Computer Architectures Empowered by Sierpinski Interconnection Networks utilizing an Optimization Assistant

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ABSTRACT

The current article discusses Sierpinski networks, which are fractal networks with certain applications in computer science, physics, and chemistry. These networks are typically used in complicated frameworks, fractals, and recursive assemblages. The results derived in this study are in mathematical and graphical format for particular classes of these networks of two distinct sorts with two invariants, K-Banhatti Sombor (KBSO) and Dharwad, along with their reduced forms. These results can facilitate the formation, scalability, and introduction of novel interconnection network topologies, chemical compounds, and VLSI processor circuits. The mathematical expressions employed in this research offer modeling insights and design guidelines to computer engineers. The derived simulation results demonstrate the optimal ranges for a certain network. The optimization assistant tool deployed in this work provides a single maximized value representing the maximum optimized network. These ranges can be put into service to dynamically establish a network according to the requirements of this paper.

Keywords-Sierpinski; Dharwad; invariants; k-Banhatti sombor indices; maple; network graph; interconnection network; inclusive innovation

I. INTRODUCTION

Sierpinski networks play an important role in the electrical and electronic design of Local Area Networks (LANs), processor interconnections, music creation, and power generation interconnections, especially where inheritance characteristics are required. Sierpinski networks are networks that have repeated factors in triangular, pentagon, or other shapes and are frequently used in interconnections. The relationship of chemical networks with various actual qualities and substance reactivity is shown by the topological index, a real number connected to the synthetic constitution [1]. The Sierpinski network is another physical layout for interconnection networks in multiprocessor systems and LANs. Sierpinski is a class of networks characterized by a few boundaries and with a consistent number of connections/hubs for a few given boundaries. As the size and intricacy of a framework increase, the dependability angles become similarly significant and should be remembered for the framework execution study. Sierpinski networks have very adaptable costs and execution. The Sierpinski can be a successful interconnection network for future parallel computer systems and LANs [2, 3]. However, the issue of interconnecting processors to achieve high computational data transfer capacity has not been completely addressed. Expanded parallelism implies more correspondence among processors and, subsequently, a significant increase in overheads. Internode distance, message traffic thickness, and fault resistance are subject to the measurement and level of the hub. The distance across the level of a hub is a great basis for gauging the cost and execution of a multiprocessor framework. An interconnection network with an enormous distance across it has an extremely low message-passing transmission capacity. Configuration systems should be effectively expandable, and progressions in the fundamental hub setup are intricate as the number of hubs increases [4]. In contrast to the large number of other existing networks, the Sierpinski network has a steady degree, simplifying its extension and scaling. It is appropriate for VLSI designs since this technology expects processors in a chip to have a steady number of ports due to VLSI pin constraints [3]. A measured development is yet conceivable, even after the processors are created onto solid chips. The

Sierpinski network is a hierarchical construction with adjoining hubs associated firmly and distant hubs associated freely. This property matches the correspondence prerequisites of many parallel application calculations in a traffic region [9].

The concept of Sombor indices was defined in [5]. The sharp lower and upper bounds of the linked network, as well as the properties reaching the boundaries, are captured using a novel vertex degree-based invariant graph, called the Sombor index. The K-Banhatti Sombor (KBSO) indices come in two versions: full and reduced [5]. The KBSO index is a numerical representation of a network graph that depicts the network's symmetry and provides a scientific language for describing its characteristics [6]. The Dharwad degree-based topological indices come in forms, such as reduced Dharwad, reduced Dharwad exponential, and δ-Dharwad index, which have been utilized to solve the topology of aromatic compounds [7]. Cheminformatics is a new field of study that combines computer science, mathematics, and chemistry. Its key components include QSAR and QSPR. The QSAR and QSPR are modeling methods that engage mathematical equations or expressions to correlate network structure features. In addition, they offer a quantifiable link between the characteristics of chemical or network architectures. Topological angles may be represented graphically employing numeric values, due to invariance and the graph's automorphism characteristic. There are several applications of graph theory in the domains of chemistry and computer science [8].

