Application of the TOPSIS Method for Multi-Objective Optimization of a Two-Stage Helical Gearbox

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Received: 19 April 2024 | Revised: 16 May 2024 | Accepted: 1 June 2024

 DOI: https://doi.org/10.48084/etasr.7551

ABSTRACT

In order to design a high-efficiency two-stage gearbox to reduce power loss and conserve energy, a Multi-Criterion Decision-Making (MCDM) method is selected for solving the Multi-Objective Optimization Problem (MOOP) in this research. The study's objective is to determine the best primary design factors that will increase gearbox efficiency and decrease gearbox mass. To that end, the first stage's gear ratio and the first and second stages' Coefficients of Wheel Face Width (CWFW) were chosen as the three main design elements. Furthermore, two distinct goals were analyzed: the lowest gearbox mass and the highest gearbox efficiency. Additionally, the MOOP is carried out in two steps: phase 1 solves the Single-Objective Optimization Problem (SOOP) to close the gap between variable levels, and phase 2 solves the MOOP to determine the optimal primary design factors. Furthermore, the TOPSIS approach was selected to address the MOOP. For the first time, an MCDM technique is used to solve the MOOP of a two-stage helical gearbox considering the power losses during idle motion. When designing the gearbox, the optimal values for three crucial design parameters were ascertained according to the study's results.

Keywords-gearbox; two-stage helical gearbox; gear ratio; multi-objective optimization; TOPSIS method

I. INTRODUCTION

Helical gearboxes are extensively utilized in a wide range of industrial applications due to their inexpensive, dependable operation, and straightforward design. Designing a high-efficiency gearbox to reduce power loss and conserve energy is therefore a key need. In addition, a gearbox must guarantee that other parameters like length, mass, or volume are minimal. As a result, it is a MOOP with high gearbox efficiency requirement among others. Numerous studies on MOOof helical gearboxes, including the maximum gearbox efficiency function, have been conducted up to this point. In order to minimize the transmission volume and power losses, authors in [1] carried out a MOO of gear pair parameters using the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) technique. Gear module, face width, pinion and wheel profile shift coefficients, and pinion tooth count were the optimization variables used in this work. Additionally, the impact of the friction coefficient, sliding velocities, and nor-mal load on the gearing efficiency were examined. They concluded that a trade-off between efficiency and volume is necessary, and that a lower gear module, a lower face width, higher profile shift coefficients, and a higher pinion tooth count produce satisfactory results for both goals. Authors in [2] also optimized a two-stage helical gearbox using the NSGA-II approach. Two goal functions were used: the lowest gearbox volume and the lowest overall gearbox power loss. Numerous limitations were also considered, including tribological limitations, pitting stress, and bending stress. It was found that the multi-objective approach reduces the gearbox's overall power loss by half and that solutions derived from single objective minimization without tribological constraints had a significant probability of wear failure. It was also demonstrated that MOO produced smaller power losses in comparison to single-objective optimization under tribological constraints. In order to reduce power loss and vibrational excitation caused by meshing, authors in [3] carried out a MOO study of a gear unit utilizing the NSGA-II method in a multi-scale approach that goes from the gear contact to the entire transmission, concluding that using both macro and micro geometry parameters simultaneously during MOO yields different results than using macro geometry parameters first and micro geometry parameters second. Additionally, a comparison was done between the total power loss in the single stage gear unit and the local power loss caused by gear tooth friction in terms of design variable values in order to investigate the significance of considering the entire gear unit.

A helical gear pair's macro shape was optimized in [4] for low weight, high efficiency, and low noise. The trends of the best solutions for five combinations of the three goals were also examined. The study's goals in this work were the gear mass, gear efficiency, and transmission error. The objectives were scored and standardized in order to examine the outcomes. It was observed that, when mass, efficiency, and transmission error were taken into account, the majority of the top rankings were from the best options. For low weight, high efficiency, and low noise, the gear optimization process should take these three goals into account. Using the NSGA-II approach, authors in [5] optimized a two-stage spur gearbox. Three goals were simultaneously studied in this work: volume, power output, and center distance. Three design constraints and eight design variables were chosen. The study's findings showed that, in comparison to power output and center distance, the variables related to the module, pinion tooth number, and face-width had a greater influence on volume.

A study to jointly optimize a gearbox and an electric motor for the purpose of designing an electric vehicle drive system was presented in [6]. The work's goal functions were to minimize the drive system's weight and overall energy loss. The optimization outcomes are contrasted with earlier findings to highlight collaborative optimization's further potential. It was observed that when the drive system was overall optimized, raising the gear ratio raises the system's overall efficiency. In order to optimize a three-stage wind turbine gearbox, authors in [7] considered two goal functions: minimizing weight and minimizing power loss while taking into account the standard mechanical design restrictions and the tribological constraints. In addition, three distinct gear tooth involute profiles—unmodified, smooth meshing, and high load capacity—were taken into account. At the recommended speed of 20 rpm, these three profiles were evaluated using various synthetic-based ISO VG PAO (polyalphaolefin) oils. Using ISO VG PAO 320, 680, and 1000 oils, the gearbox's results were compared with and without tribological limitation. According to the results, PAO 320 oil performed better than the other two grades (PAO 680 and 1000). Additionally, power loss was significantly decreased with tribological restriction for the selected model. A MOO study of a two-stage spur gearbox under a wide range of constraints was carried out in [8]. Minimum volume and minimum gearbox power losses were the study's goals. The findings suggest that solutions derived from single objective minimization have a significant likelihood of experiencing wear failure. Additionally, when utilizing multi-goal optimization as opposed to single objective optimization, the overall power loss is cut in half.

