Synthesis of an Orbit Tracking Controller for a 2DOF Helicopter based on Sequential Manifolds with Stabilization Time in the Presence of Disturbances

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ABSTRACT

This study presents the design of a controller for a two-degree-of-freedom (2-DOF) helicopter based on sequential invariant manifolds with exponential convergence. The system is decomposed into two subsystems for pitch and yaw angles, and exponentially stable manifolds are constructed for each subsystem. The control law is found based on sequential manifolds and the Analytical Design of Aggregated Regulators (ADAR) method. The controller is designed to increase the system's stability against disturbances while ensuring stability over a finite period of time. The response time of the system can be evaluated in advance through the parameters of the designed manifold. The robustness of the control law for external disturbances was proven using the Lyapunov function in the design process. Finally, the effectiveness of the proposed controller based on the synergetic control theory is demonstrated by numerical simulation results and a comparison with the backstepping controller.

Keywords-sequential manifold; backstepping controller; SMC; 2-DOF helicopter; ADAR

I. INTRODUCTION

A helicopter not only has the characteristics of small size, strong adaptability, and ease of use, but also has the functions of vertical takeoff, hovering, and low-altitude flight in a small area. However, directly using the aircraft model in control law testing is not realistic for academic researchers. Therefore, various experimental pieces of equipment in the laboratory have been specially designed for teaching and research in flight dynamics and control. The 2-DOF helicopter system is a helicopter model commonly employed in laboratories. This system operates similarly to a helicopter in certain aspects. The 2-DOF helicopter is a nonlinear and unstable multiple-input, multiple-output (MIMO) system characterized by the coupling effect between the dynamics of the rotor blades and the body structure, which is caused by the impact reaction principle originating from the acceleration and deceleration of motor-propeller groups. All these features make controlling a 2-DOF helicopter a technical problem that needs to be investigated.

Many studies have proposed backstepping and sliding mode controllers for engineering and 2-DOF helicopter systems [1-22]. In [1], a new output feedback control strategy was presented for unmodeled uncertainties in nonlinear systems. Sliding Mode Controllers (SMCs) have been introduced in [7-10]. These studies provide stable control algorithms in the presence of noise and inaccuracy in the mathematical model. The robust control algorithm is based on super-twisting sliding mode control (2-SMC), providing a solution to deal with the torque caused by center-gravity turbulence and ensure smooth flight. In [9], the SMC law was combined with a state observer for a 2-DOF helicopter system, helping to achieve asymptotic attitude adjustment for set points and follow the trajectory. The backstepping method has also been widely applied to study the control law of 2-DOF helicopter systems [11-14]. As far as is known, there are very few studies that directly use the backstepping control law, which is often combined with adaptive techniques, sliding control methods, neural networks, and fuzzy theory. In [11, 12], the backstepping method was combined with the recommendation law to synthesize the control law for this system, giving satisfactory results when there was interference. In [13,14], the design of an SMC was combined with the backstepping method to control the pitch and yaw angles of the main and tail rotor of the 2-DOF system under incorrect parameter conditions. This control strategy dealt with the problems of the 2-DOF helicopter relatively well. Although good developments have been presented in control research for nonlinear systems with uncertainty, these control strategies require very accurate system models. However, in reality, many nonlinear system models are often inaccurate.

This study proposes a method to design a finite-time control law based on sequential manifolds to ensure global stability in the presence of disturbances. The controller is established on the ADAR method presented in [23-25]. Some studies [26-28] examined and applied this method to technical objects to achieve some pretty impressive results. The proposed controller ensures system stability after a given period. The stability of the system in the presence of noise is proven through the Lyapunov function, which provides a limit on the amplitude of the noise, making the system globally stable. Finally, the simulation results displayed the superiority of the control law proposed in this study compared to the backstepping controller.

II. MATHEMATICAL MODEL OF A 2-DOF HELICOPTER

The 2-DOF helicopter includes a bar that is rotated on its base so that it can rotate freely in both the horizontal and vertical planes. There are two rotors (main rotor and tail rotor), driven by direct current motors, at each end of the rod. Two rotors, driven by variable-speed electric motors, allow the helicopter to rotate in the vertical and the horizontal plane (altitude and yaw). The mathematical model of the 2-DOF helicopter was developed according to the assumptions in [5, 6].