The planning of a topological index involves the numerical conversion of a network structure. The initial intent was to present novel computer systems and networks that benefit from efficiency and progress using topological indices. A graph can be deployed to exhibit the structure of an interconnection network analytically. The geography of a graph determines how vertices and edges are connected. Graph theory is directly related to networks, as the geography of a network and certain properties can easily be resolved by topological indices such as KBSO and Dharwad. Any two network hubs can resolve the maximum distance between them. The number of connections to the hub reveals its level. Using complex network analysis equipment and networks - from intranet to global networks, electric power connections, social networks, sexual disease transmission networks, and genetic networks - can be described with graph theory.

Topological Invariants (TIs) are boundaries that mathematically represent the connection designs (structure) between the hubs or performers in a network. These models are built on the foundation of QSAR and QSPR. The use of measurements in the network plane enables a quantitative assessment of different mapping techniques to protect geographies. It is crucial to support reliable quantitative methodologies that can disclose which important locations and landscape elements play a crucial and prominent role in the functioning of territorial mosaics to improve the efficacy of observation and conservation efforts [10].

II. LITERATURE REVIEW

In [11], the connections between math and music were investigated through fractals known as the Sierpiński Triangle and the Sierpiński Pedal Triangle. In the past 25 years, substantial attention has been paid to graphs whose illustrations may be viewed as approximations of the well-known Sierpiski triangle. These graphs are intriguing for a variety of reasons, including games such as the Tower of Hanoi or the Chinese rings, topology, physics, the study of connectivity networks, and more. In [12], the properties of Sierpiński-type graphs were classified. An extraordinary instance of summed up Sierpinski graphs is the duality of Apollonian organization, which displays the unmistakable without scale little world attributes as seen in different genuine networks. These networks have been a topic of ongoing examination for different properties [13]. This group of graphs can be produced by taking a specific number of duplicates of a similar fundamental diagram. A topological index is a number that demonstrates a few fundamental properties of compound structures, such as Sierpinski graphs [14]. The Sierpinski fractal or Sierpinski gasket Σ is a natural item contemplated in dynamical systems. In [15], an S_n diagram emphasized the interaction that prompts Σ and focused on a portion of its properties, including its cycle structure, mastery number, and pebbling number. Sierpinski and Sierpinski-type graphs are considered in fractal hypothesis and show up normally in assorted areas of mathematics and other scientific fields [16, 17]. Sierpinski graphs create a densely populated class of fractal graphs. Numerous characteristics, including physicochemical, thermodynamic, chemical motion, and organic activity, are not fixed by applying graph theory. Specific graph invariants known as topological indices can be used to represent these features. Invariants played a crucial role in the QRAR/QSPR work focusing on these graphs [18]. The m-polynomial is helpful in fictitious chemistry because it contributes significantly to the processing of the specific enunciations of different degree-based topological indices. In [18-19], the m-polynomial of these networks was calculated and then other degree-based topological indices were reconstructed. Most existing approaches rely on the neighborhood or numerical properties of pixels [20]. Recently, there has been a focus on explicit network geographic characteristics [21]. A supra-atomic synergist machine can result from a high thickness of reactant locales combined with specific spatial headings. According to the ongoing variety of MOF geographies, these porous systems are likely to prove to be effective authoritative structures for a wide range of multisite-catalyzed chemical reactions [22]. In [23], Sierpinski networks were employed for hand gesture recognition.

III. RESEARCH METHODOLOGY

A. *Objectives*

This study investigates the topological invariants of Sierpinski networks, presenting the merits of the KBSO and Dharwad indexes and their simplified variants. The findings can be exploited for the modeling and scalability of Sierpinski networks.

B. *Significance*

The results of solving existing Sierpinski networks through modern invariants can create new network topologies with distinct features, which can be utilized in the disciplines of interconnection networks, signal processing systems, image processing, multiprocessing, parallel computing, and chemical structures [24, 32-36]. The analysis provides the strength to develop error-free, failure-free, and best-performing networks.