Grey Relation Analysis (GRA) and the Taguchi technique were used in [9] to investigate the MOOP of building a two-stage helical gearbox. The aim of this study was to determine the ideal fundamental design parameters that enhance gearbox efficiency while decreasing gearbox mass. In order to identify the optimal key design elements for a two-stage helical gearbox, a MOOP was solved with the Taguchi and GRA methods in [10]. Two goals examined in this work were the lowest possible gearbox height and the highest possible gearbox efficiency. Moreover, similar methods were used to optimize a two-stage helical gearbox with second stage double gear-sets in [11] in order to increase efficiency and reduce gearbox mass. Optimum main design elements for raising gearbox efficiency can be found in [9-11], but the effect of primary design factors on gearbox efficiency has not yet been assessed. In order to classify the state of the gearbox for wind turbines into excellent or bad (broken tooth) condition, authors in [12] presented a neural network-based model that combines an auto encoder with the Bidirectional Long Short-Term Memory (BLSTM). A study was carried out in [13] on the fracture analysis of a cycloidal gearbox used as a yaw drive on a wind turbine. In order to identify a gearbox defect for a wind turbine, the Non-parametric Ensemble Empirical Mode Decomposition (NCEEMD) method for nonlinear and non-stationary signal analysis was applied in [14]. Multi-Criteria Decision-Making (MCDM) is one of the main decision-making processes, which is used to find the best option by taking into account multiple criteria during the selection process. Numerous tools and techniques of MCDM can be used in a variety of fields, including engineering design. In [15], the improved NSGA-I) and MCDM were applied to solve a Multi-objective Uncertainty Optimization Design (MUOD) problem for the planetary gear train of an electric vehicle, using the Analytic Hierarchy Process (AHP) and the entropy weight method. In order to improve both gear quality and process productivity in laser machining of stainless steel gears, the Fuzzy-MOORA hybrid technique was employed in [16] to choose the most suitable input variables. The linguistic variable utilized for each criterion determined the relative weight of each criterion. In [17], the use of the MCDM technique to determine the ideal plant layout design was suggested. The MCDM was solved using the TOPSIS and WASPAS (Weighted Aggregated Sum-Product Assessment) methods, and the weights were determined with the entropy method.

While numerous MOOs have been performed to increase gearbox efficiency, the impact of a gearbox's primary design factors on efficiency has not been studied. Also, the previously described studies did not account for the power loss that occurs while a gear is in an idle state or when a gear is immersed in lubricant during bath lubrication. Furthermore, no study has yet been conducted to solve the MOOP utilizing the MCDM technique. This work used the MCDM method to perform MOO research for a two-stage helical gearbox. Two different objectives were looked into: reducing gearbox mass and raising gearbox efficiency. This paper looked at three optimal primary design parameters for the two-stage helical gearbox. Among these are the first stage's gear ratio and the combined weight for both stages. Furthermore, the optimization task was approached using the TOPSIS method, and the weights of the criteria were determined using the entropy method. One of the main conclusions of the research is the suggestion to apply an MCDM technique to solve MOOPs in conjunction with two-step problem solving, tackling single- and multi-objective problems. Moreover, the problem's solutions are more effective than those reported in earlier studies.

II. THE OPTIMIZATION PROBLEM

The gearbox mass and efficiency are initially determined in this section in order to construct the optimization problem. Then, the constraints and objective functions are specified. The nomenclature used in the optimization issue can be seen in Table I.

A. Determination of Gearbox Mass

The mass of the gearbox m_{gb}, can be found by:

$$m_{gb} = m_{gh} + m_g + m_s \quad (1)$$

where m_{gh}, m_g, and m_s are determined in detail below.

TABLE I. NOMENCLATURE OF OPTIMIZATION OF A HELICAL GEARBOX

Parameter	Symbol	Units
Gearbox mass	m_{gb}	kg
Gear mass	m_g	kg
Shaft mass	m_s	kg
Gearbox housing mass	m_{gh}	kg
Gear mass of the first stage	m_{g1}	kg
Gear mass of the second stage	m_{g2}	kg
Weight density of gear materials	ρ_g	kg/m^3
Mass density of gearbox housing materials	ρ_{gh}	kg/m^3
Volume coefficients of the pinion	e_1	-
Volume coefficients of the gear	e_2	-
Pitch diameter of the pinion of stage 1	d_{w11}	mm
Pitch diameter of the gear of stage 2	d_{w21}	mm
Pitch diameter of the pinion of stage 2	d_{w12}	mm
Pitch diameter of the gear of stage 2	d_{w22}	mm
Center distance of stage 1	a_{w1}	mm
Center distance of stage 2	a_{w2}	mm
Gear ratio of stage 1	u_1	-
Gear ratio of stage 2	u_2	-
Gearbox ratio	u_{gb}	-
Gear width of stage 1	b_{w1}	mm
Gear width of stage 2	b_{w2}	mm
Wheel face width coefficient of stage 1	X_{ba1}	-
Wheel face width coefficient of stage 2	X_{ba2}	-
Material coefficient	k_a	Mpa$^{1/3}$
Allowable contact stress of stages i (i=1÷2)	AS_i	Mpa
Contacting load ratio for pitting resistance	$k_{H\beta}$	-
Torque on the pinion of stage i (i=1÷2)	T_{1i}	Nmm
Output torque	T_{out}	Nmm
Efficiency of a helical gear unit	η_{hg}	-
Efficiency of a rolling bearing pair	η_b	-
Gearbox housing volume	V_{gh}	dm^3
Volumes of bottom housing A	V_A	dm^3
Volumes of bottom housing B	V_B	dm^3
Volumes of bottom housing B	V_C	dm^3
Mass of shaft *j* (*j*=1÷3)	m_{sj}	kg
Mass density of shaft material	ρ_s	kg/m^3
Length of shaft *i*	l_{si}	mm
Diameter of shaft *i*	d_{si}	mm
Allowable shear stress of shaft material	$[\tau]$	MPa
Total power loss in the gearbox	Pl	
Power loss in the gears	Plg	Kw
Power loss in the bearings	Plb	Kw
Power loss in the seals	Pls	Kw
Power loss in the idle motion	Pzo	Kw
Efficiency of a helical gearbox	η_{hb}	-
Efficiency of the *i* stage of the gearbox	η_{gi}	-
Friction coefficient	f	-
Friction coefficient of bearing	f_b	-
Arc of approach on *i* stage	β_{ai}	-
Arc of recess on *i* stage	β_{ri}	-
Outside radius of the pinion	R_{e1i}	mm
Outside radius of the gear	R_{e2i}	mm
Base-circle radius of the pinion	R_{01i}	mm
Base-circle radius of the gear	R_{02i}	mm
Pressure angle	α	rad.
Sliding velocity of gear	v	m/s
Peripheral speed of bearing	v_b	m/s
Load of bearing *i*	Fi	N
ISO viscosity grade number	VG_{40}	-
Hydraulic moment of power losses	T_H	Nm

