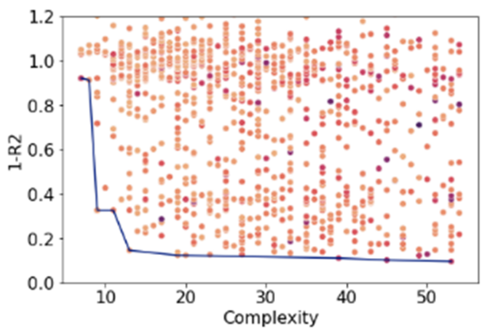

Fig. 5. The trade-off accuracy complexity of the formulas.

Table III introduces the two best-fitting equations obtained on the corresponding Pareto fronts, and the correlation coefficients with the two constructed datasets, and the independent validation dataset in [16]. It is apparent that the GP formula is slightly better than the hybrid model on the constructed dataset and displays no difference on the independent dataset. However, the GP formula violates physical laws in some instances, leading to some permeability coefficients lower than 0, and has higher complexity (Figure 6). In fact, the hybrid model has several advantages for practical application. Figure 7 illustrates the comparison of the hybrid formula with the purely analytical formulas constructed above.

TABLE III. THE GP AND HYBRID PREDICTING FORMULAS

Formula	**Equation**	**R^2 coefficient**	
		This work dataset	**Dataset in [16]**
GP	$K = 0.83 * \phi - 13.36 * (0.48 - 0.06 * MS) * (0.71 - 0.04 * \phi) * (4.91 - 13.15 * WC) * (-0.33 * MS - 0.25 * \phi + 9.21 + 33.92 * e^{-0.78*AC}) - 12.41$	0.91	0.9
Hybrid	$K = \frac{\phi^3}{(100 - \phi)^2}(3.74 * \exp(-18875500 * (0.23 - WC)^2 * (0.31 - WC)^2 * (1 - 0.056 * MS)^2 - 275.94 * (0.254 - CA)^2) + 1.96)$	0.89	0.9

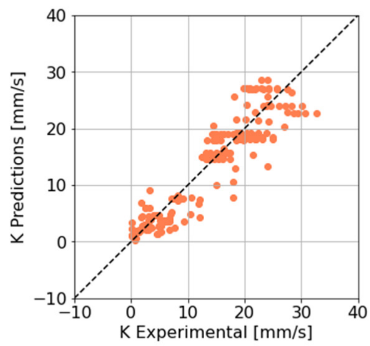

Fig. 6. Comparison of the prediction and the experimental results of the proposed formulas.

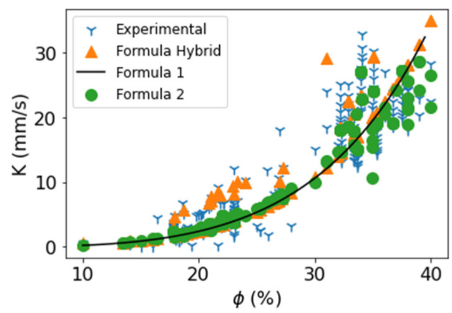

Fig. 7. Comparison of the prediction results of the proposed formulas with the experimental data.

V. CONCLUSIONS

The pursuit of developing straightforward analytical formulas to predict the permeability coefficient of pervious concrete material is a compelling subject; however, it has not produced significantly successful outcomes to date. This study introduces a novel approach for exploring mathematical formulas to predict the permeability coefficient of pervious concrete. Following this approach, a dataset consisting of 195 samples from 12 independently published sources is initially constructed. Subsequently, this study redefines the Kozeny-

Carman equation in which the permeability coefficient depends on porosity and the coefficient *A*, which characterizes the spatial structure of the material. Utilizing simple regression techniques, simplified equation forms are established and demonstrated to be effective through comparison with independent datasets and recently proposed analytical equations. Furthermore, to enhance the effectiveness of the method, for the first time, the Genetic Programming-based Symbolic Regression method is applied to compute the permeability coefficient of the pervious concrete material. When this method is combined with the original analytical equation and symbolic regression technique, a hybrid equation is formulated. This equation ensures both basic physical conditions and efficiency while being simple enough for engineering-level application. This is the main and noteworthy result of the current work. However, it is necessary to validate the proposed results through experimental testing. The computational process developed in this paper can also be applied to other types of non-conventional concrete materials.

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REFERENCES

[1] B. Ferguson, *Porous Pavements*. Boca Raton, Florida, USA: CRC Press, 2005.

[2] R. Zhong, Z. Leng, and C. Poon, "Research and application of pervious concrete as a sustainable pavement material: A state-of-the-art and state-of-the-practice review," *Construction and Building Materials*, vol. 183, pp. 544–553, Sep. 2018, https://doi.org/10.1016/j.conbuildmat.2018.06.131.

[3] A. Abdelhady, L. Hui, and H. Zhang, "Comprehensive study to accurately predict the water permeability of pervious concrete using constant head method," *Construction and Building Materials*, vol. 308, Nov. 2021, Art. no. 125046, https://doi.org/10.1016/j.conbuildmat.2021.125046.