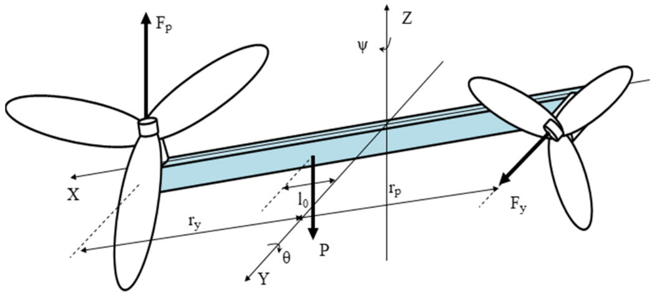

Fig. 1. The 2-DOF helicopter model.

The dynamic model of the system was developed using the Euler-Lagrange equation. Based on [4], the equations for the dynamics of a 2-DOF helicopter are:

$$\begin{cases} \ddot{\theta} = \dfrac{-M_{heli} g l_0 \cos(\theta) - B_p \dot{\theta} - M_{heli} g l_0^2 \dot{\alpha}^2 \sin(\theta)\cos(\theta)}{J_{eq_p} + M_{heli} l_0^2} \\ \qquad + \dfrac{r_p K_{pp} V_p + K_{py} V_y}{J_{eq\_p} + M_{heli} l_0^2} \\ \ddot{\alpha} = \dfrac{-B_y \dot{\alpha} + 2 M_{heli} l_0^2 \dot{\alpha} \dot{\theta} \sin(\theta)\cos(\theta)}{J_{eq_y} + M_{heli} l_0^2 \cos(\theta)^2} \\ \qquad + \dfrac{K_{yp} V_p + r_y K_{yy} V_y}{J_{eq\_y} + M_{heli} l_0^2 \cos(\theta)^2} \end{cases} \quad (1)$$

where θ and α are the pitch and yaw angles, and V_p and V_y are the voltages of the front and rear motors, respectively. M_{heli} is the total moving mass of the 2-DOF helicopter, g is the gravitational acceleration, l_0 is the distance from the 2-DOF helicopter center of gravity to the rotation axis, B_p and B_y are the coefficients of resistance to movement acting on the pitch and yaw axes, respectively, and J_{eq_p} and J_{eq_y} are the moments of inertia along the lift and rotation axes, correspondingly. The coefficients K_{pp} and K_{yy} are the constants of the front and rear engine thrust along the lifting and rotating axes. K_{py} and K_{yp} are the constants of the front and rear engine thrust along the rotation and lifting axes. The parameters r_p and r_y are the distances from each engine to the center of rotation of the helicopter.

Lets set variables $x_1 = [x_{11}, x_{12}]^T = [\theta, \alpha]^T$, and $x_2 = [x_{21}, x_{22}]^T = [\dot{\theta}, \dot{\alpha}]^T$. According to the 2-DOF helicopter system, it can be converted to a general MIMO system as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = N + Mu + d \end{cases} \quad (2)$$

where $u = [V_p, V_y]^T$ represents the control input, $d = [d_1, d_2]^T$ denotes the unknown disturbances which are bounded $|d| < D$, and D has finite positive elements. Furthermore, N and M are determined from (1):

$$N = \begin{bmatrix} \dfrac{-M_{heli} g l_0 \cos(x_{11}) - B_p x_{21} - M_{heli} g l_0^2 x_{22}^2 \sin(x_{11})\cos(x_{11})}{Jeq\_p + M_{heli} l_0^2} \\ \dfrac{-B_y x_{22} + 2 M_{heli} l_0^2 x_{22} x_{21} \sin(x_{11})\cos(x_{11})}{J_{eq\_y} + M_{heli} l_0^2 \cos(x_{11})^2} \end{bmatrix}$$

$$M = \begin{bmatrix} \dfrac{r_p K_{pp}}{Jeq\_p + M_{heli} l_0^2} & \dfrac{K_{py}}{Jeq\_p + M_{heli} l_0^2} \\ \dfrac{K_{yp}}{J_{eq\_y} + M_{heli} l_0^2 \cos(x_{11})^2} & \dfrac{r_y K_{yy}}{Jeq\_y + M_{heli} l_0^2 \cos(x_{11})^2} \end{bmatrix}$$

III. SYNTHESIS OF CONTROL LAWS AND STABILITY ANALYSIS

A. Synthesis of Control Laws Considering Stability Time based on Sequential Manifolds

The assembly control design process follows the analytical design of the aggregated regulators (ADAR) approach. The main steps of the process can be summarized as follows.

