A Lyapunov Function for Vector Control Drives in Induction Machines

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***Abstract*—In this paper a useful Lyapunov function for vector control of induction machines is introduced. To do this, some beneficial theorems are reviewed and by applying these theorems and state equations to a vector control drive, its candidate Lyapunov function is achieved. Range of set points such as reference speed, reference flux linkage values and speed controller gain is obtained for such a drive and therefore these values can be set offline to avoid unstable behavior. The simulated and experimental results are given in the last section of the paper to show the efficacy of the given Lyapunov function.**

Keywords-Lyapunov function; stable Area; vector control drives

I. Introduction

A linear system shows stable, unstable, or oscillated responses; however, in a nonlinear system other kinds of behaviors such as bifurcated, chaotic and sub harmonic may occur. Electric machines are highly nonlinear systems because they have mutual dynamics. In addition, electric drives, or other controller types, are implemented in electric machines to control their speed, torque and thus also result to a nonlinear behavior. These time responses are not acceptable except the stable cases from an application point of view. It is obvious that these nonlinear behaviors cause fluctuation in electromagnetic torque, speed of the rotor, current and voltage components, flux linkages, etc. Strange time responses in electric machines drives such as induction machines, synchronous reluctance machines, permanent magnet synchronous machines, DC machines, brushless DC machines , etc. has been reported in several paper [1-7].

Nowadays vector control drives of induction machines are more suitable for a variety of applications such as trains, conveyer belts because of their dynamical behavior as well as for having a tough structure and cheap price. As mentioned, these drives may represent stable, unstable and chaotic responses. There are some researches that show the bifurcated and chaotic response of these drives caused by different reasons like inverter malfunction [8, 9], error in parameters' values like rotor and stator resistances [10-13] or even because of an inherent nonlinearity of the system [14].

Some researchers used Poincare map to study these behaviors. The Poincare map is a powerful method to investigate the strange behaviors such as chaotic and bifurcated ones [14, 15]. This map is useful for power electronic systems such as DC-DC choppers. However, it is not useful and applicable enough to analyze the stability of other systems such as induction machine drives which are the case study of this paper. One of the most powerful methods to study the global stability of nonlinear systems is the Lyapunov method. Using a Lyapunov function is the main core of the method. Finding these functions for nonlinear systems is difficult. Suitable theorems to evaluate the stability of nonlinear systems are needed [14]. In this paper these theorems are used to achieve a useful Lyapunov function for vector control drives in induction machines.

II. Candidate Lyapunov Function for Transformed Dynamics

Finding the Lyapunov function for a nonlinear system is very important, because stability analysis or controller design such as sliding mode controller, adaptive control, etc. are based on this function. According to the Lyapunov theory there is a nonlinear system such as:

$$\dot{X} = F(X) \quad X \in R^n \tag{1}$$

If a function such as $V : R^n \to R$ with the following conditions exists then the origin ($X = 0$) is a stable equilibrium state for this system:

$$\begin{gathered} V(X) > 0 \\ \dot{V}(X) < 0 \\ \|X\| \to \infty; V(X) \to \infty \\ V(0) = 0 \,, \dot{V}(0) = 0 \end{gathered} \tag{2}$$

The shortcoming of this theorem is that there is not a unique method to obtain a Lyapunov function. Sometimes this theorem is named Lyapunov direct method. There is another theorem based on linearized dynamical equations of a nonlinear system. It talks about local stability conditions for the equilibrium states of a system. This method is called indirect Lyapunov method in nonlinear system theory.

In [14], it was shown that if there exists a Lyapunov function like $V(X)$ for nonlinear system such as (1), then what can be concluded about the stability of a system such as:

$$\dot{X} = H^{-1}F(X) \qquad (3)$$

This system is a transformed system and two theories are introduced in [16] to achieve its Lyapunov function. The origin will be stable equilibrium point for this system if there exists a positive function $g(X)$ that satisfies the following conditions.

