An Improved Form of Hazen-Williams Equation for Pressurized Flow

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ABSTRACT

This study performed a sensitivity analysis to correlate the frictional head loss calculated by the Darcy-Weisbach (D-W) and the Hazen-Williams (H-W) formulas. For a broad variety of fluid temperatures, velocities, and pipeline diameters, this study considered an extensive discussion and analysis to determine friction loss within pressurized pipelines using Microsoft Excel. Regression analysis and statistical tools were applied to improve the relationship between the two equations. A more accurate expression was developed to calculate the friction loss in terms of the H-W equation. The estimated values were compared with previous experimental and numerical studies, and a good agreement was observed. The proposed model was evaluated using WaterGEMS software in an application example of a water supply system against the D-W and H-W equations. Good agreements were recorded between predicted values and previous studies, with an error of less than 1%. These findings can be used to improve the hydraulic design of engineering applications.

Keywords-friction; Darcy-Weisbach; Hazen-Williams; computation; WaterGEMS; application example

I. INTRODUCTION

In the design of pipe networks, frictional head losses play a significant role in determining the hydraulic balance and overall cost of the network [1]. Designers determine the sizing of pipes in a network based on the value of frictional losses allowed. The operating cost is inversely correlated with the pipeline diameter. For example, by increasing the pipeline diameter, the frictional head loss increases, the pumps' power consumption is reduced, and vice versa. The Darcy-Weisbach (D-W) (1) and the Hazen-Williams (H-W) (2) theoretical and empirical equations, respectively, are commonly used to calculate energy head losses due to friction in pipes:

$$h_f = f(\frac{L}{d})(\frac{V^2}{2g}) \quad (1)$$

where h_f is the head loss caused by friction, f is the D-W friction coefficient, L is the pipeline length, V is the flow velocity, d is the pipeline diameter, and g is the gravity acceleration.

$$h_f = 10.69L(\frac{Q}{C})^{1.852}(d^{-4.87}) \quad (2)$$

where h_f is the frictional head loss (m), Q is the flow rate (m^3/sec), C is the H-W friction coefficient, L is the pipeline length (m), and d is the pipeline diameter (m).

Determining the frictional head loss by applying the D-W equation is quite challenging because it involves different variables. The friction factor (f) depends on the ratio of roughness height to the pipe diameter (e/d) and Reynolds number (R_e), which depend on the pipe size, flow velocity, fluid density, and viscosity, which depend on the fluid's temperature:

$$R_e = \frac{\rho V d}{\mu} = \frac{V d}{\upsilon} \quad (3)$$

where ρ is the fluid's density, μ is the fluid's dynamic viscosity, and υ is the fluid's kinematic viscosity. Computing the friction loss depends on the flow condition [2], whether it is laminar ($R_e < 2000$) or turbulent ($R_e > 4000$). For the laminar flow, the friction loss is determined by:

$$f = \frac{64}{R_e} \quad (4)$$

For fully turbulent and transitional flows, f can be determined by the following implicit equation (known as Colebrook-White):

$$\frac{1}{\sqrt{f}} = -2Log\left[\frac{e}{3.7d} + \frac{2.51}{R_e\sqrt{f}}\right] \quad (5)$$

where e is pipe roughness height. In [3], a formula for calculating f was introduced, yielding results consistent with those of [4]:

$$f = 0.53\left(\frac{e}{d}\right) + 0.094\left(\frac{e}{d}\right)^{0.225} + 88\left(\frac{e}{d}\right)^{0.44}(R_e)^{-1.62e^{0.134}} \quad (6)$$

In [5], an accurate formula was developed for estimating f in the D-W equation. This formula is valid for all flow regimes, as well as smooth and rough pipelines:

$$f = 8\left[\left(\frac{8}{R_e}\right)^{12} + \left\{\left(-2Log\frac{\left(\frac{e}{d}\right)}{3.7} + \left(\frac{7}{R_e}\right)^{0.9}\right)^{16} + \left(\frac{37530}{R_e}\right)^{16}\right\}^{\frac{-3}{2}}\right]^{\frac{1}{12}} \quad (7)$$

Many programs, including EPANET [6], use the explicit equations [7-8] to calculate the head losses, which can be expressed as follows:

$$f = \frac{0.25}{\left(-Log\left[\frac{e}{3.7d} + \frac{5.74}{R_e^{0.9}}\right]\right)^2} \quad (8)$$

The H-W equation is used more frequently in practice than the D-W equation due to its simplicity. However, this equation has several shortcomings, such as ignoring the fluid temperature and density and assuming a fixed roughness coefficient for all flow velocities and pipe diameters [9, 10]. In [11], a diagram was plotted, based on experimental data, between the friction factor f versus R_e on logarithmic scales for different relative roughness values for all flow regimes. In [12], the potential effects of using the H-W equation were experimentally examined for designing large-diameter pipes. In [13], an experimental investigation was conducted on the friction loss coefficient in non-circular pipes.