Suppose the controlled MIMO system is divided into s subsystems of order n, described by a system of autonomous nonlinear differential equations of the form:

$$\begin{cases} \dot{x}_i(t) = f_i(x_1, \dots, x_i) + a_{i+1} x_{i+1}, \quad i = 1 \div n-1 \\ \dot{x}_n(t) = f_n(x_1, \dots, x_n) + \sum_{j=1}^{s} b_j u_j \end{cases} \quad (3)$$

where x is the subsystem state vector, u is the control input vector of the large system, and t is time.

First, define s macro variables for the subsystems as a function of the subsystem's state variables $\psi_j (j = 1 \div s)$. The potentiometric control law will force the moving system to operate on manifolds $\psi_j = 0$. By selecting these macro variables, it is possible to ensure that the system has the characteristics according to the desired control quality parameters. The conventional selection method is based on a function of system state variables of the form:

$$\psi_j = x_n - \varphi_j(x_1, \dots, x_{n-1}, x_{sp})$$

Macro variables will have dynamic properties that satisfy the system of convergent functional equations, usually choosing a system of equations of the form:

$$T_j\dot{\psi}_j + \psi_j = 0, \quad T_j > 0, \quad j = \overline{1,s} \tag{4}$$

where T_j is a design parameter that determines the rate of convergence to the manifold specified by the macro variable. Substituting the macro variable derivative into (3) and (4) gives:

$$T_j \frac{\partial \psi_j}{\partial x}\left(f_i(x_1,\ldots,x_n) + \sum_{j=1}^{s} b_j u_j\right) + \psi_j = 0,\ T_j > 0 \tag{5}$$

These equations are utilized to synthesize the control law vector u, but in the control law, the function φ_j is unknown. The function φ_j in the s equations above is found from the subsystems. Based on [26, 28], for each SISO subsystem, manifolds are built to ensure that the final manifold converges to 0. Normally, the functional form

$$\psi_{j,n-1} = x_2 - (x_1 - x_{sp})/T_{j,n}$$

can be chosen, where x_{sp} is the desired technology invariant.

Lemma 1: When the manifold satisfies (4), in each time channel, the manifold $\psi_j(x)$ is stable from the initial position to the origin with an error of less than 5%, satisfying:

$$t \le 3T_j \tag{6}$$

Proof*:*

The solution of (4) demonstrates that the stable function is asymptotically stable to the origin. Transforming (4) gives:

$$-\frac{1}{T}dt = \frac{d\psi}{\psi} \Rightarrow -\frac{1}{T}\int_0^t dt = \int_{\psi_0}^{\psi_t}\frac{d\psi}{\psi} \Rightarrow -\frac{1}{T}t = ln\left(\frac{|\psi_t|}{|\psi_0|}\right) \Rightarrow$$
$$t = -T\,ln\left(\frac{|\psi_t|}{|\psi_0|}\right)$$

With the condition of stability about the origin with error less than 5%, we have:

$$t = -T_j\,ln\left(\frac{|\psi_t|}{|\psi_0|}\right) \le -T_j\,ln\left(\frac{|0.5\psi_0|}{|\psi_0|}\right) \le 3T_j \tag{7}$$

Lemma proven.

Lemma 2: The stabilization time to the origin for a MIMO system with s subsystems (3) with the control law u found above satisfies the following conditions.

Proof*:*

For each subsystem, the proposed control law will push the system through $n-1$ manifolds to reach the desired technological invariant, and in the first step, to make the system globally stable, it is necessary to perform an initial movement about s manifold ψ_j. From here, combined with Lemma 1, Lemma 2 is proven.

B. Synthesis of Finite-Time Control Law for 2-DOF Helicopter based on Sequential Manifolds

The purpose of the 2-DOF helicopter control problem is to certify that the helicopter's pitch and yaw channels follow the desired values θ_{sp} and α_{sp} by changing the voltage applied to the motor to create an impact torque. From the synthesis process presented above, the control signal vector function $u = [V_p, V_y]^T$ depends on the phase coordinates. The control signal vector will move the 2-DOF helicopter position according to the given signal or stabilize at the desired position when there is interference to assure the required quality of the system. From the purpose of the problem of controlling a 2-DOF helicopter following a given signal, based on integrated control theory, the first technological invariant is presented, corresponding to the control goal.