$$\int g(X)(\nabla(V^2(X)))^T H dx > 0 \qquad (4)$$

And $g(X)$ will exist if the following matrix be symmetric:

$$\begin{pmatrix} \left(\dfrac{\partial\nabla\left(V^2(X)\right)}{\partial x_1} + \dfrac{\partial \ln\left(g(X)\right)}{\partial x_1}\nabla\left(V^2(X)\right)\right)^T \\ \left(\dfrac{\partial\nabla\left(V^2(X)\right)}{\partial x_2} + \dfrac{\partial \ln\left(g(X)\right)}{\partial x_2}\nabla\left(V^2(X)\right)\right)^T \\ \vdots \\ \left(\dfrac{\partial\nabla\left(V^2(X)\right)}{\partial x_n} + \dfrac{\partial \ln\left(g(X)\right)}{\partial x_n}\nabla\left(V^2(X)\right)\right)^T \end{pmatrix} \times H \qquad (5)$$

One of the most applicable Lyapunov functions is quadratic function. Its global form is:

$$V(X) = X^T P X \qquad (6)$$

In this equation P is a positive matrix. By using the above method, if system (1) has a quadratic Lyapunov function and PH is positive and symmetric, its transformed system such as (2) has a quadratic Lyapunov function. Its Lyapunov function is :

$$W(X) = \int X^T PH dX \qquad (7)$$

It can be shown that dynamical equation of almost all rotational electric machines is:

$$\dot{X} = H^{-1}\left(AX + \begin{pmatrix} X^T N_1 X \\ X^T N_2 X \\ \vdots \\ X^T N_n X \end{pmatrix}\right) \qquad (8)$$

X , H^{-1} and A are originated from current components, inductance, moment of inertia, and resistance of machine. Also the other states ($X^T N_i X$) are originated from rotational voltages. This dynamic system is a transformed system therefore the above mentioned method can be used to study its stability. If a quadratic function such as (5) is found in a way that PH is positive and symmetric and the following condition is satisfied:

$$X^T P \begin{pmatrix} X^T N_1 X \\ X^T N_2 X \\ \vdots \\ X^T N_n X \end{pmatrix} = 0 \qquad (9)$$

Then (6) shows its Lyapunov function.

III. ROTOR FIELD ORIENTED VECTOR CONTROL SYSTEM

To investigate the stability of a system, first its model must be decided. The electrical model of squirrel cage induction machine in the synchronous reference oriented to the arbitrary $\vec{\Psi}_g$ flux linkage vector frame is shown in (10) [17].

$$\frac{d}{dt}\begin{pmatrix} \vec{\Psi}_{s\Psi_g} \\ \vec{\Psi}_{r\Psi_g} \end{pmatrix} = \begin{pmatrix} \vec{u}_{s\Psi_g} - r_s \vec{i}_{s\Psi_g} - j\omega_g \vec{\Psi}_{s\Psi_g} \\ -r_s \vec{i}_{r\Psi_g} - j(\omega_g - \omega_r)\vec{\Psi}_{r\Psi_g} \end{pmatrix} \qquad (10)$$

In these equations, $\vec{i}_{s\Psi_g}$, $\vec{i}_{r\Psi_g}$, $\vec{\Psi}_{s\Psi_g}$ and $\vec{\Psi}_{r\Psi_g}$ are current and flux linkage vectors of stator and rotor, respectively which can be expressed according to direct and quadrature components of stator current (i_{sx} , i_{sy}) and direct and quadrature components of rotor current (i_{rx} , i_{ry}).

$$\begin{aligned} \vec{\Psi}_{s\Psi_g} &= L_s i_{s\Psi_g} + L_m i_{r\Psi_g} \\ \vec{\Psi}_{r\Psi_g} &= L_r i_{r\Psi_g} + L_m i_{s\Psi_g} \\ \vec{i}_{s\Psi} &= i_{sx} + j i_{sy} \\ \vec{i}_{r\Psi} &= i_{rx} + j i_{ry} \end{aligned} \qquad (11)$$

Also $\vec{u}_{s\Psi_g}$ is voltage vector of stator, ω_g and ω_r are angular speed of the frame and rotor respectively, which can be achieved by mechanical equation.

$$J\frac{d\omega_r}{dt} = 1.5PL_m\left(\vec{i}_{r\Psi_g} \otimes \vec{i}_{s\Psi_g}\right) - B\omega_r - T_L \qquad (12)$$