C. Research Contributions

This study aims to improve current networks by maximizing their flexibility. Previous studies modeled specific networks using inferences made from topological invariants. The findings are developed deploying newly created topological indices to visually solve networks. In addition, it is examined how the network's or graph's lower and upper boundaries relate to each other. These connections are shown through optimization. The findings can offer guidelines for the design of advanced networks.

D. Method

Using TIs, this study takes an existing Sierpinski network, connects it to a graph, performs mapping over it, and resolves its topology. Formula-based outcomes are compared to earlier findings. This study also optimized the results with an optimization assistant. Figure 1 represents the study's progression for mapping, connecting, resolving, contrasting, and modeling the network using inferred outcomes [25-26, 40-41].

IV. RESULTS

Sierpinski graphs S(a, 3) are the graphs of the Tower of Hanoi with n disks. On the other hand, Sierpinski gasket graphs Sa are graphs typically portrayed by the set number of repetitions that lead to the Sierpinski gasket graph. The number of vertices and edges of the Sierpinski Gasket Graph are $3a/2$ - $3/2$ in addition, to $3a$ exclusively. Equations (1) and (2) show the KBSO index and its reduced form that will be used for the solution of the Sierpinski computer network mentioned in Figures 2, 4, and 6 [27]. Table I presents the frequency distributions of the edge partitions of the Sierpinski graph G.

$$KBSO(G) = \sum_{ue} \sqrt{d_u^{\,2} + d_e^{\,2}} \quad (1)$$

$$KBSO_{red}(G) = \sum_{ue} \sqrt{(d_u - 1)^2 + (d_e - 1)^2} \quad (2)$$

TABLE I. EDGE PARTITION OF SIERPINSKI (SN)

E	$\varepsilon(d_u, d_v)$	d_e	$\varepsilon(d_u, d_e)$	Recurrence
ε_1	ε(2, 4)	4	ε(2, 4)	6
ε_2	ε(4, 4)	6	ε(4, 6)	3^n-6