$$x_{sp} = [\theta_{sp} \quad \alpha_{sp}]^T \tag{7}$$

In the first step, based on the system mathematical model, when the control signal vector u changes, it affects the dynamics of the pitch and yaw channels, so the first manifold vector is selected as:

$$\psi_1 = [\psi_{11} \quad \psi_{21}]^T =$$
$$[x_{21} - \varphi_{11}(x_{11},\theta_{sp}) \quad x_{22} - \varphi_{21}(x_{12},\alpha_{sp})]^T \tag{8}$$

where the function vectors φ_{11} and φ_{21} determine the desired characteristics of the change in angular velocity of the pitch and yaw channels at the intersection with the invariant manifolds $\psi_1 = 0$ and $\psi_2 = 0$. The functions φ_{11} and φ_{21} are determined in the process of synthesizing the control law, proceeding from the first technological invariant condition (7). To ensure the global stability of the manifold (8), according to the design method above, the macro variables ψ_1 and ψ_2 must satisfy the solution of the system of basic functional equations:

$$T\dot{\psi}_1 + \psi_1 = 0 \tag{9}$$

where $T = [T_1\ 0; 0\ T_2]$ is a positive definite constant matrix to ascertain the conditions for asymptotic stability of the system motion. Substituting (8) into (9) and (2) gives the system:

$$Mu + N =$$
$$\begin{bmatrix} -\frac{1}{T_1}\left(x_{21} - \varphi_{11}(x_{11},\theta_{sp})\right) + \frac{\partial\varphi_{11}}{\partial x_{11}}x_{21} + \frac{\partial\varphi_{11}}{\partial\theta_{sp}}\dot{\theta}_{sp} \\ -\frac{1}{T_2}\left(x_{22} - \varphi_{21}(x_{12},a_{sp})\right) + \frac{\partial\varphi_{21}}{\partial x_{12}}x_{22} + \frac{\partial\varphi_{j,1}}{\partial a_{sp}}\dot{a}_{sp} \end{bmatrix} \tag{10}$$

The vector of the internal control signal u is:

$$u = M^{-1}\left(\begin{bmatrix} -\frac{1}{T_{11}}\psi_{11} + \frac{\partial\varphi_{11}}{\partial x_{11}}x_{21} + \frac{\partial\varphi_{11}}{\partial\theta_{sp}}\dot{\theta}_{sp} \\ -\frac{1}{T_{21}}\psi_{21} + \frac{\partial\varphi_{21}}{\partial x_{12}}x_{22} + \frac{\partial\varphi_{21}}{\partial\alpha_{sp}}\dot{\alpha}_{sp} \end{bmatrix}\right) \tag{11}$$

When the system enters the manifold, the point representing the system touches the intersection of the manifold $\psi_1 = 0$, the system will experience dynamic separation of the subsystems in (2), and the dynamics of the subsystems will occur. The subsystem according to the channels is modeled by:

$$\dot{x} = [\varphi_{11}(x_{11},\theta_{sp}) \quad \varphi_{21}(x_{12},\alpha_{sp})]^T \tag{12}$$

where the functions $\varphi_{11}(x_{11},\theta_{sp})$ and $\varphi_{21}(x_{12},\alpha_{sp})$ in the decomposition systems (11) and (12) can be considered internal control signals.

In the second step of the synthesis, to search for control and to determine the functions $\varphi_{11}(x_{11},\theta_{sp})$and $\varphi_{21}(x_{12},\alpha_{sp})$, an additional invariant manifold is introduced, which will ensure the stability of the closed-loop system and the fulfillment of technological invariance (7). System dynamics (11) and (12) to certify stability and satisfy technological invariance (7) will choose the internal control signals:

$\varphi_{11}(x_{11}) = -(x_{11} - \theta_{sp})/T_{11}$ and

$\varphi_{21}(x_{12}) = -(x_{12} - \theta_{sp})/T_{21}$

so that the system of separation equations has the following form:

$$\dot{x} = \left[-\frac{1}{T_{11}}\left(x_{11} - \theta_{sp}\right) \quad -\frac{1}{T_{21}}\left(x_{12} - \alpha_{sp}\right)\right]^T \tag{13}$$