In this equation $\otimes$ shows the vector product. Using this model, rotor field oriented vector control of induction machines can be achieved. Suppose the frame is oriented tor the rotor flux linkage so:

$$\Psi_{rx} = \vec{\Psi}_{r\Psi_g} \qquad (13)$$

$$\Psi_{ry} = 0 \Rightarrow i_{ry} = -\frac{L_m}{L_r} i_{sy} \qquad (14)$$

The electrical equation of the rotor can be decoupled to the real and imaginary parts.

$$-r_r i_{rx} - (\omega_g - \omega_r)\Psi_{ry} + \frac{d}{dt}\Psi_{rx} = 0 \qquad (15)$$

$$-r_r i_{ry} + (\omega_g - \omega_r)\Psi_{rx} + \frac{d}{dt}\Psi_{ry} = 0 \qquad (16)$$

Using (13), (14), (15), and (16) it can be written as:

$$-r_r i_{rx} + \frac{d}{dt}\Psi_{rx} = 0 \qquad (17)$$

$$\left(\omega_g - \omega_r\right) = \frac{\Psi_{rx}}{r_r i_{ry}} \qquad (18)$$

The first equation shows i_{rx} is affected by direct component of rotor flux linkage, therefore when its variation is slow or even it is constant in the steady state, i_{rx} becomes zero. As a result the motor torque can be written as:

$$T_e = 1.5 P L_m \left(\vec{i}_s \otimes \vec{i}_r\right) \qquad (19)$$

$$\Psi_{rx} = \vec{\Psi}_{r\Psi_r} = L_m i_{sx} \qquad (20)$$

Equation (20) shows that rotor flux can be controlled by direct component of stator current and therefore from (19) the torque can be controlled by quadrature component of stator current. As a result the goal of the vector control is satisfied.

To implement this control two problems remain. The first one is that the rotor flux linkage vector must be tracked. To do this, (18) can be used to produce:

$$\rho_r = \int \omega_g \, dt = \int \left(\omega_r + \frac{i_{sx}}{\tau_r i_{sy}}\right) dt \qquad (21)$$

The second one is that the direct and quadrature current components must be produced. If the voltage source inverter is used to produce these components, a control strategy will be needed to produce them. To do this, each component of stator voltages such as u_{sx} is divided into two components ($u_{sx} = u_x + u_{dx}$). One of these components (u_x) is used to control the same current component (i_{sx}) and the other is supplied to control the other component (i_{sy}):

$$u_{sx} = u_x + u_{dx} = r_s i_{sx} + \frac{d}{dt}\Psi_{sx} - \omega_g \Psi_{sy} \qquad (22)$$

$$u_{sy} = u_y + u_{dy} = r_s i_{sy} + \frac{d}{dt}\Psi_{sy} + \omega_g \Psi_{sx} \qquad (23)$$

$$u_{dx} = -\omega_g \Psi_{sy} = -\omega_g L'_s i_{sy} \qquad (24)$$

$$u_{dy} = \omega_g \Psi_{sx} = \omega_g L_s i_{sx} \qquad (25)$$

In these equations $L'_s = \left(L_s - \frac{L_m^2}{L_r}\right)$ is the stator transient inductance. By using reference values of current instead of their actual values, the voltage references for inverter are achieved. Flux linkage is controlled by reference direct current component, but reference quadrature current component is adjusted to control the speed because this component controls torque. Therefore reference current components are defined as:

$$i_{sxref} = \frac{\Psi_{ref}}{L_m} \qquad (26)$$

$$i_{syref} = k_{p\omega}\left(\omega_{ref} - \omega_r\right) + k_{i\omega}\int\left(\omega_{ref} - \omega_r\right)dt \qquad (27)$$

A rotor flux oriented vector control model can be written according to the above discussion.

$$\frac{d}{dt}\begin{pmatrix} i_{sx} \\ i_{sy} \\ i_{rx} \\ i_{ry} \\ \omega_r \\ \omega_i \end{pmatrix} = D^{-1} \times$$

$$\times \begin{pmatrix} r_s i_{sxref} - M L_s i_{syref} - r_s i_{sx} + M\left(L_s i_{sy} + L_m i_{ry}\right) \\ -r_s i_{syref} - M L_s i_{sxref} - r_s i_{sy} - M\left(L_s i_{sx} + L_m i_{rx}\right) \\ -r_r i_{rx} + \dfrac{i_{syref}}{\tau_r i_{sxref}}\left(L_m i_{sy} + L_r i_{ry}\right) \\ -r_r i_{ry} - \dfrac{i_{syref}}{\tau_r i_{sxref}}\left(L_m i_{sx} + L_r i_{rx}\right) \\ \dfrac{1}{J}\left(1.5 P L_m\left(i_{sy} i_{rx} - i_{sx} i_{ry}\right) - B\omega_r - T_L\right) \\ \omega_{ref} - \omega_r \end{pmatrix} \qquad (28)$$