$d_e = d_u + d_v - 2$

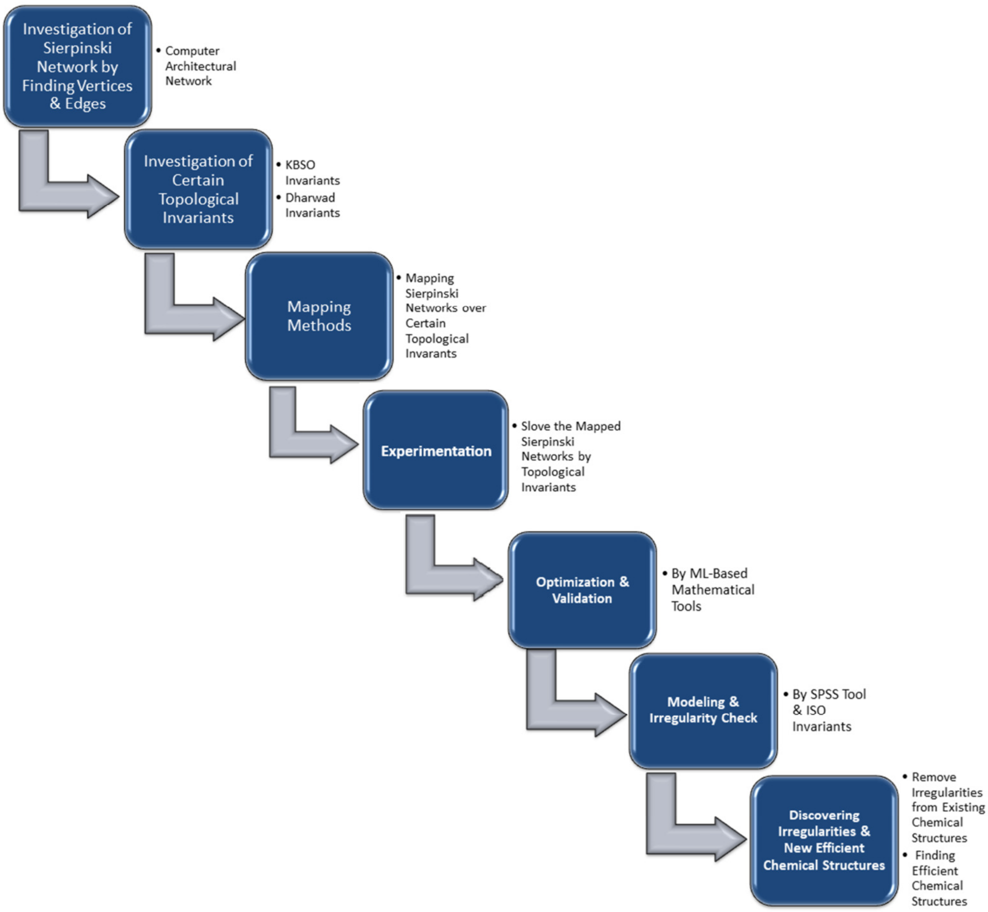

Fig. 1. Methodology flow diagram: represents the road map for the precedence of mapping, linking, solving, comparing, and modeling the network by deduced results [37-39].

A. Main Results

In computer science and electronics, the Sierpinski network with dimension S_n is utilized to implement the loop notion. It is particularly effective in ICs, as portrayed in Figure 2.

1) Sierpinski Network Graph

The Sierpinski graph has two different types of edges, as observed in Figure 2. The edges' division is described in Table I.

2) Theorem 1

Let G be an S_n graph, then, *KBSO* and $KBSO_{red}$ indices are:

$$KBSO(G) = 12\sqrt{5} - 2\sqrt{13}(3^n - 6) \quad (3)$$

$$KBSO_{red}(G) = 6\sqrt{10} + \sqrt{34}(3^n - 6) \quad (4)$$

The established findings of the S_n graph, seen in Figure 2, are represented by (3) and (4) to enhance current networks and create new designs using the *KBSO* and $KBSO_{red}$ invariants [28-29].

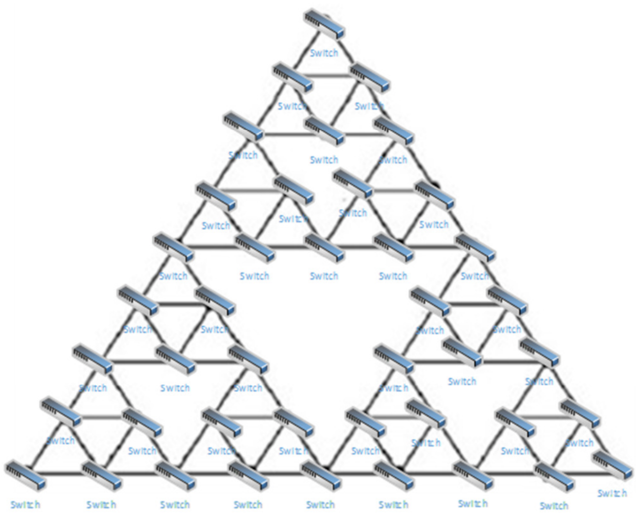

Fig. 2. Sierpinski network S_n.