For (13), the asymptotic stability conditions at $x_{11} = \theta_{sp}$, $x_{12} = \alpha_{sp}$ and T_{11}, T_{21} are positive constants. From (13) and the manifolds $\varphi_{11}(x_{11}, \theta_{sp})$ and $\varphi_{21}(x_{12}, \alpha_{sp})$ selected above, the control law u for the 2-DOF helicopter can be described as:

$$u = M^{-1} \times \left(\begin{bmatrix} -\frac{1}{T_1}\left(x_{21} + \frac{1}{T_{11}}\left(x_{11} - \theta_{sp}\right)\right) - \frac{1}{T_{11}}x_{21} + \frac{1}{T_{11}}\dot{\theta}_{sp} \\ -\frac{1}{T_2}\left(x_{22} + \frac{1}{T_{21}}\left(x_{12} - \alpha_{sp}\right)\right) - \frac{1}{T_{21}}x_{22} + \frac{1}{T_{21}}\dot{\alpha}_{sp} \end{bmatrix} - N\right) \tag{14}$$

With control law (12) and Lemma 2, the time for the system to stabilize is:

$$t_{sys\_heli} \le max\left(\left(max\left(3T_{j,1}\right) + max\left(3T_j\right)\right)\Big|_{j=1}^{2}\right) \tag{15}$$

C. Synthesis of the Backstepping Controller for 2-DOF Helicopter

To prove its effectiveness, a control law was synthesized deploying the Backstepping method [12] to compare the numerical results of the system's response. The design process using the backstepping method is carried out as follows:

1) Step 1

First, the angular error on the Pitch and Yaw channels is selected:

$$z_1 = x_1 - x_{sp} \tag{16}$$

The virtual control variable is defined as:

$$\alpha_1 = -C_1 z_1 \tag{17}$$

where C_1 is a positive definite diagonal matrix. Then consider the error given by:

$$z_2 = x_2 - \alpha_1 - \dot{x}_{sp} \tag{18}$$

After that, $\dot{z}_1 = \dot{x}_1 - \dot{x}_{sp} = x_2 - \dot{x}_d = z_2 + \alpha_1$.

According to the backstepping control, the first Lyapunov function is defined as:

$$V_1 = \frac{1}{2} z_1^T z_1 \tag{19}$$

Differentiating (19) with respect to time gives:

$$\dot{V}_1 = z_1^T \dot{z}_1 = -C_1 z_1^T z_1 + z_1^T z_2 \tag{20}$$

Obviously, if $z_2 = 0$ then $\dot{V}_1 \le 0$, but it cannot be guaranteed that $z_2 = 0$ all the time. Therefore, the virtual control variable α_2 is introduced to make $z_2 = 0$.

2) Step 2

From (18) and (17) $\dot{\alpha}_1 = -C_1 \dot{z}_1 = -C_1(\dot{x}_1 - \dot{x}_{sp})$. The derivative of z_2 is expressed as:

$$\dot{z}_2 = N + Mu + C_1(x_2 - \dot{x}_{sp}) - \ddot{x}_{sp} \tag{21}$$

Choose the positive definite Lyapunov function:

$$V_2 = V_1 + \frac{1}{2} z_2^T z_2 \tag{22}$$

Differentiating (22) with respect to time gives:

$$\dot{V}_2 = \dot{V}_1 + z_2^T \dot{z}_2 = -C_1 z_1^T z_1 + z_2^T (z_1 + \dot{z}_2) \tag{23}$$

Similarly, to $\dot{V}_2 \le 0$ for all states of the system:

$$z_1 + \dot{z}_2 = -C_2 z_2 \tag{24}$$

where C_2 is a positive definite diagonal matrix. From (21) and (24), the control law becomes:

$$u = M^{-1}\left(-C_2 z_2 - C_1(x_2 - \dot{x}_{sp}) - z_1 - N + \ddot{x}_{sp}\right) \tag{25}$$

D. Stability Analysis

To check the stability of the control law u when the system has unknown and noisy components, such as (2), the positive definite Lyapunov function is chosen, having the form:

$$V = 0.5 \psi_1^T \psi_1 \tag{26}$$

Taking the Lyapunov derivative (26) when the control law (14) acts on (2), gives:

$$\dot{V} = \psi_1^T \left(N + Mu + d - \begin{bmatrix} \frac{\partial \varphi_{11}}{\partial x_{11}} x_{21} + \frac{\partial \varphi_{11}}{\partial \theta_{sp}} \dot{\theta}_{sp} \\ \frac{\partial \varphi_{21}}{\partial x_{12}} x_{22} + \frac{\partial \varphi_{j,1}}{\partial \alpha_{sp}} \dot{\alpha}_{sp} \end{bmatrix}\right) \tag{27}$$