In these equations $M = \left(\omega_r + \frac{i_{syref}}{\tau_r i_{sxref}}\right)$ and D^{-1} is :

$$D^{-1} = \frac{1}{L_L}\begin{pmatrix} L_r & 0 & -L_m & 0 & 0 & 0 \\ 0 & L_r & 0 & -L_m & 0 & 0 \\ -L_m & 0 & L_s & 0 & 0 & 0 \\ 0 & -L_m & 0 & L_s & 0 & 0 \\ 0 & 0 & 0 & 0 & L_L & 0 \\ 0 & 0 & 0 & 0 & 0 & L_L \end{pmatrix} \qquad (29)$$

$$L_L = L_s L_r - L_m^2$$

Figure (1) depicts a rotor field oriented vector control drive.

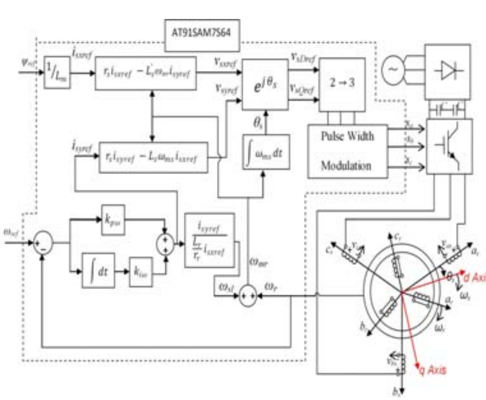

Fig. 1. Rotor field oriented vector control drive.

IV. STABILITY ANALYSIS OF THE SYSTEM

To study the stability analysis of a rotor field oriented vector control at first the equilibrium point of (28) must be obtained. The zero state of the dynamical equation of the system must be transformed to the origin. When the speed controller is only proportional controller, the equilibrium point of this system is [14]:

$$i_{sx}^* = i_{sxref}\ ,\ i_{rx}^* = 0\ ,\ i_{sy}^* = -\frac{L_r}{L_m} i_{ry}^*\ ,\ i_{ry}^* = -\frac{T_L + B\omega_r^*}{1.5PL_m i_{sxref}}$$
$$\omega_r^* = \frac{\left(\dfrac{1.5PL_m^2 i_{sxref} k_{p\omega}}{L_r}\right)\omega_{ref} - T_L}{B + \dfrac{1.5PL_m^2 i_{sxref} k_{p\omega}}{L_r}} \quad (30)$$

But when the speed controller is proportional-integral, the equilibrium is [14]:

$$i_{sx}^* = i_{sxref}\ ,\ i_{rx}^* = 0\ ,\ i_{sy}^* = -\frac{L_r}{L_m} i_{ry}^*\ ,\ i_{ry}^* = -\frac{T_L + B\omega_r^*}{1.5PL_m i_{sxref}}$$
$$\omega_r^* = \omega_{ref}\ ,\ \omega_i^* = \frac{i_{sy}^*}{k_{i\omega}} \quad (31)$$

For simplicity the proportional speed controller is considered. The new variables for the new system are chosen as:

$$x_1 = i_{sx} - i_{sx}^*\ ,\ x_2 = i_{sy} - i_{sy}^*\ ,\ x_3 = i_{rx} - i_{rx}^*$$
$$x_4 = i_{ry} - i_{ry}^*\ ,\ x_5 = \omega_r - \omega_r^* \quad (32)$$

Therefore dynamical equations of the new system are:

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = L^{-1}\begin{pmatrix} SL_s k_{p\omega} x_5 - r_s x_1 + S(L_s x_2 + L_m x_4) \\ -r_s k_{p\omega} x_5 - r_s x_2 - S(L_s x_1 + L_m x_3) \\ -r_r x_3 + T(L_r x_4 + L_m x_2) + \dfrac{k_{p\omega} L_m}{\tau_r} x_5 \\ -r_r x_4 - T(L_r x_3 + L_m x_1) \end{pmatrix}$$
$$\frac{dx_5}{dt} = \frac{1}{J}(T_{Te} - Bx_5) \quad (33)$$
$$T_{Te} = 1.5PL_m\left(x_3\left(x_2 + i_{sy}^*\right) - x_1 x_4 - x_1 i_{ry}^* - x_4 i_{sxref}\right)$$
$$S = \frac{k_{p\omega}\left(\omega_{ref} - \left(x_5 + \omega_r^*\right)\right)}{\tau_r i_{sxref}} + \left(x_5 + \omega_r^*\right)$$
$$T = \frac{k_{p\omega}\left(\omega_{ref} - \left(x_5 + \omega_r^*\right)\right)}{\tau_r i_{sxref}}$$

L is machine inductances given bellow:

$$L = \begin{pmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{pmatrix} \quad (34)$$

This system's equation is similar to equation (7). In this equation H and N_i are given below:

$$H = \begin{pmatrix} L & 0 \\ 0 & \dfrac{1}{J} \end{pmatrix}$$

$$N_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{ML_s}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{ML_m}{2} \\ 0 & \frac{ML_s}{2} & 0 & \frac{ML_m}{2} & 0 \end{pmatrix}$$

$$N_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{ML_s}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{ML_m}{2} \\ 0 & -\frac{ML_s}{2} & 0 & -\frac{ML_m}{2} & 0 \end{pmatrix} \quad (35)$$

$$N_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{ML_s}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{ML_m}{2} \\ 0 & -\frac{ML_s}{2} & 0 & -\frac{ML_m}{2} & 0 \end{pmatrix}$$

$$N_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{ML_s}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{ML_m}{2} \\ 0 & \frac{ML_s}{2} & 0 & \frac{ML_m}{2} & 0 \end{pmatrix}$$

$$N_5 = \begin{pmatrix} 0 & 0 & 0 & -\frac{3PL_m}{2J} & 0 \\ 0 & 0 & -\frac{3PL_m}{2J} & 0 & 0 \\ 0 & -\frac{3PL_m}{2J} & 0 & 0 & 0 \\ -\frac{3PL_m}{2J} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M = \left(1 - \frac{k_{p\omega}}{\tau_r i_{sxref}}\right)$$

$$N = \frac{k_{p\omega}}{\tau_r i_{sxref}}$$

According to (7) matrix P can be:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{2J}{3P} \end{pmatrix} \qquad (36)$$

This matrix satisfies the conditions mentioned in section2.The Lyapunov function of the system is obtained according to (7):

$$W^2(X) = L_s x_1^2 + L_s x_2^2 + L_r x_3^2 + L_r x_4^2 + + 2L_m x_1 x_3 + 2L_m x_1 x_3 + \frac{2J}{3P} x_5^2 \qquad (37)$$

Boundary of different parameters such as speed controllers values, the reference speed, or flux values to have stable response can be achieved using this Lyapunov function. To do this time derivation of this function must be negative.

V. Experimental approach

An experimental prototype was prepared by the authors to verify the analysis. A laboratory 175 W and four pole three phase induction motor whose parameters and characteristics are shown in Table I is used. AT91SAM7S64 microcontroller is used to implement the rotor speed calculation and filtering, speed controller, and processing operation including the calculation of reference voltage values for inverter, pulse-width-modulation (PWM), DC bus voltage control, reference values setting such as reference speed and reference flux linkage, etc.

TABLE I. INDUCTION MOTOR PARAMETERS

Parameter	Description	Value
r_s	stator resistor	43.1 Ω
r_r	rotor resistor	72 Ω
L_s	stator inductance	1.995 H
L_r	rotor inductance	1.995 H
L_m	Mag. inductance	1.96 H
J	moment of inertia	0.0024 Kgm
B	friction coefficient	0.001 Kgm/s
T_L	no-load torque	0.1 Nm

This microcontroller can work with 55 MHz clock signal but it was used with 48 MHz clock and has ARM core therefore it can be used for this drive which needs high speed processing operation. To measure the speed of the rotor a 2000-pulse/revolution speed encoder is connected to the shaft of rotor. Its pulses after a voltage level adjusting are used for counter of the microcontroller. Microcontroller gives the actual speed by calculating the speed and filtering it by an FIR filtering and a low-pass filter to neutralize the quantization error and noise. A logic operation board is designed to operate on PWM pulses produced by microcontroller. These pulses after high-speed optocouplers are delivered to the power section. An Inverter with IGBT switches, RC snubber circuit, and high-speed anti-parallel power diodes is designed to supply the motor.