3) Investigation of S_n Graphs by KBSO Indices

Proof:

$$KBSO(G) = \sum_{ue} \sqrt{d_u{}^2 + d_\varrho{}^2}$$

$$KBSO(G) = \sqrt{2^2 + 4^2}\,(6) + \sqrt{4^2 + 6^2}\,(3^n - 6)$$

$$KBSO(G) = 12\sqrt{5} - 2\sqrt{13}(3^n - 6)$$

$$KBSO_{red}(G) = \sum_{ue} \sqrt{(d_u - 1)^2 + (d_\varrho - 1)^2}$$

$$KBSO_{red}(G) = \sqrt{(2-1)^2 + (4-1)^2}(6)$$
$$+\sqrt{(4-1)^2 + (6-1)^2}\,(3^n - 6)$$

$$KBSO_{red}(G) = 6\sqrt{10} + \sqrt{34}(3^n - 6)$$

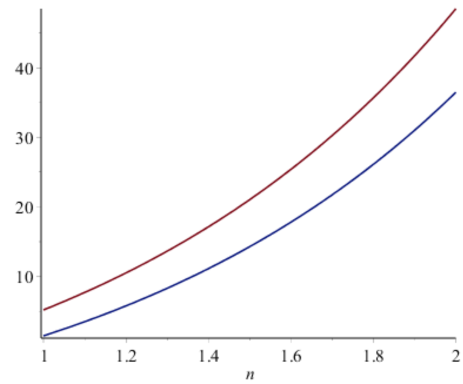

Fig. 3. Sierpinksi network S_n: KBSO (red) and $KBSO_{red}$ (blue) indices.

Figure 3 demonstrates the results of the *KBSO* index and its reduced form for the Sierpinski network S_n in red and blue lines, respectively. The diagram depicts the upper and lower bounds to calculate the efficiency of the network.

$$D(G) = \sum_{ue} \sqrt{d_u^3 + d_v^3} \quad (5)$$

$$RD\,(G) = \sum_{ue} \sqrt{(d_u - 1)^3 + (d_v - 1)^3} \quad (6)$$

These equations reflect the Dharwad index and its simplified form, two novel vertex-degree-based invariants. They are used to solve the Sierpinski network to enhance current networks and create new designs.

4) Theorem 2

Let G be a graph of S_n, then, the Dharwad and reduced Dharwad indices are:

$$D(G) = 15r + 8 + \sqrt{35}\,r + 3\sqrt{6}\,(r - 3) \quad (7)$$

$$RD(G) = 8r + \sqrt{2}\,(3r + 2) - 12 \quad (8)$$

Equations (7) and (8) are the proven results of the Sierpinski network by the Dharwad index and its reduced form, respectively.

5) Investigation of Sierpinski Network by Dharwad Indices

Proof:

$$D(G) = \sum_{ue} \sqrt{d_u^3 + d_v^3}$$

$$D(G) = \sqrt{2^3 + 4^3}\,(6) + 9\sqrt{4^3 + 4^3}\,(3^n - 6)$$

$$D(G) = 36\sqrt{2} + 8\sqrt{2}\,(3^n - 6)$$

$$RD\,(G) = \sum_{ue} \sqrt{(d_u - 1)^3 + (d_v - 1)^3}$$

$$RD\,(G) = \sqrt{(2-1)^3 + (4-1)^3}\,(6)$$
$$+\sqrt{(4-1)^3 + (4-1)^3}\,(3^n - 6)$$

$$RD\,(G) = 12\sqrt{7} + 3\sqrt{6}\,(3^n - 6)$$

Using the Dharwad index and its simplified version, Figure 4 displays the upper and lower limits of a solved network S_n.

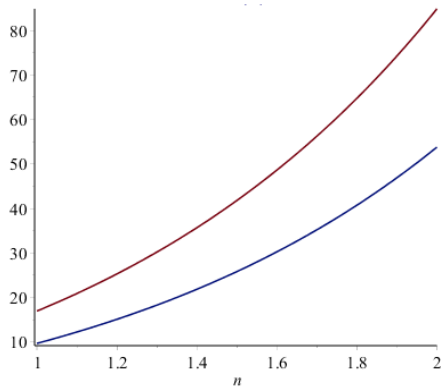

Fig. 4. Sierpinksi network: Dharwad (red) and $Dharwad_{red}$ (blue) for S_n.