Substituting the proposed control law (14) into (27) gives:

$$\dot{V} = \psi_1^T \left(\begin{matrix} N + \delta - \begin{bmatrix} \frac{\partial \varphi_{11}}{\partial x_{11}} x_{21} + \frac{\partial \varphi_{11}}{\partial \theta_{sp}} \dot{\theta}_{sp} \\ \frac{\partial \varphi_{21}}{\partial x_{12}} x_{22} + \frac{\partial \varphi_{j,1}}{\partial \alpha_{sp}} \dot{\alpha}_{sp} \end{bmatrix} + \\ \begin{bmatrix} -\frac{1}{T_{11}} \psi_{11} + \frac{\partial \varphi_{11}}{\partial x_{11}} x_{21} + \frac{\partial \varphi_{11}}{\partial \theta_{sp}} \dot{\theta}_{sp} \\ -\frac{1}{T_{21}} \psi_{21} + \frac{\partial \varphi_{21}}{\partial x_{12}} x_{22} + \frac{\partial \varphi_{21}}{\partial \alpha_{sp}} \dot{\alpha}_{sp} \end{bmatrix} - N \end{matrix} \right) \Rightarrow$$

$$\dot{V} = -\psi_1^T (T^{-1} \psi_1 + d) \tag{28}$$

The proposed control law (21) will bring the moving system to the ψ_1 manifold. From (9), the solution of ψ_1 can be found:

$$\psi_1(t) = \psi_0 e^{-T^{-1} t} \tag{29}$$

where $\psi_0 = [\psi_{10} \;\; \psi_{20}]$ is the value vector at the initial point in each manifold. Substituting (36) into (35) gives:

$$\dot{V} = -\left(\psi_0 e^{-T^{-1} t}\right)^T \left(T^{-1} \psi_0 e^{-T^{-1} t} + d\right) = \\ = -e^{-T^{-1} t} {\psi_0}^T \left(T^{-1} \psi_0 e^{-T^{-1} t} + d\right) \tag{30}$$

The system is asymptotically stable if $\dot{V} \leq 0$. So, d must satisfy:

$$|d| < T^{-1}|\psi_0|e^{-T^{-1}t} \tag{31}$$

Therefore, with the proposed control law, when interference is blocked, the system is still stable near the desired state but is not able to eliminate interference. The parameters of the T matrix affect the sustainability when there is interference.

IV. RESULTS AND DISCUSSION

A. Test Description

In-depth computer-based simulations were carried out to demonstrate the potential of the proposed control law for the 2-DOF helicopter system. The parameters of the 2-DOF helicopter system were: M_{heli} = 1.0750 kg , g = 9.8 m/s^2, B_p = 0.0071 N/V, B_y = 0.0220 N/V, J_{eq_p} = 0.0215 kgm^2, J_{eq_y} = 0.0237, K_{pp} = 0.022 Nm/V, K_{yy} = 0.0022 Nm/V , K_{py} = 0.0221 Nm/V, K_{yp} = -0.0227 Nm/V, l_0 = 0.002 m, r_p =0.25 m, and r_y = 0.2 m. The proposed control law parameters were selected based on the desired stabilization time through the parameters T_1, T_2, T_{11} and T_{21}. These parameters reflect the convergence time to the selected manifolds and the total time from the initial position to the desired point can be calculated in advance. In this study, a set of parameters was selected with the scenario where the system is expected to be stable for 1 s as follows: $T_1 = 0.01$, $T_2 = 0.01$, $T_{11} = 0.25$, $T_{21} = 0.25$. Backstepping controller parameters C_1 and C_2 were selected on the basis of [12] by trial and error. The first method for implementing the backstepping control law and the proposed control law was carried out with 3 scenarios: the first scenario is when the initial state of the system is at the origin $x_1(0) = [0; 0]T$ rad, $x_2(0) = [0; 0]T$ rad and will stabilize to the expected values $x_{sp}(t) = [0.3; -0.8]T$ rad. The second scenario is when the initial state of the origin system will track the orbit with the desired tracking signal of the form:

$x_{sp}(t) = [1.4\sin(0.5t); 1.4\cos(0.5t)]T$ rad [5]. The third scenario is when the initial state of the system at the origin will stabilize to the expected values $x_{sp}(t) = [0.2; 0.3]T$ rad when the system is affected by random disturbance d in the range [-5; 5].