1) Calculation of m_{gh}

The gearbox housing mass m_{gh} is determined by:

$$m_{gh} = \rho_{gh} \cdot V_{gh} \quad (2)$$

where V_{gh} can be found by:

$$V_{gh} = 2 \cdot V_A + 2 \cdot V_B + 2 \cdot V_C \quad (3)$$

In which:

$$V_A = L \cdot H \cdot S_G \quad (4)$$

$$V_B = L \cdot B_1 \cdot 1.5 \cdot S_G \quad (5)$$

$$V_C = B_2 \cdot H \cdot S_G = (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \quad (6)$$

where L, H, B_1, and S_G are determined by [18]:

$$L = (d_{w11} + \frac{d_{w21}}{2} + \frac{d_{w12}}{2} + d_{w22}/2 + 22.5)/0.975 \quad (7)$$

$$H = \max(d_{w21}; d_{w22}) + 8.5 \cdot S_G \quad (8)$$

$$B_1 = b_{w1} + b_{w2} + 6 \cdot S_G \quad (9)$$

$$S_G = 0.005 \cdot L + 4.5 \quad (10)$$

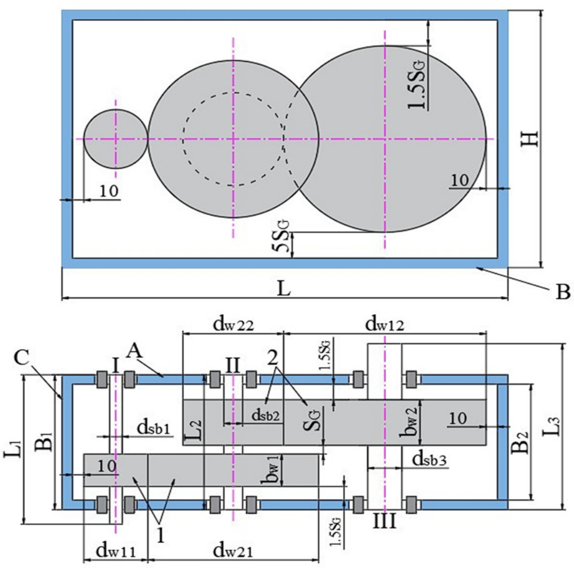

Fig. 1. Calculated schema.

2) Calculation of m_g

$$m_g = m_{g1} + m_{g2} \quad (11)$$

where:

$$m_{g1} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}}{4}\right) \quad (12)$$

$$m_{g2} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}}{4}\right) \quad (13)$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \quad (14)$$

$$b_{w2} = X_{ba2} \cdot a_{w2} \quad (15)$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \quad (16)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \quad (17)$$

where i=1÷2; $\rho_g = 7800$ (kg/m^3) as the gear material is steel, $e_1 = 1$ and $e_2 = 0.6$ [19], and a_{wi} can be found by [19]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (18)$$

in which:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3}(u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \quad (19)$$

3) Calculation of m_s

The mass of all shafts of the gearbox is calculated by:

$$m_s = \sum_{j=1}^{3} m_{sj} \quad (20)$$

in which:

$$m_{sj} = \rho_s \cdot \pi \cdot d_{sj}^2 \cdot l_{sj}/4 \quad (21)$$

In (21), l_{sj} can be found by (Figure 1):

$$l_{s1} = B_1 + 1.2 \cdot d_{s1} \quad (22)$$

$$l_{s2} = B_1 \quad (23)$$

$$l_{s3} = B_1 + 1.2 \cdot d_{s3} \quad (24)$$

The diameter of the shaft j (j=1÷3) is determined by [19]:

$$d_{sj} = [T_{1j}/(0.2 \cdot [\tau])]^{1/3} \quad (25)$$

where $\rho_g = 7800$ (kg/m^3) and $\rho_s = 7800$ (kg/m^3) as the gear and the shaft materials are steel, $[\tau]$= 17 MPa [19].

B. Determination of Gearbox Efficiency

The gearbox efficiency (%) can be calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (26)$$

where P_l is determined by [20]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (27)$$

wherein P_{lg}, P_{lb}, P_{ls}, and P_{zo} are determined below.

1) Calculation of P_{lg}

$$P_{lg} = \sum_{i=1}^{2} P_{lgi} \quad (28)$$

Where:

$$P_{lgi} = P_{gi} \cdot (1 - \eta_{gi}) \quad (29)$$

η_{gi} is found by [21]:

$$\eta_{gi} = 1 - \left(\frac{1 + 1/u_i}{\beta_{ai} + \beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot (\beta_{ai}^2 + \beta_{ri}^2) \quad (30)$$

where β_{ai} and β_{ri} are calculated by [21]:

$$\beta_{ai} = \frac{(R_{e2i}^2 - R_{02i}^2)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}} \quad (31)$$

$$\beta_{ri} = \frac{(R_{e1i}^2 - R_{01i}^2)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}} \quad (32)$$

and *f* can be determined by [9]:

- If v ≤ 0.424 m/s:

$$f = -0.0877 \cdot v + 0.0525 \quad (33)$$

- If *v* > 0.424 m/s:

$$f = 0.0028 \cdot v + 0.0104 \quad (34)$$

2) Calculation of P_{lb}

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \quad (35)$$

where *i*=1÷6 and $f_b = 0.0011$ as the radical ball bearings with angular contact were used [20].

3) Calculation of P_s

$$P_s = \sum_{i=1}^{2} P_{si} \quad (36)$$

where *i* is the ordinal number of seal (*i*=1÷2); and P_{si} is calculated by [20]:

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\,log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (37)$$

4) Calculation of P_{zo}

$$P_{Z0} = \sum_{i=1}^{k} T_{Hi} \cdot \frac{\pi \cdot n_i}{30} \quad (38)$$

where *k* is the total number of gear pairs (*k*=2), *n* is the number of revolutions of thedriven gear, and T_{Hi} is determined by [20]:

$$T_{Hi} = C_{Sp} \cdot C_1 \cdot e^{\frac{C_2 \cdot v}{v_{to}}} \quad (39)$$

where $C_{Sp} = 1$ for the stage 1 when the involved oil has to pass until the mesh and C_{Sp} is calculated by th(4) for the stage 2 (Figure 2):

$$C_{Sp} = \left(\frac{4 \cdot e_{max}}{3 \cdot h_C}\right)^{1.5} \cdot \frac{2 \cdot h_C}{l_{hi}} \quad (40)$$

In which, l_{hi} is calculated by[20]:

$$l_{hi} = (1.2 \div 2.0) \cdot d_{a2i} \quad (41)$$

In (39), C_1 and C_2 are determined by [20]:

$$C_1 = 0.063 \cdot \left(\frac{e_1 + e_2}{e_0}\right) + 0.0128 \cdot \left(\frac{b}{b_0}\right) \quad (42)$$

$$C_2 = \frac{e_1 + e_2}{80 \cdot e_0} + 0.2 \quad (43)$$

where $e_0 = b_0 = 10$ mm.