TABLE II. EDGE PARTITION OF SIERPINSKI NETWORK S(N, K)

ε	ε(d_u, d_v)	d_e	ε(d_u, d_e)	Recurrence
$ε_1$	ε(2, k)	K	ε(2, k)	2k
$ε_2$	ε(3, 3)	4	ε(3, 4)	$(K^{n+1}-5k)/2$

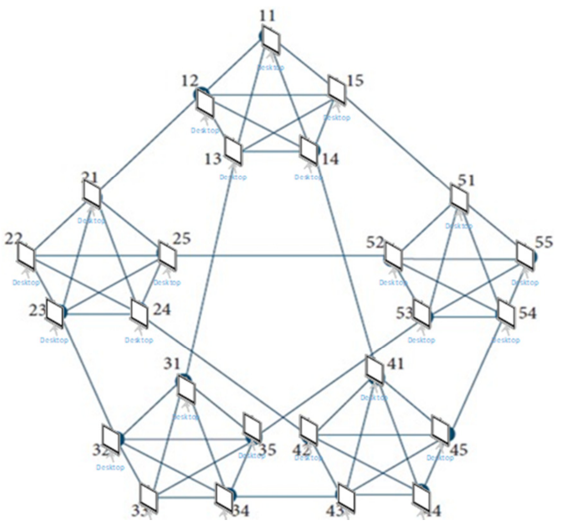

Fig. 5. Sierpinski network S(*n*, *k*).

B. Main Results

To generalize the network, Figure 5 illustrates the Sierpinski Network S(*n*, *k*) with two unique edges labeled *n* and *k*. The network shows looping themes in which multiple fully connected or mesh networks are connected to form a close bridge-like network. These types of network structures are used in memory, processors, chemical compounds, etc.

1) Sierpinski Network S(n, k)

If the edge set is represented by E(*G*), the graph of S(*n*, *k*) has two different types of edges, as noticed in Figure 5. Table II describes the edge division.

2) Theorem 3

Let *G* be a graph of S(*n*, *k*) Then *KBSO* and $KBSO_{red}$ are:

$$KBSO(G) = 2\sqrt{k^2+4}\,k + \frac{5}{2}k^{n+1} - \frac{25}{2}k \quad (9)$$

$$KBSO_{red}(G) = $$
$$2\sqrt{1+(k-1)^2}\,k + \sqrt{13}\left(\frac{1}{2}k^{n+1} - \frac{5}{2}k\right) \quad (10)$$

Equations (9) and (10) employ the *KBSO* index and its reduced version, respectively, to describe the conclusions that were demonstrated for the graph S(*n*, *k*) shown in Figure 4. These results are manifested graphically in Figure 6, in context with bounds.

3) Investigation of S(n, k) Graphs by KBSO Indices

Proof:

$$KBSO(G) = \sum_{ue} \sqrt{d_u{}^2 + d_e{}^2}$$

$$KBSO(G) = \sqrt{2^2+k^2}(2k) + \frac{\sqrt{3^2+4^2}(k^{n+1}-5k)}{2}$$

$$KBSO\,(G) = 2\sqrt{k^2+4}\,k + \frac{5}{2}k^{n+1} - \frac{25}{2}k$$

$$KBSO_{red}(G) = \sum_{ue} \sqrt{(d_u-1)^2 + (d_e-1)^2}$$

$$KBSO_{red}(G) = \sqrt{(2-1)^2+(k-1)^2}(2k)$$
$$+\sqrt{(3-1)^2+(4-1)^2}\left(\frac{(k^{n+1}-5k)}{2}\right)$$

$$KBSO_{red}(G) = 2\sqrt{1+(k-1)^2}\,k + \sqrt{13}\left(\frac{1}{2}k^{n+1} - \frac{5}{2}k\right)$$

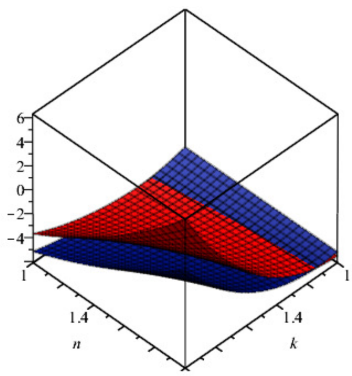

Fig. 6. *KBSO* (red) and $KBSO_{red}$ (blue) for S(*n*, *k*).