B. Numerical Test Results

The simulation scenarios were performed in Matlab. In the first scenario, Figures 2 and 3 depict the response of the pitch and yaw angles of the 2-DOF helicopter system with the two control laws. It can be noted that both the backstepping controller and the proposed control law are stable about the origin. The actual value of the angle in the channels of the 2-DOF helicopter system is the desired value. However, the graph clearly shows that the proposed controller's transient time is better. The transition time from the pitch angle θ to the desired value with an error of 0.015 rad is 0.76 s, while the backstepping controller takes 5.28 s, and for the yaw angle channel α the time transient is 0.76 s, while the backstepping controller takes 5.30 s. In addition, it can be noticed that the quality of the proposed controller is much better, as there are no overshoots and fluctuations. In contrast, the backstepping controller has large overshoot and oscillations. The transient time of the system with the proposed control law to meet the requirements of the scenario is less than 1 s and satisfies inequality (20). This demonstrates the effectiveness of the control law design following this method.

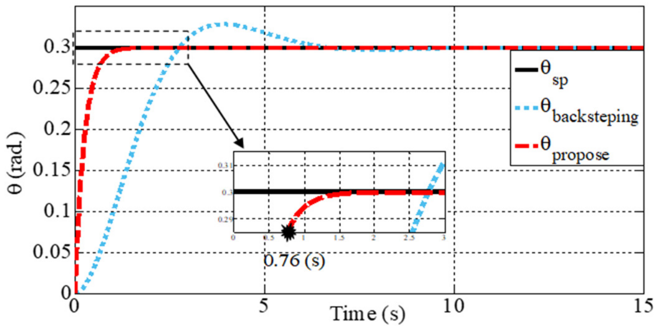

Fig. 2. Pitch angle response of the 2-DOF helicopter.

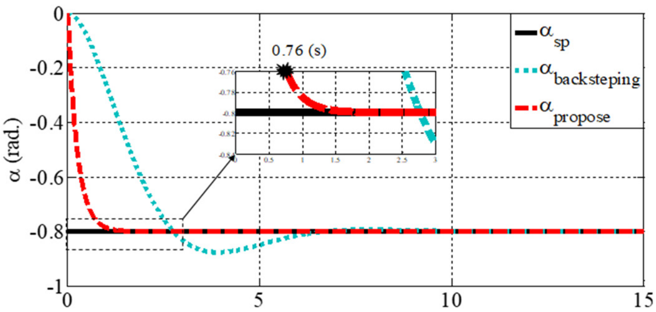

Fig. 3. Yaw angle response of the 2-DOF helicopter.

Figures 4 and 5 portray the responses of the pitch and yaw angles of the 2-DOF helicopter system with the two control laws.

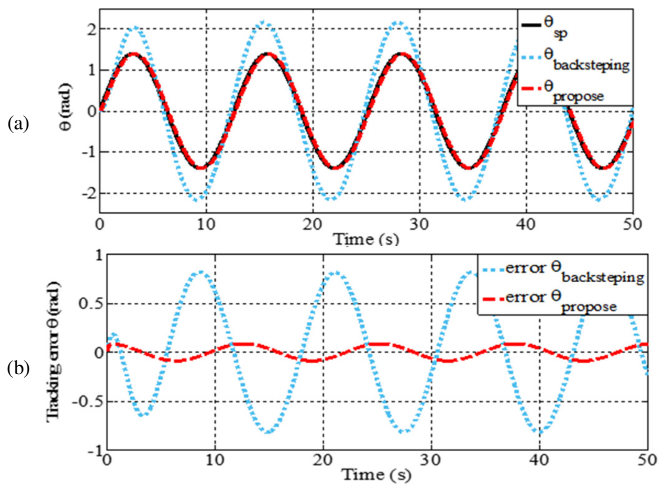

Fig. 4. The pitch angle response of the 2-DOF helicopter: (a) trajectory tracking signal, (b) error signal of angle compared to set value.

Figures 4(a) and 5(a) manifest that if the setting signal is oscillating, the quality of the proposed and the backstepping controller are both worse than in the first scenario. These two controllers both give results in phase tracking with small errors. The recommended controller is out of phase before the set signal (0.13 rad), and the backstepping controller is in phase before the set signal (0.14 rad). However, with the proposed controller, the system response results are better in terms of amplitude than with the backstepping controller. With the proposed control law, the amplitude tracking error is in the range [-0.174; 0.174] for the pitch channel and [-0.173; 0.173] for the yaw channel. For the backstepping controller, it is [-0.85; 0.85] for the pitch channel and [-0.84; 0.84] for the yaw channel. In addition, the error graphs in Figures 4(b) and 5(b) also exhibit that the proposed controller gives the largest error at $nT/2$ signal cycles, while the backstepping controller gives the largest error at $(2n-1)T/4$ set signal cycles. Thus, it can be concluded that the backstepping controller is sensitive to changes in set value amplitude. The proposed controller responds well to changes in the set signal and is in a stable state.