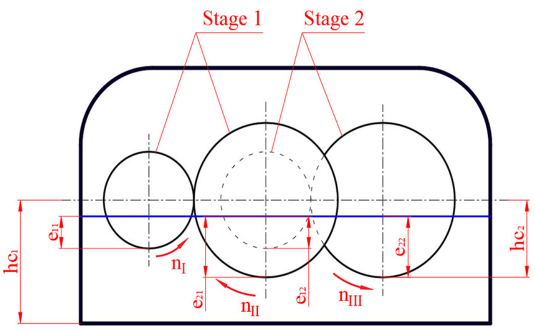

Fig. 2. Schema for the calculation of lubrication factors.

C. Objective Functions and Constrains

1) Objective Functions

In this work, the MOOP consists of two objectives:

- Minimizing the gearbox mass:

$$\min f_1(X) = m_{gb} \quad (44)$$

- Maximizing the gearbox efficiency:

$$\min f_2(X) = \eta_{gb} \quad (45)$$

where *X* is the vector reflecting the design variables. For a two-stage helical gearbox, there are five main design parameters namely u_1, Xba_1, Xba_2, AS_1, and AS_2 [9]. Besides, it was noted that the optimal values of AS1 and AS2 are their maximum values [9]. Therefore, in this work, three main design factors, i.e. u_1, Xba_1, Xba_2 were selected as variables for the optimization problem a:

$$X = \{u_1, Xba_1, Xba_2\} \quad (46)$$

2) Constrains

The multi-objective function needs to adhere to the following constraints:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9 \quad (47)$$

$$0.25 \le Xba_1 \le 0.4 \text{ and } 0.25 \le Xba_2 \le 0.4 \quad (48)$$

III. METHODOLOGY

A. Method to Solve the MOOP

As stated above, three main design factors are selected as variables for the MOOP. Table II describes these factors and their minimum and maximum values. Actually, it is challenging to address the MOOP using an MCDM approach, because there are a lot of options or potential solutions when it comes to dealing with a MOOP. To ensure the accuracy of the parameters and avoid missing the optimization problem's solution, the three parameters in this work have limits as shown in Table II, and the step between variables is 0.02. As a result, the number of options (or experimental runs) that must be determined and compared is:

$$(9-1)/0.02 \cdot (0.4-0.25)/0.02 \cdot (0.4-0.25)/0.02 = 22.500 \text{ runs.}$$

It is not viable to solve the MOOP using the MCDM method directly due to the large amount of options. In this work, the TOPSIS method was employed to address the MOOP with two objectives: minimum gearbox mass and maximum gearbox efficiency to find optimum values of three main design variables. A simulation experiment was constructed in order to address the MOOP for a two-stage helical gearbox. Furthermore, as this is a simulation experiment, the number of experiments can be increased by using the full factorial design, which is not constrained by the budget of each experiment. As a result, we will have $5^3 = 125$ experiments as there are 3 experimental variables and 5 levels for each variable. But out of the three variables listed, u_1 has the widest distribution (Table II shows that u_1 ranges from 1 to 9). Consequently, even with five levels, the difference between the levels of this variable remained large (in this instance, the gap is ((9-1)/4=2). A method for resolving multi-objective problems was put forth in an effort to close this gap, save time, and increase the precision of the results (Figure 3). This process is divided into two stages: phase 1 solves the SOOP to reduce the gap between levels, and phase 2 solves the MOOP to identify the ideal primary design. Furthermore, when solving the MOOP, if the variable's levels are not sufficiently close to one another or if the best solution is not suitable for the requirement, the TOPSIS issue will be rerun with the smaller distance between two levels of the u_1 (refer to Figure 3). Figure 4 illustrates the approach of determining the ideal value when utilizing the TOPSIS method.

TABLE II. INPUT FACTORS

Factor	Symbol	Lower limit	Upper limit
Gearbox ratio of first stage	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.4
CWFW of stage 2	X_{ba2}	0.25	0.4

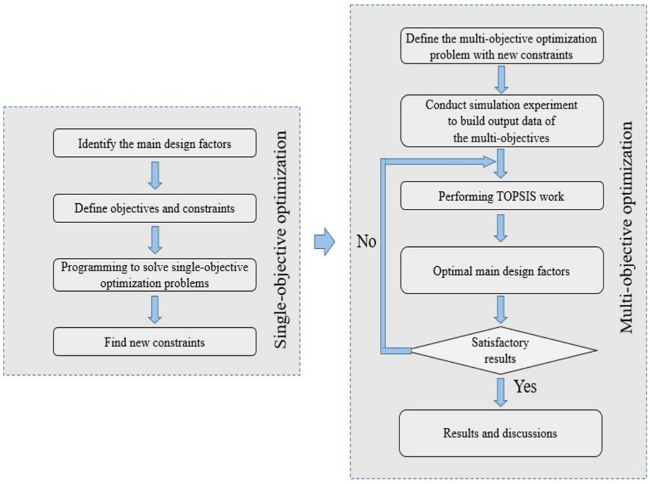

Fig. 3. The procedure to solve the MOOP.

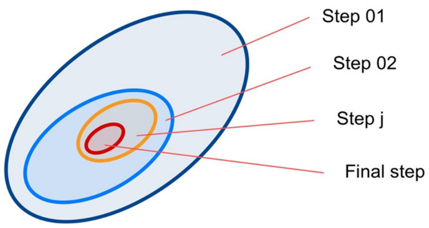

Fig. 4. Strategy for finding the optimal values.

B. Method to Solve the MCDM Problem

The following steps need to be followed in order to apply the TOPSIS technique [22]:

Create the initial decision-making matrix:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \quad (49)$$

where n and m represnt the criterion and alternative numbers.

Calculate the normalized values k_{ij} by:

$$k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad (50)$$

Finding the weighted normalized decision matrix by:

$$l_{ij} = w_j \times k_{ij} \quad (51)$$

Determine the best alternative A^+ and the worst s alternative A^- by:

$$A^+ = \{l_1^+, l_2^+, \dots, l_j^+, \dots, l_n^+\} \quad (52)$$

$$A^- = \{l_1^-, l_2^-, \dots, l_j^-, \dots, l_n^-\} \quad (53)$$

where l_j^+ and l_j^- are the best and worst values of criterion j (j=1,2, ..., n).

Calculating best and worst options D_i^+ and D_i^- by:

$$D_i^+ = \sqrt{\sum_{j=1}^{n}\left(l_{ij} - l_j^+\right)^2} \quad (54)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n}\left(l_{ij} - l_j^-\right)^2} \quad (55)$$

Finding the closeness coefficient R_i of each alternative by:

$$R_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (56)$$

Rank the alternatives, starting with the maximum value of R.