Figure 6 illustrates the results of the *KBSO* index and its reduced form in red and blue, respectively, demonstrating the S(*n*, *k*) network's sharp upper and lower boundaries.

4) Theorem 4

Let *G* be a graph of S(*n*, *k*), then, Dharwad and $Dharwad_{red}$ indices are:

$$D(G) = 15r + 8 + \sqrt{35}\,r + 3\sqrt{6}\,(r-3) \quad (11)$$

$$RD(G) = 8r + \sqrt{2}\,(3r+2) - 12 \quad (12)$$

Equations (11) and (12) result from the S(*n*, *k*) network by the Dharwad index and its reduced form, respectively. Graphical forms of these results are presented in Figure 7, where the red and blue planes depict the Dharwad index and its condensed form, respectively. The S(*n*, *k*) network's sharp upper and lower boundaries can also be observed.

5) Investigation of Sierpinski Network by Dharwad Indices

Proof:

$$D(G) = \sum_{ue} \sqrt{d_u^3 + d_v^3}$$

$$D(G) = \sqrt{2^3 + k^3}\,(2k) + \sqrt{3^3 + 3^3}\,\left(\frac{(k^{n+1} - 5k)}{2}\right)$$

$$D(G) = 2\sqrt{k^3 + 8}\,k + 3\sqrt{6}\left(\frac{1}{2}k^{n+1} - \frac{5}{2}k\right)$$

$$RD\ (G) = \sum_{ue} \sqrt{(du-1)^3 + (dv-1)^3}$$

$$RD\ (G) = \sqrt{(2-1)^3 + (k-1)^3}\,(2k)$$

$$+5\sqrt{(3-1)^3 + (3-1)^3}\,\left(\frac{(k^{n+1} - 5k)}{2}\right)$$

$$RD\ (G) = 2\sqrt{1 + (k-1)^3}\,k + 2k^{n+1} - 10k)$$

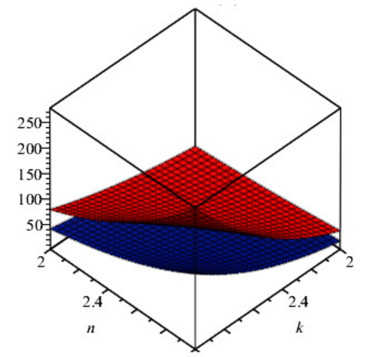

Fig. 7. Dharwad and Dharwad$_{red}$ for S(n, k).

C. Discussion

The Sierpinski S_n network graph shows sharp lower and upper bounds when drawing the simulation according to mathematical results after applying KBSOs and Dharwad topological invariants. On the other hand, the Sierpinski S(n, k) network does not exhibit sharp lower and upper bounds in the simulation. So, the study checked this network graph by ISO TI to find irregularities in the results [30-31].

Figure 8 demonstrates the results of anomalies discovered in the topology of the Sierpinski S(n, k) network using the ISO index, according to the following mathematical relation:

$$ISO(G) = 2\sqrt{|\mathrm{k}^2 - 4|}\,\mathrm{k}$$

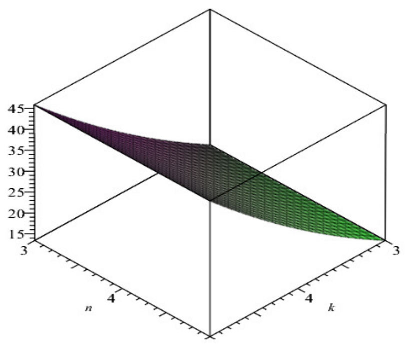

Fig. 8. Irregularity Sombor index for S(n, k).

