# Pile Design using the Modified Unified Method combined with Monte Carlo Simulation

**Hoa Cao Van**
Department of Construction Technology, Faculty of Civil Engineering, Ho Chi Minh City University of Architecture, Vietnam
hoa.caovan@uah.edu.vn (corresponding author)

*Received: 13 March 2024 | Revised: 1 April 2024 | Accepted: 9 April 2024*

 *DOI: https://doi.org/10.48084/etasr.7247*

## ABSTRACT

**Piles are typically designed to ensure the bearing capacity and settlement elastic behavior. However, some projects seem over-designed, leading to unnecessary waste, whereas others experience excessive settlement. This could be caused by various factors, such as site investigation, sampling and testing methods, selection of soil behavior model, and calculation programs. To achieve a successful pile design, engineers must consider, among others, the loads applied to the pile, the resistance capacity of the piles, the pile material's bearing capacity, the pile's displacement, and the soil's settlement. On the other hand, the input parameters for geotechnical problems, in general, and pile design problems, in particular, often do not reflect the actual behavior of the soil due to its heterogeneous and anisotropic nature. To address these challenges, an Artificial Neural Network (ANN) approach is proposed for pile design, using a relatively wide range of soil input data. This study establishes a numerical program for pile design combined with the ANN approach, validated by verifying the pile design of a project constructed in Vietnam. The results indicate that the proposed program can reasonably simulate pile group behavior and assist engineers in deploying appropriate safety factors.**

***Keywords-pile design; Monte Carlo simulation; artificial neural networks; hybrid model; modified unified method***

## I. INTRODUCTION

Piles have been a widely used foundation solution for various civil structures [1]. There are different design approaches to predict the ultimate capacities and settlement of piles, ranging from simple empirical formulations to more advanced finite element analyses. However, these approaches, whether simple or rigorous, depend on various factors, such as soil stratification, soil-pile interaction, and the distribution of shaft resistance along the pile depth. These factors result in a significant level of uncertainty and obstacles to the implementation of simple regression analyses. Therefore, more extensive and sophisticated approaches are necessary to ensure appropriate structural and serviceability performance.

A new research approach would be to apply metaheuristic optimization algorithms. These algorithms are widely used to solve complex problems in various fields and offer several advantages. Firstly, they rely on simple concepts and are easy to implement. Secondly, they do not require information on the gradient of the objective function. Thirdly, they can bypass local minima. Lastly, they can be deployed to solve various problems in different fields [2, 3]. Since the application of metaheuristics depends on computers, advances in the processing power of computers have accelerated the development of metaheuristics. Exploration and exploitation are the two main phases of a metaheuristic algorithm. The main differences among metaheuristics lie in how they balance those two processes [4-6]. Single-solution-based or population-based metaheuristics is a fundamental distinction of metaheuristic algorithms. Basic single-solution-based metaheuristics are more exploitation-oriented than exploration-oriented, while population-based metaheuristics are more exploration-oriented [7].

Artificial Neural Networks (ANNs) [8] are a type of computing architecture that solves complex problems by working with interconnected yet simple computing components. These components, also known as processing elements, are analogous to the neurons in a brain, consisting of many basic computational units arranged in layers. The interest in ANNs has increased tremendously over the last few years. This is mainly due to their nonlinear and parallel processing capabilities. In most ANN applications, the back-propagation algorithm is utilized. This algorithm employs the gradient-descent method to minimize the error function [8].

In the field of material modeling, some researchers [9-11] have used a back-propagation ANN to model the behavior of concrete in the state of plane stress under monotonic biaxial loading and compressive uniaxial cycle loading. Their findings appear to be very promising. Authors in [12] demonstrated the effectiveness of ANNs in characterizing composite materials. They employed a back-propagation ANN to predict composite thermal properties accurately. The network was trained utilizing basic information about the constituent materials, component ratios, and environmental conditions used to create the composite.

In the field of geotechnics [13, 14], ANNs have been deployed to predict the ultimate capacity of driven piles based on in situ tests. However, most of these models were limited to large displacement-driven piles. Very little work has been done on forecasting the capacity of low-displacement piles. On the other hand, some researchers [15-17] have adopted the ANN approach in conjunction with different techniques, such as evolutionary computation and probabilistic techniques, to develop more sophisticated and integrated systems.