C. Method to Find the Criteria Weights

The weights of the criteria were determined in this work using the Entropy technique. This method can be implemented by [23]:

Determining the indicator normalized values:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2} \quad (57)$$

Finding the Entropy for each indicator:

$$me_j = -\sum_{i=1}^{m}[p_{ij} \times ln(p_{ij})] - \left(1 - \sum_{i=1}^{m} p_{ij}\right) \times ln\left(1 - \sum_{i=1}^{m} p_{ij}\right) \quad (58)$$

Calculating the weight of each indicator:

$$w_j = \frac{1 - me_j}{\sum_{j=1}^{m}(1 - me_j)} \quad (59)$$

IV. SINGLE-OBJECTIVE OPTIMIZATION

In this present work, the SOOP is solved by the direct search approach. Additionally, a computer program was developed in Matlab to address two SOOPs: optimizing gearbox efficiency and lowering gearbox mass. Figures and observations from the program's results include the following: The link between u_1 and m_{gb} is depicted in Figure 5. m_{gb} reaches its lowest value when u_1 is at its ideal value. Figure 6 illustrates the relation between u_1 and η_{gb} and exhibits the ideal value of u_1 at which η_{gb} reaches its maximum. The relationship between X_{ba1} and X_{ba2} with m_{gb} and η_{gb}, respectively, are depicted in Figures 7 and 8. It is clear from these findings Xthat m_{gb} will increase when X_{ba1} and X_{ba2} increase. On the other hand, η_{gb} falls as X_{ba1} and X_{ba2} rise. The relationship between the overall gearbox ratio, u_t, and the optimal gear ratio, u_1, for the first stage is shown in Figure 9. New limitations derived for the variable u_1 are shown in Table III.

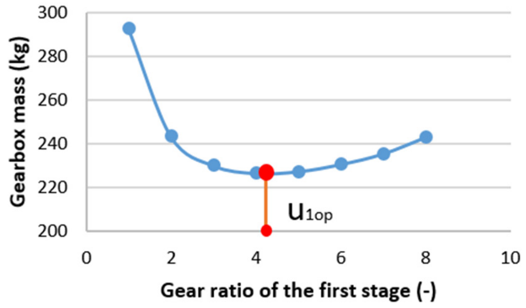

Fig. 5. Gearbox mass versus first stage gear ratio.

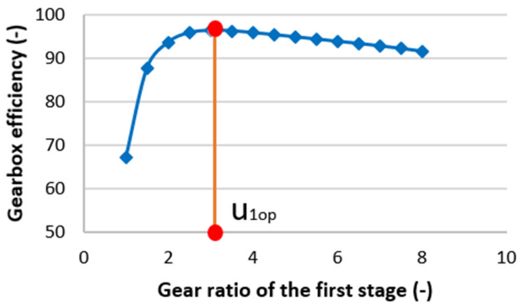

Fig. 6. Gearbox efficiency versus first stage gear ratio.

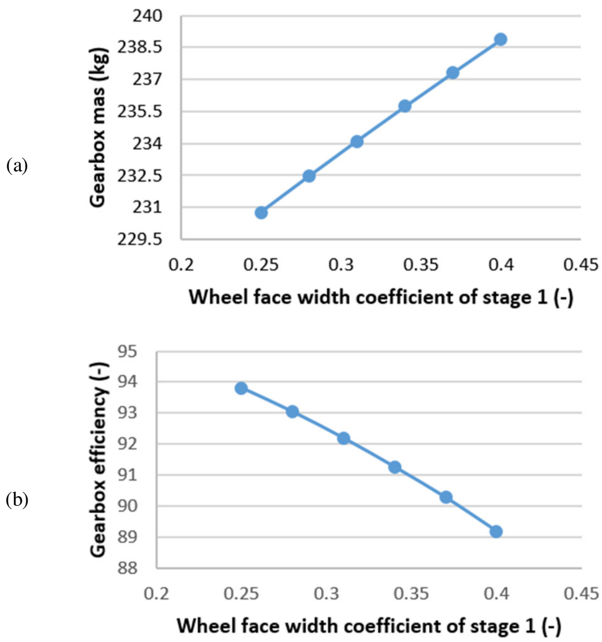

Fig. 7. Relation between (a) X_{ba1} and gearbox mass and (b) gearbox efficiency.

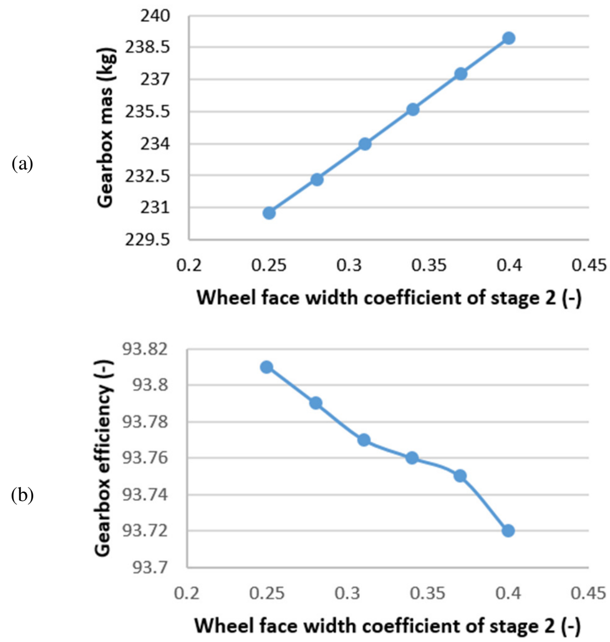

Fig. 8. Relation between X_{ba2} and (a) gearbox mass and (b) gearbox efficiency.

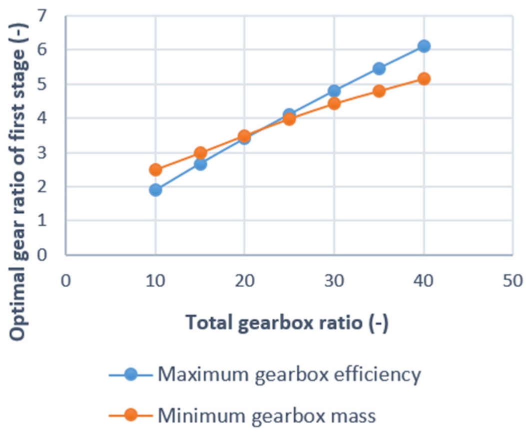

Fig. 9. Optimum gear ratio of the first stage versus total gearbox ratio.