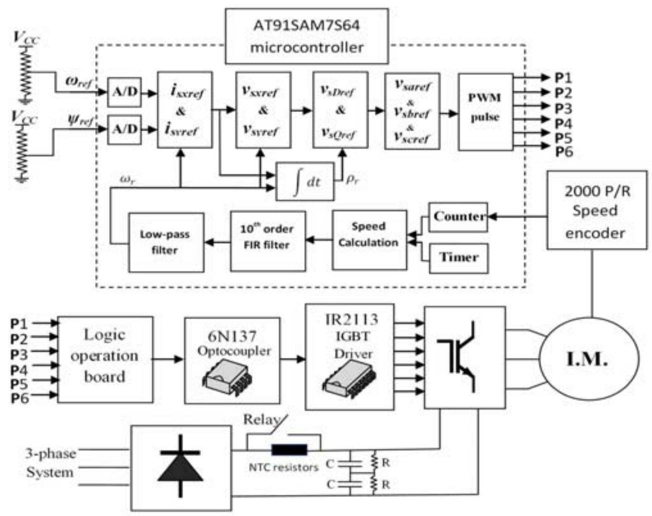

Fig. 2. Experimental set-up diagram.

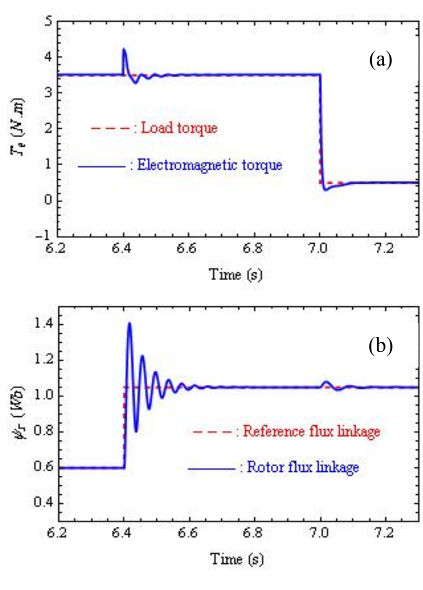

Fig. 3. Vector control response: (a) Flux linkage response, (b) Torque response

PWM pulses are delivered to IGBT driver circuits to turn the IGBT switches on or off. Also a 530-volt DC bus voltage is designed using three phase voltages, rectifiers, NTC resistors, relay, capacitors, discharge circuit, etc. The output current is monitored using a shunt resistor and a digital oscilloscope. Figure (2) shows the block diagram of experimental setup. Figure (3) shows dynamical response of the system depicted in Figure (1). As illustrated in Figure (3a) when the value of flux reference is changed, electromagnetic torque of the motor does not affected. Also, as seen in Figure (3b) when the Load torque is changed, the value of flux fluctuates negligibly.

TABLE II. EQUILIBRIUM POINTS FOR THREE SET POINTS

CASE	ω_{ref}	ψ_{ref}	$k_{p\omega}$	i_{sy}^*	i_{rx}^*	i_{ry}^*	ω_r^*	ω_i^*
1	230	0.95	2	0.48	0.11	-0.11	230	1.18
2	230	0.1	5	0.05	1.12	-1.12	230	1.12
3	130	0.95	2	0.48	0.08	-0.08	130	0.82

Table II shows three set point and speed controller values for rotor field oriented vector control drive and also their equilibrium states calculated by (30). Figures (4a) and (4b) illustrates dynamical responses of the system by changing the set points from case 1 to case 2 at t=6.98 sec. As shown in these figures, the system response is stable in case 1 and is unstable in case 2. These results are shown in [14] that the case 2 has a chaotic response. Figure (5) portrays the Lyapunov time derivative for two cases. As shown in case 1, time derivation of the Lyapunov function becomes negative and remains until zero which shows that this set point is stable. When the set point changes to case 2, time derivative of Lyapunov function becomes positive and negative alternatively which shows that the system response converges to the equilibrium point and then diverges from it.