D. Optimization of the Results of Sierpinski Networks

The optimization of the results of the Sierpinski network by TIs was performed with the help of an optimization assistant. The green dot is the highest optimized point of the network. Figure 9 illustrates the highest optimized value found by Dharwad (48.4661) for S_n. The optimization of all results of the Sierpinski network S(n, k) by topological invariants was also performed with the help of the optimization assistant. Figure 10 portrays the highest optimized value found by $KBSO_{red}$ (19.957) for S(n, k).

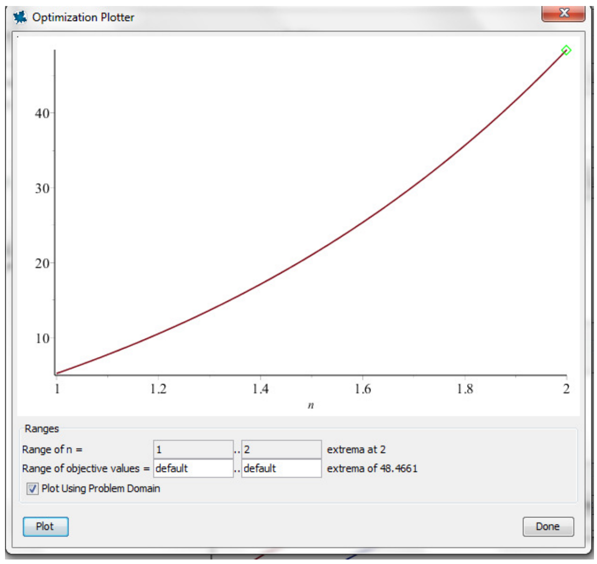

Fig. 9. Highest optimized results of the Sierpinski networks found by Dharwad (48.4661) for S_n.

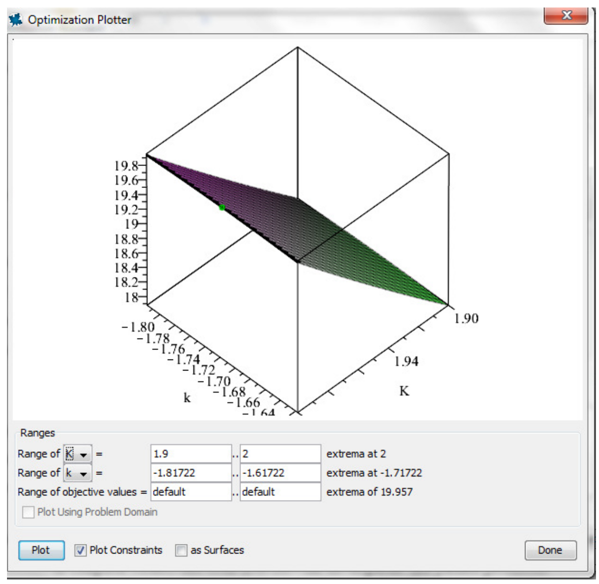

Fig. 10. Highest optimized results of Sierpinski networks found by $KBSO_{red}$ (19.957) for S(n, k).

V. CONCLUSION

Numerous applications and implementations of topological indices can be detected in the domains of computer science, chemistry, informatics, arithmetic, electronics, and other disciplines. Nonaccurate QSPR and QSAR have the greatest degree of achievable massive applicability. This study delved into the recently published KBSO invariants, which have outstanding estimate properties for various types of Sierpinski graphs or networks. Graph representations of KBSO indices for Sierpinski network graphs can be found in Figures 3 and 6. The recently introduced Dharwad indices can provide certain physical qualities. The Dharwad indices for two variations of the Sierpinski network are graphically displayed in Figures 4 and 7. The demonstration and modeling of interconnected networks employed in computer science and other disciplines can use these findings. The study also examined the Sierpinski network for irregularities and produced mathematical results to remove them. These results were optimized with the help of an optimization assistant to find the best range and single value for the modeled network.

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