TABLE III. NEW CONSTRAINTS OF u_1

	u_1	
u_t	Lower limit	Upper limit
10	1.79	2.59
15	2.57	3.08
20	3.31	3.59
25	3.88	4.22
30	4.32	4.91
35	4.69	5.57

V. MULTI-OBJECTIVE OPTIMIZATION

A computer program has been developed to carry out the simulation studies. For the analysis, the gearbox ratios 10, 15, 20, 25, 30, and 35 were taken into account. The answers to this issue with u_t=10 are shown in the following. A total of 125 initial testing runs were carried out using this overall gearbox ratio (as indicated in Section III). The gearbox mass and gearbox efficiency, which are the experiment's output values, will be used as input parameters by TOPSIS to solve the MOOP. Until the distance between two levels of each variable is less than 0.02, this process will be repeated. Table IV presents the main design factors and output responses for u_t = 10 in the 4th run (the final run) of TOPSIS. Based on the entropy method, the criteria's weights were determined as follows: To start, find the normalized values of p_{ij} using (57). The entropy value for each indicator m_{ej} is obtained by (58). Lastly, the weight of the criteria w_j is determine by (59). For the last run of TOPSIS, the weights of m_{gb} and η_{gb} were found to be 0.4885 and 0.5115, respectively. Section III.B provides guidance for applying the TOPSIS technique to multi-objective decision making. As a result, (50) yields the normalized values of k_{ij}, and (51) yields the normalized weighted values of l_{ij}. Equations (52) and (53) yield the A^+ and A^- values for m_{gb} and η_{gb}, respectively. It is observed that for A^+, m_{gb} and η_{gb} are equal to 0.0423 and 0.0462, and for A^-, they are equal to 0.0451 and 0.0452. Furthermore, D_i^+ and D_i^- values were determined by (54) and (55). Lastly, (56) was used to get the ratio R_i. The results of the ranking of the options and the computation of several parameters using the TOSIS approach are displayed in Table V (for the final run of TOPSIS work). The Table indicates that option 26 is the most optimal choice out of all the options considered. As a result, u_1 = 2.398, X_{ba1} = 0.25, and X_{ba2} = 0.25 are the ideal values for the primary design elements (see Table IV).

TABLE IV. MAIN DESIGN FACTORS AND OUTPUT RESPONSES FOR u_T =10 IN THE 4TH RUN OF TOPSIS

Trial	u_1	X_{ba1}	X_{ba2}	m_{gb} (kg)	η_{gb}(%)
1	2.38	0.25	0.25	233.28	96.61
2	2.38	0.25	0.2875	235.3	96.8
3	2.38	0.25	0.325	237.4	96.78
4	2.38	0.25	0.3625	239.53	96.77
5	2.38	0.25	0.4	241.67	96.75
6	2.38	0.2875	0.25	235.02	96.26
25	2.38	0.4	0.4	248.31	95.12
26	**2.40**	**0.25**	**0.25**	**233.27**	**96.73**
27	2.40	0.25	0.2875	235.3	96.79
51	2.42	0.25	0.25	233.27	96.72
52	2.42	0.25	0.2875	235.29	96.77
53	2.42	0.25	0.325	237.4	96.76
75	2.43	0.4	0.4	248.4	95.1
76	2.43	0.25	0.25	233.26	96.69
77	2.43	0.25	0.2875	235.3	96.76
100	2.43	0.4	0.4	248.4	95.1
101	2.45	0.25	0.25	233.26	96.67
102	2.45	0.25	0.2875	235.3	96.74
123	2.45	0.4	0.325	244.18	95
124	2.45	0.4	0.3625	246.3	95.01
125	2.45	0.4	0.4	248.44	95.07

TABLE V. CALCULATED RESULTS AND RANKING OF ALTERNATIVES BY TOPSIS METHOD FOR u_T =10

Trial	kij		lij		Si+	Si-	Ri	Rank
	m_{gb}	η_{gb}	m_{gb}	η_{gb}				
1	0.0866	0.0901	0.0423	0.0461	0.0001	0.0029	0.9694	5
2	0.0874	0.0903	0.0427	0.0462	0.0004	0.0026	0.8737	7
3	0.0882	0.0902	0.0431	0.0462	0.0008	0.0022	0.7460	22
4	0.0889	0.0902	0.0435	0.0462	0.0011	0.0019	0.6205	41
5	0.0897	0.0902	0.0438	0.0461	0.0015	0.0015	0.5005	68
6	0.0873	0.0898	0.0426	0.0459	0.0004	0.0025	0.8603	11
25	0.0922	0.0887	0.0450	0.0454	0.0028	0.0001	0.0455	122
26	**0.0866**	**0.0902**	**0.0423**	**0.0461**	**0.0000**	**0.0029**	**0.9886**	**1**
27	0.0874	0.0902	0.0427	0.0462	0.0004	0.0026	0.8736	8
51	0.0866	0.0902	0.0423	0.0461	0.0000	0.0029	0.9870	2
52	0.0874	0.0902	0.0427	0.0462	0.0004	0.0026	0.8740	6
53	0.0882	0.0902	0.0431	0.0461	0.0008	0.0022	0.7456	23
75	0.0922	0.0887	0.0451	0.0454	0.0029	0.0001	0.0416	124
76	0.0866	0.0902	0.0423	0.0461	0.0001	0.0029	0.9822	3
77	0.0874	0.0902	0.0427	0.0461	0.0004	0.0026	0.8733	9
100	0.0922	0.0887	0.0451	0.0454	0.0029	0.0001	0.0416	123
101	0.0866	0.0901	0.0423	0.0461	0.0001	0.0029	0.9790	4
102	0.0874	0.0902	0.0427	0.0461	0.0004	0.0026	0.8729	10
123	0.0907	0.0886	0.0443	0.0453	0.0022	0.0008	0.2645	109
124	0.0915	0.0886	0.0447	0.0453	0.0025	0.0004	0.1362	120
125	0.0923	0.0886	0.0451	0.0453	0.0029	0.0001	0.0368	125

To evaluate the reliability of the found results (option 26 is the best), the MOOP was solved using two more MCDM techniques: MARCOS and SAW. Also, the entropy method was applied to calculate the weights. In the end, option 26 works best when both of these approaches are used to address the MOOP (Table I). The best option, 26, has been generated by all three MCDM methods, proving that the choice of decision-making technique has no influence on the best alternative.

Following with the prior conversation, Table VI presents the ideal values for the primary design parameters that correlate to the remaining u_t values of 10, 20, 25, 30, and 35. Table VI data allow for the following conclusions to be made:

X_{ba1} = 0.25 and X_{ba2} = 0.25 are the minimal values that represent the ideal values for X_{ba1} and X_{ba2}. This outcome also aligns with the remarks made in [9]. This is because the coefficients X_{ba1} and X_{ba2} must be as little as feasible in order to obtain the desired minimum gearbox mass. The gear widths (expressed by (16) and (17)) and, consequently, the gear mass (represented by (14) and (15)) can be decreased by lowering these coefficients.