The Modified Unified Pile Design Method [18-21] is a geotechnical problem that relies on a set of input parameters. These parameters are determined from numerous experimental samples and often exhibit a significant degree of variation. The objective of this research is to find a reliable solution and a suitable ANN simulation tool that can be put into service to achieve the desired outcome.

Numerical methods based on the Monte Carlo simulation can be loosely defined in general terms similarly to any methods that rely on random sampling to estimate the solutions. Monte Carlo methods are often applied to problems that are either too complicated to be described by a mathematical model or whose parameter space is too large to be explored systematically. Due to the emergence of big data problems, Monte Carlo methods have become powerful tools for analyzing the problem.

In this paper, the Monte Carlo simulation is engaged to optimize the pile length based on the Modified Unified Method concept [18-21]. The problem modeling and a real-life example are presented to establish a basis for future, more detailed research.

## II. MATERIALS AND METHODS

### *A. The Mathematical Formulas*

In pile design, it is essential to simultaneously determine the pile's resistance capacity and displacement. Authors in [18] proposed the unified method for pile design, emphasizing the importance of understanding how loads are transferred from the pile to the soil and vice versa. There are three aspects involved in designing piles: (i) The sum of dead and live loads should be less than the resistance capacity of the pile divided by the safety factor. Negative friction loads should not be included. (ii) The sum of dead load and negative friction should be less than the material strength divided by the safety factor. Live loads do not coexist with negative friction. (iii) The pile settlement must not exceed the allowable value. This analysis excludes live loads and negative friction. Loads from a super-structure usually do not cause significant settlements. If the neutral plane is located in or above the compressive soil layer, the pile group will experience settling even if the overall safety factor appears satisfactory. The load at the pile head can be calculated based on the raft load transmission mechanism. However, the load can significantly differ between piles depending on the pile tip resistance or length.

The behavior of piles, according to [18], can be seen in Figure 1. Pile design involves four main aspects. Firstly, the load from the superstructure, which includes negative friction that creates a down-drag force (curve No.1), distributes along the pile depth. The down-drag force is determined using formulas similar to those utilized for calculating positive friction. Right after construction, there is no negative friction, but over time, the latter develops and increases the down-drag force while the load at the pile head remains the same. Secondly, the resistance capacity (curve No.2) includes shaft resistance (positive friction) and toe resistance, calculated according to Vietnamese National Standards (VNS) and distributed along the pile depth. The allowable pile head displacement (curve No.3) includes pile toe displacement and body deformation. Employing Mindlin's first solution, soil settlement (curve No.4) is calculated based on pile-soil interaction.

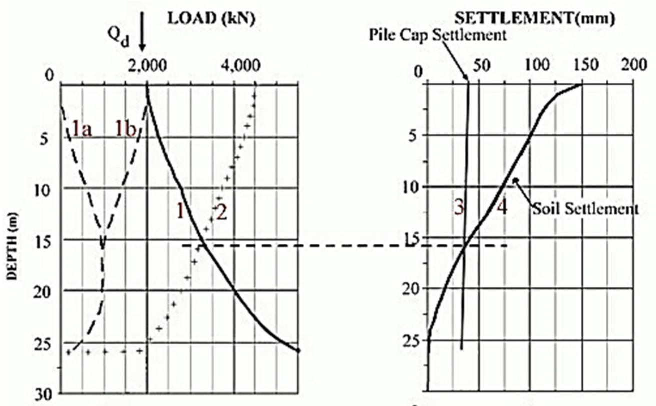


Fig. 1 The unified method.

In this study, the load and resistance will be calculated in accordance with the VNS using (1) and (2). The displacement of the pile and the settlement of the soil will be calculated with (3) and (4). The load at the pile head is typically determined deploying the SAFE program. Still, in this case, the program will utilize the calculated loads from the design document for later comparison with the designed resistance and settlement. In order to simplify the initial steps of the research, the SPT index will be employed.