In [14], an equation was presented that links the H-W and D-W equations for plastic and metallic pipelines, which is limited to pipe diameters of 15-50 mm and flow rates of 0.25-2 l/s. In [15], WaterCAD was used to develop an empirical formula for estimating head loss within treated wastewater pipes in Kerbala City, Iraq, using the H-W and D-W equations. In [16], the impact of pipe aging was examined for various pipe materials on head losses for the unicentral network in Malaysia using the H-W and D-W equations. In [17], the results of 29 explicit equations that calculate the friction coefficient were compared to the one calculated by the Colebrook-White equation. This study concluded that each equation has specific limitations and significant variations in its output. In [18], a hydraulic analysis was performed to compare the friction factor for the H-W and D-W equations in irrigation lines. The friction correction of the coefficients was evaluated and compared with field data collected from center-pivot irrigation systems with polyvinyl chloride laterals [19]. The results revealed a poor agreement between the observed data and those computed from the typical H-W and D-W equations.

This study aims to analyze the relationships between the H-W and D-W formulas considering high flow rates, large pipeline diameters, and a high Reynolds number. The analysis included different pipe materials and flow rates at varying temperatures. The results were verified with those of previous experimental and numerical studies. The analysis procedures were simplified using a Microsoft Excel worksheet. In addition, a case study was presented to compare the results of the proposed solution with those obtained by WaterGEMS.

II. MATERIALS AND METHODS

This study performed a sensitivity analysis to identify the correlation between the head losses calculated by the H-W and D-W equations. A wide range of values was taken for each variable considered in the field of hydraulic engineering applications, as shown in Table I. For this analysis, only one parameter was modified at a time while keeping the others constant. Microsoft Excel was used to simplify the process. Under identical conditions, the head losses per meter (h_f/L) were calculated using both the H-W and D-W equations.

The fluid density (ρ) and the dynamic viscosity (μ) depend on the fluid temperature. Therefore, ρ (kg/m^3) and μ (kg.m^{-1}.s^{-1}) were calculated according to the specified temperatures (T in ^{o}C) by [20]:

$$\rho = 1.54 \times 10^{-8}T^3 - 1.85 \times 10^{-5}T^2 + 6.65 \times 10^{-8}T + 2.47 \times 10^{-1} \quad (9)$$

$$\mu = 3.38 \times 10^{-8}T^4 - 4.63 \times 10^{-5}T^3 + 2.37 \times 10^{-2}T^2 + 5.45 \times T + 4.70 \times 10^2 \quad (10)$$

The findings of this study were compared with previous ones. Furthermore, a case study is presented to compare the obtained results to the outputs of the WaterGEMS software. In addition, some statistical information is presented.

TABLE I. VALUES OF PARAMETERS APPLIED IN THE COMPUTATIONS

Parameter	Value	Units
Fluid temperature (T)	10 – 60	°C
Fluid density (ρ)	983 – 1000	kg/m^3
Fluid dynamic viscosity (μ)	0.001307 – 0.000467	kg.m^{-1}.s^{-1}
Fluid kinematic viscosity (υ)	4.75*10^{-7} – 1.31*10^{-6}	m^2/s
Fluid velocity (V)	0.25 – 2	m/s
Pipeline diameter (d)	25 – 2000	mm
Roughness height (e)	0.001, 0.01, 0.05, 0.1, 0.5, 1, 5, 10	mm
Reynolds number (R_e)	9564 - 497507	---
H-W coefficient (C)	80, 100, 120, 130, 140, 150, 160	---

III. RESULTS AND DISCUSSIONS

A. Correlation between H-W and D-W Equations for the Frictional Head Losses

A sensitivity analysis was carried out to establish a correlation between the friction head losses calculated using the H-W and D-W equations. During the analysis, all variables outlined in Table I were kept constant, except for one varied variable. Figure 1 shows the relationship between the head losses calculated by the D-W and H-W formulas. According to Figure 1, there is a good agreement between the D-W and H-W equations, with a correlation coefficient (R^2) of 99.66%. The correlation between the two formulas can be expressed as follows:

$$h_{(D-W)} = 0.9372 \times \left[h_{(H-W)}\right]^{1.0328} \quad (11)$$

where $h_{(D\text{-}W)}$ and $h_{(H\text{-}W)}$ are the frictional head loss by D-W and H-W in meters per one-meter length of the pipeline. To validate the obtained results, the values of head loss by D-W were plotted against those of (11), as shown in Figure 2. There is a high agreement between the proposed and the D-W equations (R^2 = 1). A modified H-W equation can be obtained as follows:

$$h_f = 10.828(L)^{1.0328}\left(\frac{Q}{C}\right)^{1.91275}\left(d^{-5.0297}\right) \quad (12)$$