The ideal values of u_1 and u_t exhibit a clear first-order relationship, as illustrated in Figure 10. Furthermore, the regression equation that follows (with R^2 = 0.9904) was discovered to determine the ideal values of u_1:

$$u_1 = 0.1206 \cdot u_t + 1.0938 \quad (60)$$

TABLE VI. RANKINGS OF OPTIONS BY TOPSIS, MARCOS, AND SAW

Trial	Ranking		
	TOPSIS	MARCOS	SAW
1	5	5	5
2	7	6	6
3	22	17	17
4	41	31	31
5	68	46	46
6	11	11	11
25	122	122	122
26	**1**	**1**	**1**
27	8	7	7
51	2	2	2
52	6	8	8
53	23	18	18
75	124	124	124
76	3	3	3
77	9	9	9
100	123	123	123
101	4	4	4
102	10	10	10
123	109	110	110
124	120	120	120
125	125	125	125

TABLE VII. OPTIMUM VALUES OF MAIN DESIGN FACTORS

No.	u_t					
	10	15	20	25	30	35
u_1	2.40	2.89	3.44	4.04	4.59	5.48
X_{ba1}	0.25	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25

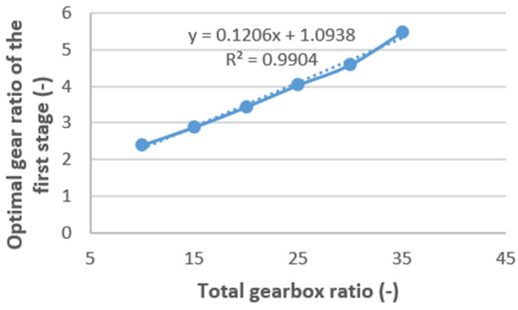

Fig. 10. Optimum gear ratio of the first stage versus total gearbox ratio.

Using the following formula, the ideal value of u_2 can be established after determining u_1.

The results of this study are compared with those obtained using the Taguchi and Gray Relational Analysis method (old method) in [9] in order to evaluate the outcomes of the model for identifying the ideal values when determined using the TOPSIS method (new method). Figure 11 describes the optimal values of u_1 corresponding to various u_t derived by the two techniques. The gearbox mass and efficiency values as determined by the old and new approaches are displayed in Figures 12 and 13, respectively. These figures make it clear that, in comparison to calculations made using the previous method, the new method yields a smaller gearbox mass and improved efficiency. For instance, with u_t = 10, the gearbox efficiency is greater $(96.73 - 94.62) \cdot 100/96.73 = 2.18\,(\%)$ and the gearbox mass is reduced $(255.24 - 233.27) \cdot 100/255.24 = 8.61\,(\%)$.

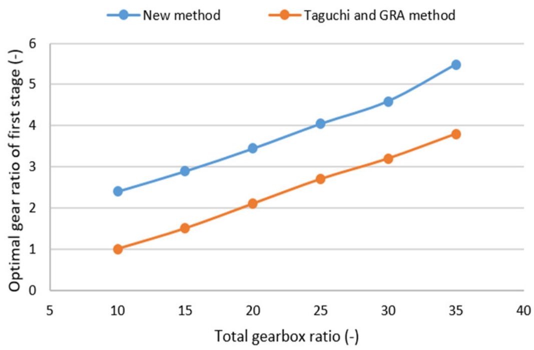

Fig. 11. Optimum values of u_1 found by the old and the new method.

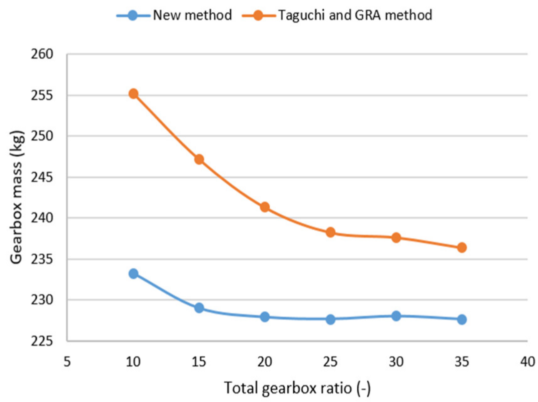

Fig. 12. Minimum gearbox mass values found by the old and the new method.

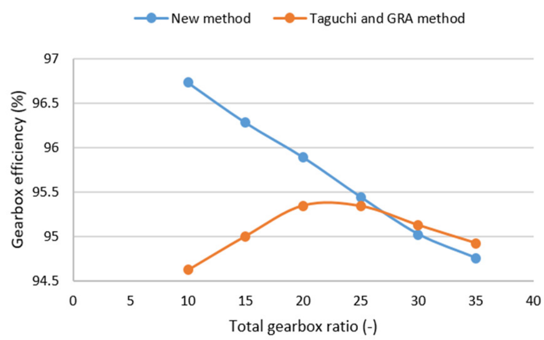

Fig. 13. Maximum gearbox efficiency values found by the old and the new method.

VI. CONCLUSIONS

In this study, the multi-objective optimization problem in the design of a two-stage helical gearbox was solved using the TOPSIS approach. Finding the ideal key design parameters to optimize gearbox efficiency while minimizing gearbox mass is the aim of the study. Three key design elements were selected to do this: the first stage gear ratio and the CWFW for the first and second stages. Furthermore, there are two steps involved in solving the multi-objective optimization problem. While phase 2 is concerned with identifying the ideal primary design factors, phase 1 is focused on addressing the single-objective optimization problem of narrowing the gap between variable values. This work led to the following conclusions:

- By bridging the gap between variable levels, the single-objective optimization problem facilitates and expedites the solution of the multi-objective optimization issue.
- Based on the study's findings, ideal values were suggested for (60) and the the three primary design parameters for a two-stage helical gear gearbox.
- The two objectives were evaluated in relation to the key design parameters.
- The multi-objective optimization issue can be solved more accurately by applying the TOPSIS technique repeatedly until the desired outcomes are obtained (u_1 has an accuracy of less than 0.02).
- The remarkable degree of consistency between the suggested model of u_1 and the experimental results validates their dependability.
- The outcomes indicate that the new method of solving the multi-objective optimization problem produces better outcomes than the previous approaches (Taguchi and GRA).
- The proposed method to solve the MOOP using an MCDM method can be used in designing helical gearboxes either in academic institutions or in industries.

ACKNOWLEDGMENTS

The authors would like to thank Thai Nguyen University of Technology for their assistance with this work.