The load acting along the pile length, including down-drag load, is defined by VNS as:

$$f_{1i}\left(z\right) = P_i + A_{sj} \times f_{rj}^{-} \tag{1}$$

The load-bearing capacity of a pile along its length is defined by VNS as.

$$f_{2i}\left(z\right) = Q_{utt,i} - A_{sj} \times f_{rj}^{+} \tag{2}$$

where:

$$Q_{ult,i} = A_{ti} \times f_{ti} + A_{sL} \times f_{rj}^{+} \tag{3}$$

$$\left|f_{rj}^{+}\right| = \left|f_{rj}^{-}\right| = \beta \times a \times N_{jc} + \gamma \times b \times N_{js} \tag{4}$$

$$f_{ti} = \alpha \times c \times N_t \tag{5}$$

where $j$ is the node number, $i$ is the pile number, $P_i$ is the load at the $i^{th}$ pile, $A_{sj}$ is the shaft area of the $j^{th}$ element, $A_{sL}$ is the shaft area of the whole pile, $A_{ti}$ is the area of the pile toe, $\alpha$, $\beta$, $\gamma$ are coefficients taken from national codes, and $a$, $b$, $c$ are variables of the ANN algorithm. The cumulative pile settlement along its length can be estimated by:

$$f_{3i}(z) = S_a - 2 \times P_{i,j} \times \Delta / 3E_{pi}A_{pi} \tag{6}$$

Soil settlement along pile length is estimated [2-4] by:

$$f_{4i}(z) = \sum_{o}^{z_i} w_i \tag{7}$$

where the settlement at any point in the subsoil is a sum of settlement caused by load acting at all pile's nodes.

$$w_i = \sum_{1}^{j} a_{ij} P_{ij} \tag{8}$$

$$a_{ij} = \frac{1}{16\pi\overline{G}(1-v)} \begin{bmatrix} \dfrac{3-4v}{R_{1,ij}} + \\ \dfrac{8(1-v)^2-(3-4v)}{R_{2,ij}} + \\ \dfrac{(z_i-c_j)^2}{R_{1,ij}^3} + \\ \dfrac{(3-4v)(z_i+c_j)^2-2c_jz_k}{R_{2,ij}^3} + \\ \dfrac{6c_jz_i(z_i+c_j)^2}{R_{2,ij}^5} \end{bmatrix} \tag{9}$$

where $S_a$ is the allowable settlement, depending on national code, $P_{ij}$ is the external load acting at the pile node, $E_{pi}$ is the Young's modulus of pile material, $\Delta$ is the length of pile element, and $A_{pi}$ is $i^{th}$ pile's area, $z_i$, $z_k$, $c_j$, $R_{1,ij}$, $R_{2,ij}$ are parameters defined in the Mindlin formula, $d$ and $e$ are variables of the ANN algorithm, and $\overline{G}$ is the average shear modulus:

$$\overline{G} = d \times \overline{N}^e \tag{10}$$

*B. Hybrid Models combining the Elastic Theory and Finite Element Method Modeling*

Many researchers have adopted a hybrid model that combines the elastic method with the finite element method to simulate the behavior of the raft-pile-soil system. This model simulates the pile cap's behavior as a 2-D plate element, the pile's behavior as a 1-D bar element, and the ground's behavior as elastic springs. This model was extensively studied by in [17, 21]. Figure 2 shows the pile group's hybrid model, where piles are divided into 1-D elements and pile-soil interaction is modeled as springs using Mindlin's first solution.

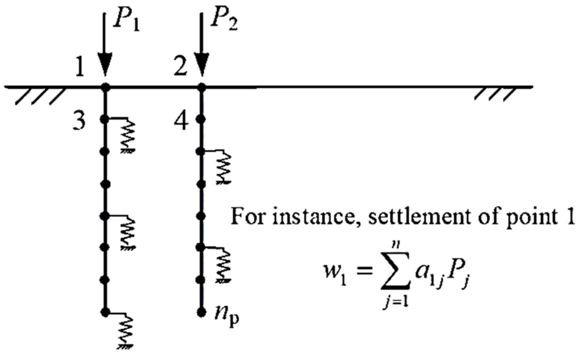


Fig. 2 Hybrid model of piles in pile group.