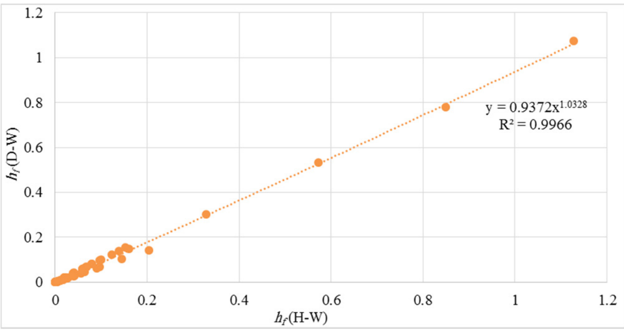

Fig. 1. Friction head losses by D-W against H-W.

B. Validation of the Proposed Equation

Figure 3(a) presents the validation of the values of friction losses of (12) against those measured by [10, 12, 13, 21, 22, 23]. Figure 3(b) compares the current results with those predicted by D-W (1), H-W (2), and other numerical studies, such as [3] (6), [5] (7), and [7] (8). Figure 3 clearly shows that the results of the proposed equation perfectly align with all previous studies that were considered. R^2 ranges from 99.97% to 99.34%, and the standard error (SE) ranges from 0.00292 to 0.00442.

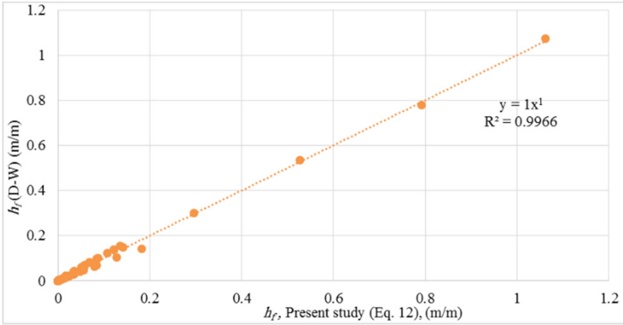

Fig. 2. Friction head losses by D-W against (12).

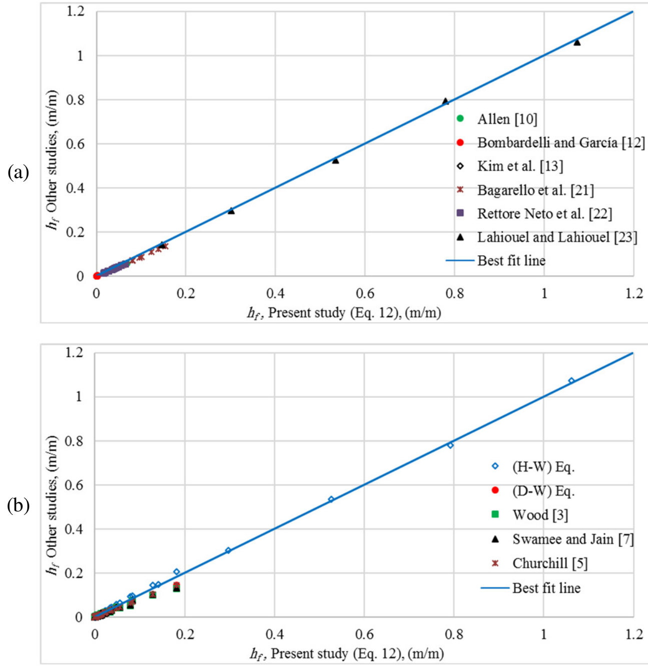

Fig. 3. Comparison between the friction head loss by (12) and: (a) experimental studies, and (b) numerical studies.

C. Illustrating Example (Application)

A case study is presented to demonstrate the accuracy of (12). The values predicted using the proposed equation were compared to those calculated by the D-W and H-W equations. The example consists of two working pumps that supply the system by 110.87 L/s against a difference in water levels of 63 m [24]. The network consists of 1475 m of pipelines with diameters ranging from 250 to 350 mm, and its elevation ranges from 16 to 53 m. The lengths and diameters of all pipelines, the elevation of the junctions, and the pumps' characteristics were extracted from the software's manual [24]. Figure 4 shows a layout of the modeled system. The WaterGEMS Connect Edition software [25] was applied in the analysis, which uses the gradient method to solve the continuity and energy principles within a pressurized conduit. Four different hydraulic systems were created to consider two distinct pipeline materials, polyvinyl chloride (PVC) (C = 150 and e = 0.0015 mm) and ductile iron (DI) (C = 120, e = 0.15 mm) [26], at two fluid temperatures (20 and 60°C).