REFERENCES

[1] D. Miler, D. Žeželj, A. Lončar, and K. Vučković, "Multi-objective spur gear pair optimization focused on volume and efficiency," *Mechanism and Machine Theory*, vol. 125, pp. 185–195, Jul. 2018, https://doi.org/10.1016/j.mechmachtheory.2018.03.012.

[2] M. Patil, P. Ramkumar, and K. Shankar, "Multi-objective optimization of the two-stage helical gearbox with tribological constraints," *Mechanism and Machine Theory*, vol. 138, pp. 38–57, Aug. 2019, https://doi.org/10.1016/j.mechmachtheory.2019.03.037.

[3] E. B. Younes, C. Changenet, J. Bruyère, E. Rigaud, and J. Perret-Liaudet, "Multi-objective optimization of gear unit design to improve efficiency and transmission error," *Mechanism and Machine Theory*, vol. 167, Jan. 2022, Art. no. 104499, https://doi.org/10.1016/j.mechmachtheory.2021.104499.

[4] S. Kim, S. Moon, J. Sohn, Y. Park, C. Choi, and G. Lee, "Macro geometry optimization of a helical gear pair for mass, efficiency, and transmission error," *Mechanism and Machine Theory*, vol. 144, Feb. 2020, Art. no. 103634, https://doi.org/10.1016/j.mechmachtheory.2019.103634.

[5] E. S. Maputi and R. Arora, "Multi-objective optimization of a 2-stage spur gearbox using NSGA-II and decision-making methods," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 42, no. 9, Aug. 2020, Art. no. 477, https://doi.org/10.1007/s40430-020-02557-2.

[6] G. Istenes and J. Polák, "Investigating the Effect of Gear Ratio in the Case of Joint Multi-Objective Optimization of Electric Motor and Gearbox," *Energies*, vol. 17, no. 5, Jan. 2024, Art. no. 1203, https://doi.org/10.3390/en17051203.

[7] A. Kumar, P. Ramkumar, and K. Shankar, "Multi-objective 3-stage wind turbine gearbox (WTG) with tribological constraint," *Mechanics Based Design of Structures and Machines*, pp. 1–27, https://doi.org/10.1080/15397734.2023.2249987.

[8] M. Patil, P. Ramkumar, and K. Shankar, "Multi-Objective Optimization of Spur Gearbox with Inclusion of Tribological Aspects," *Journal of Friction and Wear*, vol. 38, no. 6, pp. 430–436, Nov. 2017, https://doi.org/10.3103/S1068366617060101.

[9] X.-H. Le and N.-P. Vu, "Multi-Objective Optimization of a Two-Stage Helical Gearbox Using Taguchi Method and Grey Relational Analysis," *Applied Sciences*, vol. 13, no. 13, Jan. 2023, Art. no. 7601, https://doi.org/10.3390/app13137601.

[10] V.-T. Dinh *et al.*, "Multi-Objective Optimization of a Two-Stage Helical Gearbox with Second Stage Double Gear-Sets Using TOPSIS Method," *Processes*, vol. 12, no. 6, Jun. 2024, https://doi.org/10.3390/pr12061160, Art. no. 1160.

[11] T. Q. Huy, N. V. Binh, D. V. Thanh, T. H. Danh, and N. V. Trang, "Optimization of a Two-stage Helical Gearbox with Second Stage Double Gear Sets to Reduce Gearbox Mass and Increase Gearbox Efficiency," *WSEAS Transactions on Applied and Theoretical Mechanics*, vol. 18, pp. 287–298, 2023, https://doi.org/10.37394/232011.2023.18.27.

[12] M. Sreenatha and P. B. Mallikarjuna, "A Fault Diagnosis Technique for Wind Turbine Gearbox: An Approach using Optimized BLSTM Neural Network with Undercomplete Autoencoder," *Engineering, Technology & Applied Science Research*, vol. 13, no. 1, pp. 10170–10174, Feb. 2023, https://doi.org/10.48084/etasr.5595.

[13] J. A. Martins and E. C. Romao, "Fracture Analysis of a Cycloidal Gearbox as a Yaw Drive on a Wind Turbine," *Engineering, Technology & Applied Science Research*, vol. 14, no. 1, pp. 12640–12645, Feb. 2024, https://doi.org/10.48084/etasr.6613.

[14] Y. Berrouche, "A Non-Parametric Empirical Method for Nonlinear and Non-Stationary Signal Analysis," *Engineering, Technology & Applied Science Research*, vol. 12, no. 1, pp. 8058–8062, Feb. 2022, https://doi.org/10.48084/etasr.4651.

[15] X. Xu *et al.*, "Optimization Design for the Planetary Gear Train of an Electric Vehicle under Uncertainties," *Actuators*, vol. 11, no. 2, Feb. 2022, Art. no. 49, https://doi.org/10.3390/act11020049.

[16] T. C. Phokane, K. Gupta, and C. Anghel, "Optimization of Gear Manufacturing for Quality and Productivity," *Jurnal Optimasi Sistem Industri*, vol. 21, no. 1, pp. 20–27, May 2022, https://doi.org/10.25077/josi.v21.n1.p20-27.2022.

[17] A. Shivade and S. Sapkal, "Selection of optimum plant layout using AHP-TOPSIS and WASPAS approaches coupled with Entropy method," *Decision Science Letters*, vol. 11, no. 4, pp. 545–562, Jan. 2022, https://doi.org/10.5267/j.dsl.2022.5.002.

[18] I. Römhild and H. Linke, "Gezielte Auslegung von Zahnradgetrieben mit minimaler masse auf der basis neuer Berechnungsverfahren," *Konstruktion*, vol. 44, no. 7–8, pp. 229–236, 1992.

[19] T. Chat and L. Van Uyen, *Design and Calculation of Mechanical Transmissions Systems*, vol. 1, Hanoi, Vietnam: Educational Republishing House, 2007.

[20] D. Jelaska, *Gears and Gear Drives*. John Wiley & Sons, Ltd, 2012.

[21] E. Buckingham, *Analytical Mechanics of Gears*. Dover Publications, 2011.

[22] C.-L. Hwang, Y.-J. Lai, and T.-Y. Liu, "A new approach for multiple objective decision making," *Computers & Operations Research*, vol. 20, no. 8, pp. 889–899, Oct. 1993, https://doi.org/10.1016/0305-0548(93)90109-V.

[23] T. T. Hieu, N. X. Thao, L. Thuy, "Application of MOORA and COPRAS models to select materials for mushroom cultivation," *Vietnam Journal of Agricultural Sciences*, vol. 17, no. 4, pp. 32-2331, 2019.