*C. The Monte Carlo Method*

The Monte Carlo method is a problem-solving approach that utilizes random numbers and probability. In this study, the modified unified pile design method was integrated using the Monte Carlo algorithm. The problem is formulated as follows:

$$z_{1,2} = \left(f_{1i}(z) = f_{2i}(z)\right) \tag{11}$$

$$z_{3,4} = \left(f_{4i}(z) = \sum_{o}^{z_i} w_i\right) \tag{12}$$

Subject to:

$$z_{1,2} = z_{3,4} \tag{13}$$

$$45(m) = L_{\min} \le L \le L_{\max} = 57(m) \tag{14}$$

$$1 = a_{\min} \le a \le a_{\max} = 2 \tag{15}$$

$$1 = b_{\min} \le b \le b_{\max} = 2 \tag{16}$$

$$1 = c_{\min} \le c \le c_{\max} = 2 \tag{17}$$

$$0.8 = d_{\min} \le d \le d_{\max} = 1.2 \tag{18}$$

$$0.8 = e_{\min} \le e \le e_{\max} = 1.2 \tag{19}$$

$$0 = s_{a\min} \le s_a \le s_{a\max} = 0.01 \sim 0.1 \text{ (m)} \tag{20}$$

The depth of the neutral plane is a function of the following independent variables:

- $L$: Pile length (m) is calculated from the designed pile length to the expected reduced length.
- $a$, $b$, $c$: represent the change in the SPT index of 10 soil layers. N-SPT varies between the field-measured value and the N60-adjusted value.
- $d$, $e$: are related to the elastic shear modulus of the soil. These values are taken from [23].
- $S_a$ is the allowable elastic pile displacement (m).

The program was coded in MATLAB. The code can be seen below.

```
Monte_carlo.m
function monte_carlo1
var_min = [L a b c d e sa];
var_max = [L a b c d e sa];
for nr=1:n
vari=var_min+rand(1,14).*( var_max-
var_min);
[z12 z34]=obj_caohoa(vari);
z12=z34;
save()
end
end
Obj_caohoa.m
function [z12 z34]=obj_caohoa(x)
lc=x(1);
DCx=x(2:11);
a=x(12);
b=x(13);
salw=x(14);
[z12 z34]=obj_func();
end
function
[z12z34]=obj_func()
t=147;
Econ=30e+6;
nuy=0.45;
y1=load();
y2=resistant();
y3=piledisp();
y4=soilsett();
A=[];
for j=1:11
Ai=doctd(t,j,lc);
A=[A;Ai];
  end
l=A(:,3);
[yo12, xo12]=lookvalue(l,y1,y2);
[yo34, xo34]=lookvalue(l,y3,y4);
z12=abs(yo12);
z34=abs(yo34);
end
```

## III. CASE STUDY

### *A. The Connect 2 Project*

The design of the piles for the Connect 2 apartment project is explained in detail in [22]. A Monte Carlo optimization algorithm was applied in this paper to optimize the length of specific piles in the pile group. There are two types of piles in this project: (1) 122 piles with a diameter of 1 m, which were initially designed with a length of 52 m, arranged under columns or shear walls, and (2) 50 piles with a diameter of 1.2 m and a length of 57 m, arranged under the core area. All the piles have been designed according to VNS. Table I illustrates the geotechnical profile and the N-SPT values for each subsoil layer beneath the rafts.

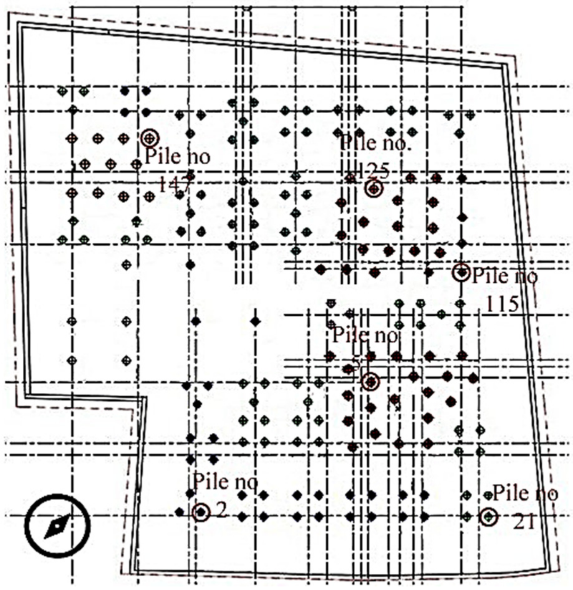


Fig. 3 Pile layout of Connect 2.