Table II presents the results of the WaterGEMS analysis for the four studied scenarios (steady state), considering both the D-W and H-W frictional head losses along with those calculated with (12). Two statistical parameters, particularly the Standard Error (SE) and the correlation coefficient (R^2), were used to evaluate the obtained results. The predicted head pressure values with (12) regarding the pipeline materials (PVC and DI) and the applied temperatures (20 and 60°C) were highly in agreement with those calculated by the H-W and D-W equations. The correlation factor (R^2) obtained considering the introduced formula ranges from 99.98% to 100%, whereas for the classical H-W formula, it ranges from 99.88% to 99.92%. Furthermore, SE for the proposed equation has the lowest values (4.6~4.7), while it is 4.70~4.80 and 4.80~4.90 for the D-W and H-W equations, respectively.

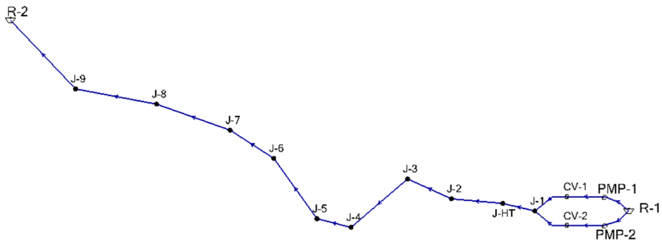

Fig. 4. A schematic of the modeled system using WaterGEMS [24].

TABLE II. COMPARISON OF PRESSURE HEAD VALUES USING D-W, H-W, AND (12) FOR THE APPLICATION EXAMPLE

Label (Node)	**Pressure head (m H_2O)**						**Pressure head (m H_2O)**					
Pipe material	**PVC (C = 150, e = 0.0015 mm)**						**DI (C = 120, e = 0.15 mm)**					
Temperature	**T = 20°C**			**T = 60°C**			**T = 20°C**			**T = 60°C**		
Equation	**WaterGEMS**		**This study (12)**	**WaterGEMS**		**This study (12)**	**WaterGEMS**		**This study (12)**	**WaterGEMS**		**This study (12)**
	D-W	**H-W**		**D-W**	**H-W**		**D-W**	**H-W**		**D-W**	**H-W**	
J-1	51.56	51.76	51.66	51.19	51.76	51.29	52.38	53.24	52.49	52.25	53.24	52.36
J-HT	49.48	49.67	49.59	49.12	49.67	49.22	50.28	51.12	50.39	50.15	51.12	50.26
J-2	48.2	48.38	48.36	47.87	48.38	47.99	48.94	49.71	49.08	48.82	49.71	48.95
J-3	35.94	36.11	36.08	35.64	36.11	35.76	36.62	37.33	36.77	36.51	37.33	36.64
J-4	45.36	45.5	45.50	45.11	45.50	45.29	45.91	46.49	46.14	45.83	46.49	46.01
J-5	44.16	44.28	44.30	43.93	44.28	44.12	44.66	45.19	44.91	44.58	45.19	44.78
J-6	27.84	27.94	28.00	27.65	27.94	27.83	28.26	28.71	28.51	28.2	28.71	28.39
J-7	21.64	21.73	21.80	21.47	21.73	21.65	22.02	22.41	22.28	21.96	22.41	22.15
J-8	16.09	16.15	16.30	15.98	16.15	16.19	16.34	16.61	16.56	16.30	16.61	16.53
J-9	11.54	11.57	11.80	11.48	11.57	11.52	11.66	11.80	11.80	11.64	11.8	11.80
R^2	------	99.92	99.99	------	99.89	99.98	------	99.94	1.00	------	99.88	99.98
SE	4.70	4.80	4.60	4.70	4.80	4.60	4.80	4.90	4.70	4.80	4.90	4.70

IV. CONCLUSIONS

This study presents a numerical analysis to determine the frictional head loss based on the D-W and H-W equations. Considering a wide range of specific parameter values, such as fluid temperatures, flow velocities, and pipeline diameters, this study carried out a large-scale analysis to calculate friction losses. Statistical tools were employed to determine a relationship between the head loss values derived from both the H-W and D-W equations. Following the analysis and discussion of the results, a predictive equation was formulated to calculate friction head loss in a pressurized conduit based on parameters such as diameter, friction coefficient, and discharge. The predicted values were then compared with experimental and numerical results from previous studies. WaterGEMS software was utilized to validate the proposed formula against the D-W and H-W equations for an application example. A high level of agreement was observed between the predicted friction loss values and those of the previously recorded results, with an accuracy exceeding 99%. Further research is needed to broaden the range of case studies and include real-life scenarios. Moreover, it is advisable to validate the results of the formulated equation through experimental testing and/or in situ measurements.

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