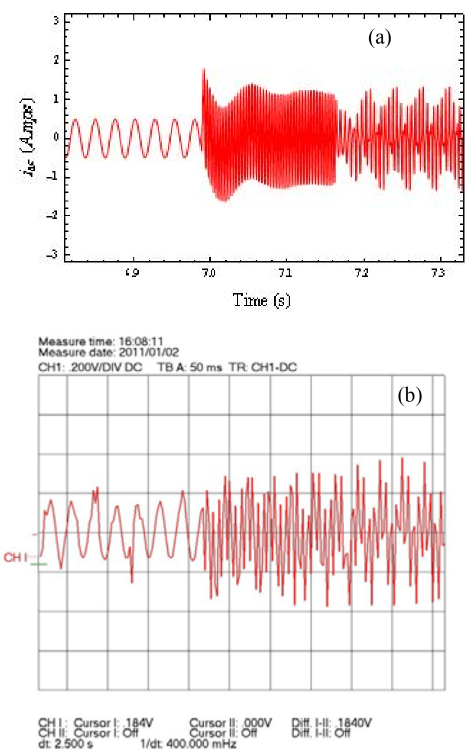

Fig. 4. Current responses of cases 1 and 2: (a) simulation, (b) experimental

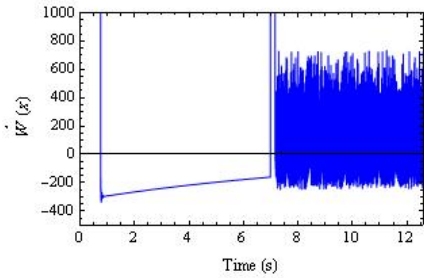

Fig. 5. Derivation value of Lyapunov function for cases 1 and 2.

It is clear that there is a stable region in the phase plane. It is obtained numerically for both cases. $\dot{W}(X)$ for $\omega_{ref} = 230\, rad/s$, $\Psi_{ref} = 0.95\, Wb$, $k_{p\omega} = 2$ and $k_{i\omega} = 0.1$ is:

$$\begin{aligned}\dot{W}(X) = &-20.93x_1^2 - 20.93x_2^2 - 20.93x_3^2 - 20.93x_4^2 \\ &- 0.06x_5 + 0.04x_3x_5 + 40.63x_4x_5 - 0.000067x_5^2 \\ &- 63.41x_2 + 41.59x_2x_5 - 124.804x_2x_3 + 0.48x_2x_3x_5 \\ &- 2.09x_2x_6 - 2.06x_4x_6 - x_5x_6 - 9.95x_1 + 124.8x_1x_4 \\ &- 0.09x_1x_5 - 0.048x_1x_4x_5 + 0.007x_1x_6\end{aligned} \tag{38}$$

Figures (6) shows the area in which the $\dot{W}(X)$ is negative for case 1 and 2, respectively. For these cases the other three states vary between -0.1 and 0.1. According to the Lyapunov theorem the case 1 is stable and the case 2 is not stable because in case 1 there is an area around the origin in which $\dot{W}(X)$ is negative. But in case 2 there is not such an area around the origin, so it is not stable. The phase plane trajectories for these cases are also shown in these figures. Note that these trajectories do not intersect with each other in the space.

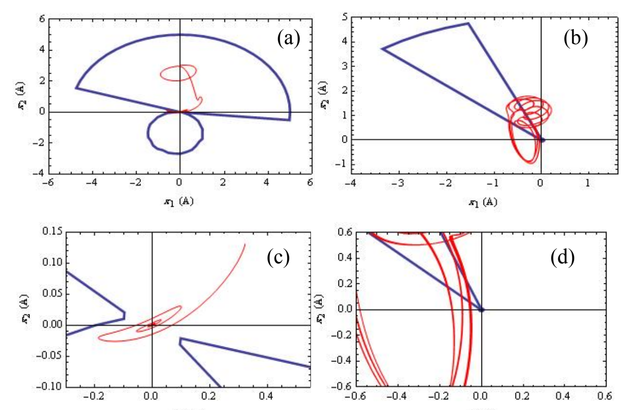

Fig. 6. Area of negative Lyapunov time derivative. (a) stable area for case 1. (b) stable area for case 2. (c) stable area for case 1 (magnified). (d) stable area for case 2 (magnified).