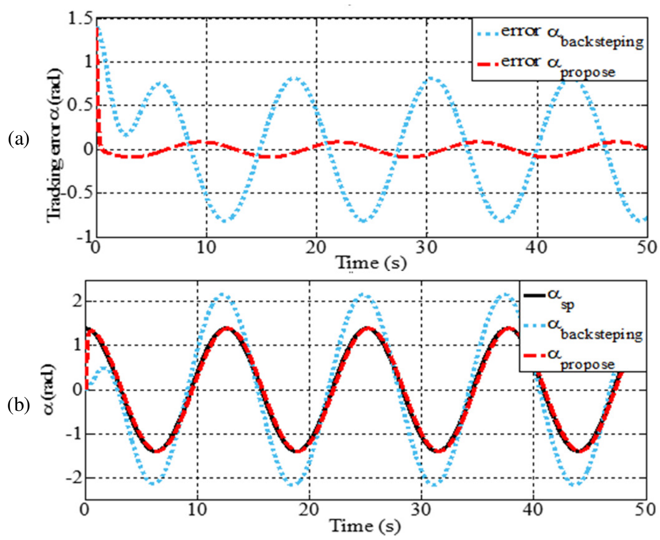

Fig. 5. The yaw angle response of the 2-DOF helicopter: (a) trajectory tracking signal, (b) Signal of angle error compared to set value.

In the third scenario, when there is noise, Figures 6 and 7 show the responses of the two channels' pitch and yaw. It can be observed that the proposed controller gives good response results when there is noise, the system is stable at the desired value on the two amplitude channels, and the noise is very small in both [-0.0006; 0.0006]. The backstepping controller is very sensitive to random noise with large fluctuation amplitudes for both channels [-0.2; 0.2]. These results suggest that the response of the proposed control law provides much better findings. This can be explained by the fact that in the proposed control law, it is only necessary to introduce the first manifold, and (31) proves the simulation results in this scenario.

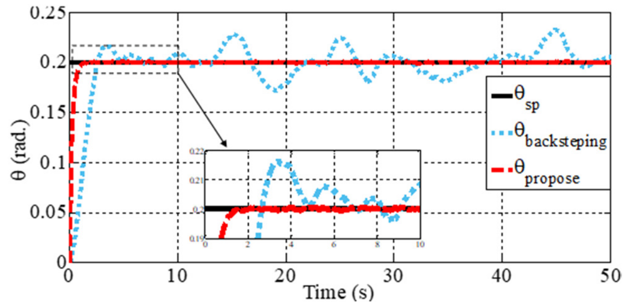

Fig. 6. Pitch angle response of a 2-DOF helicopter when there is interference.

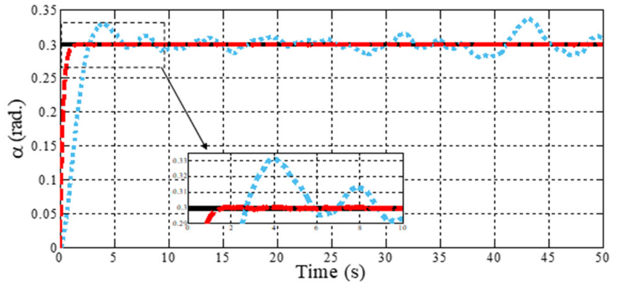

Fig. 7. Yaw angle response of a 2-DOF helicopter in response to noise.

V. CONCLUSION

This study developed a finite-time trajectory control law for a 2-DOF helicopter in the presence of interference, based on sequential manifold and combinatorial control theory. According to the analysis of the results, the proposed controller exhibits good responses to different types of set values and good trajectory-tracking ability with trigonometric set signals. When the system has noise, the proposed controller has good anti-interference ability and small fluctuation amplitudes. In addition, the present study has disclosed the stability time of the proposed control law, given the inequality testing by numerical simulations. Furthermore, this work also reveals stability conditions through stability analysis using the Lyapunov function. Finally, the simulation results confirmed the superiority and effectiveness of the proposed control law. In the future, focus will be placed on control research with different manifold shapes for 2-DOF helicopters, taking into account the physical properties of the manifold, with specified performance when model-clogging uncertainties and external disturbances exist.

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