TABLE I. SOIL PROPERTIES

| Layer | Soil layer | Depth (m) | N-SPT |
|---|---|---|---|
| 1 | Sandy clay, medium stiff | 10-16 | 13.5 |
| 2 | Fine sand, medium-dense | 17-32 | 16.1 |
| 3 | Clay, stiff | 33-36 | 20.25 |
| 4 | Fine sand, medium-dense | 37-39 | 17.5 |
| 5 | Clay, red-brown, stiff | 40-50 | 32.7 |
| 6 | Sandy clay, medium stiff | 51-53 | 32 |
| 7 | Coarse sand, very dense | 54-56 | 41.3 |
| 8 | Clay, stiff | 57-58 | 64 |
| 9 | Medium sand, dense | 59-67 | 40 |

### *B. Analysis Results*

The study analyzed 6 out of 177 piles. Two of these piles, pile No. 2 and pile No. 21, were located at the raft corner and measured 52 m in length and 1.0 m in diameter. The other 4 piles, namely piles 51, 125, 147, and 115, were situated in the core area and measured 57 m in length and 1.2 m in diameter. At the beginning of the optimization process, there were 14 design variables consisting of 1 variable $L$, 11 variables of $N$, 2 variables of $G$, and 1 variable of $S_a$. A simple random search could have got a near-optimal solution with this number of variables. However, since each calculation cycle took 4 minutes, the program generated 6,000 random combinations of the 14 design variables between the minimum and maximum bounds based on formulas (14) to (20) to ensure an accurate solution. The $z12$ , $z34$ were then predicted using the augment-neuron network based on these input models. The result is demonstrated in Figure 4. It is evident that utilizing only 6,000 combinations is insufficient to achieve near-optimal results. To achieve even near-optimal outcomes, a significant number of random combinations is necessary, at 50,000 or more. As a result, it would take a computer approximately 3,500 hours to process one pile, which is not practical. Figure 4 illustrates that very few solutions satisfy (13). However, the computational process takes a significant amount of time, so adjustments to the program are necessary.

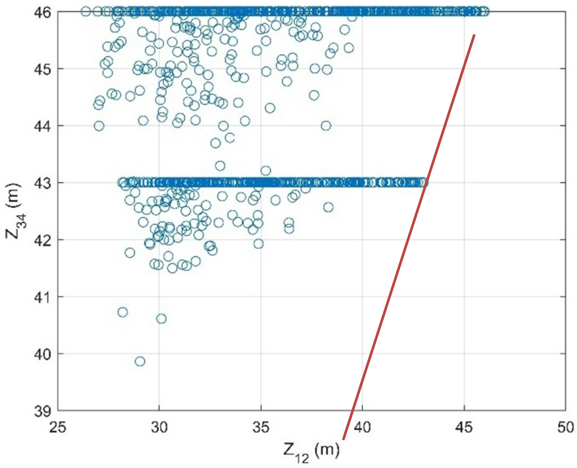


Fig. 4 FEP elevation ( $z12$ ) and SEP elevation ( $z34$ ) Scatter diagram based on 6,000 random samples.

To reduce computer running time and overcome the limitations mentioned above, (13) was inserted into the program codes. The analysis results are displayed in Figure 5, which reveals that they are convergent and comply with the set requirements. This approach can reduce the number of variable combinations to 100 or 200, with a running time of approximately 5 or 10 hours, respectively, depending on the computer configuration. It should be noted that analyzing a single pile takes between 5 to 10 hours, meaning that analyzing all of the piles would require a significant amount of time. This is a known limitation of the Monte Carlo simulation. However, this study has demonstrated that multi-objective optimization is a promising approach to the problem. The research findings suggest that this approach is moving in the right direction. The analysis results are presented in Tables II to IV.

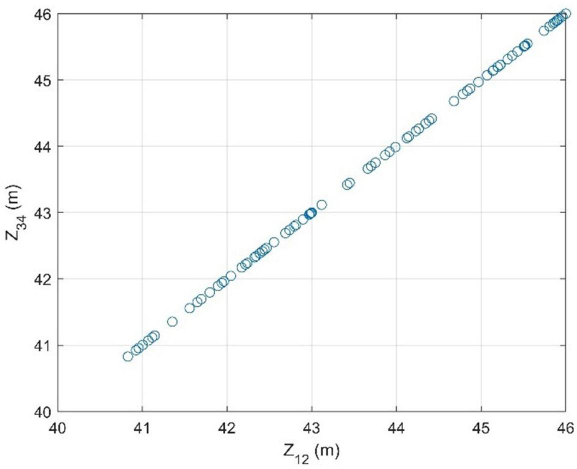


Fig. 5 Scatter diagram based on 100 random samples for $z12 = z34$.