As discussed above, the trajectories of the system cannot settle down to the origin in case 2 because the derivation of Lyapunov function is not negative even in a small area around the origin. Generally speaking, this fact can be used to determine whether or not a case has a stable condition. As an example, consider the following case (case 3). Figure (7) shows the area in which time derivative of Lyapunov function is negative. As it is obvious there is an area around the origin in which the time derivative of Lyapunov function is negative. Therefore this case is a stable case. Figures (8a) and (8b) illustrate the simulated and experimental response of the drive for case 3, respectively. The state space trajectory is given in Figure (7).

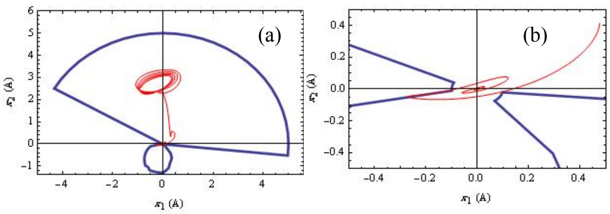

Fig. 7. Stable areas for case 3: (a) stable area for case 3, (b) stable area for case 3 (magnified).

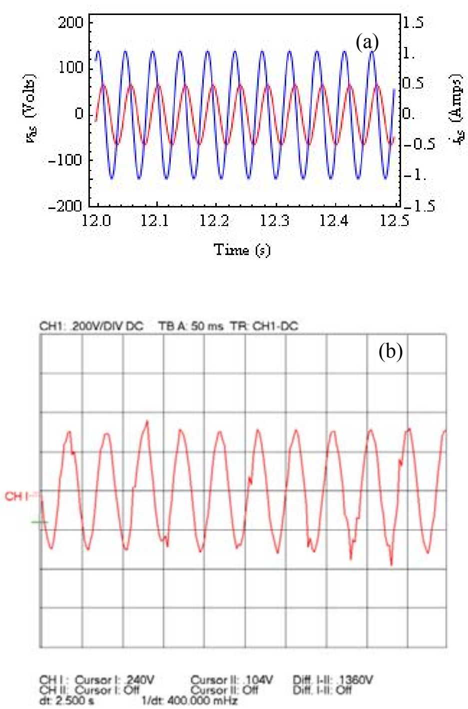

Fig. 8. Current responses for case 3: (a) simulation (b) experimental

This method can be used to find the range of speed controller's gain values, reference flux, reference speed, etc. for stable responses. The Figure (9a) and (9c) illustrate the area in which time derivative of the Lyapunov function is negative for different speed proportional controller gains. As shown, there is a range of this parameter which makes the system to be stable. Also Figure (9b) and (9d) show the areas for different reference flux linkages. Also in these cases there is a range of flux linkages which makes the system stable.

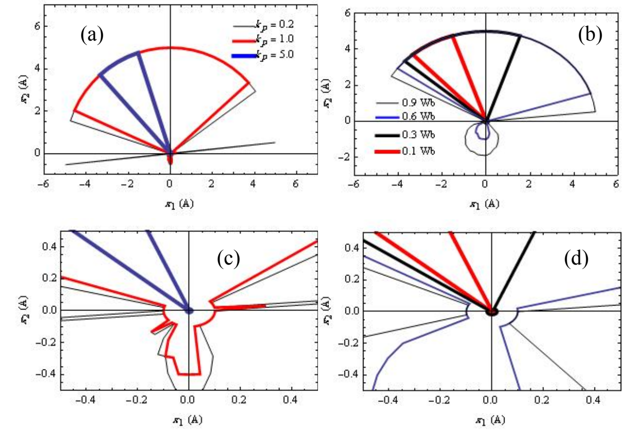

Fig. 9. Area of negative Lyapunov derivation for different parameters. (a) different $k_{p\omega}$ (b) different ψ_{ref} (c) different $k_{p\omega}$ (magnified) (d) different ψ_{ref} (magnified).

VI. CONCLUSION

In this paper a useful Lyapunov function is achieved for vector control drives of induction machines. Stable range of set points such as reference flux linkage, speed or other parameters of the system can be investigated using this function. By obtaining this stable condition for each drive, they can be set offline to avoid unstable response or chaotic behavior.

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