[4] X. Yang, J. Liu, H. Li, and Q. Ren, "Performance and ITZ of pervious concrete modified by vinyl acetate and ethylene copolymer dispersible powder," *Construction and Building Materials*, vol. 235, Feb. 2020, Art. no. 117532, https://doi.org/10.1016/j.conbuildmat.2019.117532.

[5] I. Y. Amir, A. M. Yusuf, and I. D. Uwanuakwa, "A Metaheuristic Approach of predicting the Dynamic Modulus in Asphalt Concrete," *Engineering, Technology & Applied Science Research*, vol. 14, no. 2, pp. 13106–13111, Apr. 2024, https://doi.org/10.48084/etasr.6808.

[6] A. Rezaei Lori, A. Bayat, and A. Azimi, "Influence of the replacement of fine copper slag aggregate on physical properties and abrasion resistance of pervious concrete," *Road Materials and Pavement Design*, vol. 22, no. 4, pp. 835–851, Apr. 2021, https://doi.org/10.1080/14680629.2019.1648311.

[7] H. Wang, H. Li, X. Liang, H. Zhou, N. Xie, and Z. Dai, "Investigation on the mechanical properties and environmental impacts of pervious concrete containing fly ash based on the cement-aggregate ratio," *Construction and Building Materials*, vol. 202, pp. 387–395, Mar. 2019, https://doi.org/10.1016/j.conbuildmat.2019.01.044.

[8] H. Zhou, H. Li, A. Abdelhady, X. Liang, H. Wang, and B. Yang, "Experimental investigation on the effect of pore characteristics on clogging risk of pervious concrete based on CT scanning," *Construction and Building Materials*, vol. 212, pp. 130–139, Jul. 2019, https://doi.org/10.1016/j.conbuildmat.2019.03.310.

[9] W. Yeih and J. J. Chang, "The influences of cement type and curing condition on properties of pervious concrete made with electric arc furnace slag as aggregates," *Construction and Building Materials*, vol. 197, pp. 813–820, Feb. 2019, https://doi.org/10.1016/j.conbuildmat.2018.08.178.

[10] J. T. Kevern, D. Biddle, and Q. Cao, "Effects of Macrosynthetic Fibers on Pervious Concrete Properties," *Journal of Materials in Civil Engineering*, vol. 27, no. 9, Sep. 2015, Art. no. 06014031, https://doi.org/10.1061/(ASCE)MT.1943-5533.0001213.

[11] A. Ibrahim, E. Mahmoud, M. Yamin, and V. C. Patibandla, "Experimental study on Portland cement pervious concrete mechanical and hydrological properties," *Construction and Building Materials*, vol. 50, pp. 524–529, Jan. 2014, https://doi.org/10.1016/j.conbuildmat.2013.09.022.

[12] "Report on Pervious Concrete (Reapproved 2011) Reported by ACI Committee 522," American Concrete Institute (ACI), ACI 522 R10, 2010.

[13] S. V. Thai, H. V. Viet, C. N. Tuan, A. T. D. Thao, and V. T. Bao, "Predicting the permeability of pervious concrete based on a data-driven approach," *Transport and Communications Science Journal*, vol. 73, no. 2, pp. 176–188, 2022, https://doi.org/10.47869/tcsj.73.2.7.

[14] F. Montes and L. Haselbach, "Measuring Hydraulic Conductivity in Pervious Concrete," *Environmental Engineering Science*, vol. 23, no. 6, pp. 960–969, Nov. 2006, https://doi.org/10.1089/ees.2006.23.960.

[15] V.-H. Vu, B.-V. Tran, B.-A. Le, and H.-Q. Nguyen, "Prediction of the relationship between strength and porosity of pervious concrete: A micromechanical investigation," *Mechanics Research Communications*, vol. 118, Dec. 2021, Art. no. 103791, https://doi.org/10.1016/j.mechrescom.2021.103791.

[16] B.-A. Le *et al.*, "Predicting the Compressive Strength and the Effective Porosity of Pervious Concrete Using Machine Learning Methods," *KSCE Journal of Civil Engineering*, vol. 26, no. 11, pp. 4664–4679, Nov. 2022, https://doi.org/10.1007/s12205-022-1918-z.

[17] B.-A. Le, B.-V. Tran, T.-S. Vu, V.-H. Vu, and V.-H. Nguyen, "Predicting the Compressive Strength of Pervious Cement Concrete based on Fast Genetic Programming Method," *Arabian Journal for Science and Engineering*, vol. 49, no. 4, pp. 5487–5504, Apr. 2024, https://doi.org/10.1007/s13369-023-08396-2.

[18] J. R. Koza, "Genetic programming as a means for programming computers by natural selection," *Statistics and Computing*, vol. 4, no. 2, pp. 87–112, Jun. 1994, https://doi.org/10.1007/BF00175355.

[19] B. Burlacu, G. Kronberger, and M. Kommenda, "Operon C++: an efficient genetic programming framework for symbolic regression," in *Proceedings of the 2020 Genetic and Evolutionary Computation Conference Companion*, Cancun, Mexico, Jul. 2020, pp. 1562–1570, https://doi.org/10.1145/3377929.3398099.