Table II displays a comparison between the actual length and the optimized length of piles No. 2, No. 21, No. 51, No.115, No.125, and No.147. The data reveal that the designed pile lengths of 52 and 57 m are longer than recommended by the verification consultant, which suggested 45 m and 52 m for these piles, respectively. The computed lengths are approximately 45.2 m to 45.5 m. If these pile lengths are chosen, the settlement of the piles varies from 3.7 mm to 8.3 mm.

TABLE II. PILE LENGTH OPTIMIZATION BASED ON THE SHORTEST PILE LENGTH

| Pile no. | Optimization calculation | | | Pile length as per design (m) | Pile length as recommended by the verification consultant (m) |
|---|---|---|---|---|---|
| | Neutral plane elevation (m) | Pile length (m) | Pile settlement (m) | | |
| 2 | 42.3 | 45.1 | 0.004918 | 52.0 | 45 |
| 21 | 42.4 | 45.4 | 0.003741 | 52.0 | 45 |
| 51 | 43.0 | 45.2 | 0.007284 | 57.0 | 54 |
| 115 | 42.0 | 45.2 | 0.007284 | 57.0 | 54 |
| 125 | 42.8 | 45.5 | 0.005467 | 57.0 | 54 |
| 147 | 42.2 | 45.1 | 0.008306 | 57.0 | 54 |

Table III presents the lengths of piles No. 2, No. 21, No. 51, No. 115, No. 125, and No. 147 that have settled by approximately 5 mm. The requirement for even settlement emerges because of the raft's thickness design. The results manifest that the lengths of the piles vary from 45 m to 52 m, which may necessitate categorizing the piles into several groups. The elevation of the neutral plane ranges from 42.3 to 45.5, which is a considerable difference.

TABLE III. PILE LENGTH OPTIMIZATION BASED ON THE SAME SETTLEMENT

| Pile no. | Optimization calculation | | | Pile length as per design (m) |
|---|---|---|---|---|
| | Neutral plane elevation (m) | Pile length (m) | Pile settlement (m) | |
| 2 | 42.3 | 45.1 | 0.004918 | 52.0 |
| 21 | 42.4 | 46.7 | 0.004929 | 52.0 |
| 51 | 42.8 | 46.0 | 0.004822 | 57.0 |
| 115 | 45.5 | 52.0 | 0.004984 | 57.0 |
| 125 | 42.8 | 47.0 | 0.004907 | 57.0 |
| 147 | 42.7 | 45.1 | 0.004918 | 57.0 |

Table IV depicts the lengths of the piles required for maintaining an even elevation of neutral planes. The neutral elevation of all piles is 42.3 m. To meet this requirement, the piles vary in length from 45.1 to 49.9 m, and their settlement ranges from 3.9 mm to 9.8 mm.

TABLE IV. PILE LENGTH OPTIMIZATION BASED ON THE SAME NEUTRAL PLANES ELEVATION

| Pile no. | Optimization calculation | | | Pile length as per design (m) |
|---|---|---|---|---|
| | Neutral plane elevation (m) | Pile length (m) | Pile settlement (m) | |
| 2 | 42.3 | 45.1 | 0.004918 | 52.0 |
| 21 | 42.3 | 45.9 | 0.003989 | 52.0 |
| 51 | 42.3 | 49.9 | 0.007984 | 57.0 |
| 115 | 42.3 | 49.0 | 0.005597 | 57.0 |
| 125 | 42.3 | 48.1 | 0.007184 | 57.0 |
| 147 | 42.3 | 49.0 | 0.009813 | 57.0 |

## *C. Discussion*

The conventional approach to pile design involves two steps. The first step is to estimate the pile capacity using a large safety factor, ensuring that the pile-soil behavior remains elastic. The second step is to estimate the long-term settlement of the piles and piled foundation following the consolidation calculation method of the soil layers under the pile tips.

This study proposes a method for estimating the pile capacity that combines the modified unified pile design approach with a semi-empirical formula which includes a large safety factor to determine the pile capacity. The approach also considers the pile-soil interaction based on Mindlin's first solution to calculate the elastic subsoil settlement. This method enables engineers to assess the elastic pile-soil behavior based on the computed elastic settlement value rather than relying solely on a large safety factor.

After analyzing the results presented in Tables II-III, it is clear that the pile lengths under the column's rafts can be shortened to about 45 m, while the pile lengths under the core area can be reduced to about 50 m. This conclusion aligns with the recommendations provided by the verification consultant, who suggested shortening the lengths to 45 m and 52 m, respectively. It can be seen that the piles in the pile group of Connect 2, which are 52 and 57 m in length are capable of bearing the designed loads and are still behaving elastically. The settlement of the pile group in the long term was estimated to be between 5.75 and 7.39 cm through hand calculations, while Plaxis 3D estimated it to be approximately 1.05 cm. That indicates that the pile foundation design of Connect 2 complies with the code requirement.

The program that integrates the modified unified pile design method and the Monte Carlo approach can be a practical solution for rational pile design. However, this program is time-consuming. Hence, the next research step is to adopt a multi-objective optimization approach for pile design to make the process more efficient.

## IV. CONCLUSIONS

The modified Unified Pile Design method offers a means to verify the belief of the existence of an elastic pile-soil behavior in conventional pile design. It does so by assessing the safety factor value and validating the elastic behavior through computed settlements. This approach can help clarify why some projects seem overdesigned while others experience excessive settlement despite all of them using large safety factors.

Integrating the Monte-Carlo optimization approach with the modified Unified method is quite effective in designing piles, especially when dealing with parameters influenced by various uncertainties, such as sampling, testing methods, soil models, and computer-based programs. The results of this analysis for the pile group are very promising.

Initially, the pile lengths for the project were designed at 52 and 62 m. However, the Verification Consultant suggested optimizing their lengths to 45 and 52 m, respectively. Eventually, piles of 52 and 57 m in length were used for the project. The settlement of the pile group has been monitored for two years and is deemed safe. The optimization of the pile lengths based on conventional calculation methods with calibration of soil properties, as per the designers and consultants, seems unreliable though. However, the results from this analysis indicate that the piles with lengths of 45 and 52 m still behave elastically, and the consultant's suggestion can be accepted.

Further research must be conducted on the proposed approach and its possible application to construction practice. The primary focus of future research is to shortenthe calculation time and standardize the input soil data.

## ACKNOWLEDGMENT

The author would like to thank Dr. D.-N. Truong for his invaluable support and guidance.

## REFERENCES

[1] A. Benali, B. Boukhatem, M. N. Hussien, A. Nechnech, and M. Karray, "Prediction of axial capacity of piles driven in non-cohesive soils based on neural networks approach," *Journal of Civil Engineering and Management*, vol. 23, no. 3, pp. 393–408, Mar. 2017, https://doi.org/10.3846/13923730.2016.1144643.

[2] S. Gao and C. W. de Silva, "Estimation distribution algorithms on constrained optimization problems," *Applied Mathematics and Computation*, vol. 339, pp. 323–345, Dec. 2018, https://doi.org/10.1016/j.amc.2018.07.037.

[3] M. S. Kıran, M. Gündüz, and Ö. K. Baykan, "A novel hybrid algorithm based on particle swarm and ant colony optimization for finding the global minimum," *Applied Mathematics and Computation*, vol. 219, no. 4, pp. 1515–1521, Nov. 2012, https://doi.org/10.1016/j.amc.2012.06.078.

[4] M. Birattari, L. Paquete, T. Stützle, and K. Varrentrapp, "Classification of Metaheuristics and Design of Experiments for the Analysis of Components," Technical Report AIDA-01-05, 2001. Available: http://hdl.handle.net/2013/.

[5] J. Liu, C. Wu, G. Wu, and X. Wang, "A novel differential search algorithm and applications for structure design," *Applied Mathematics and Computation*, vol. 268, pp. 246–269, Oct. 2015, https://doi.org/10.1016/j.amc.2015.06.036.

[6] Y. Wei, "A Hybrid evolutionary algorithm based on Artificial Bee Colony algorithm and Differential Evolution," in *2021 2nd International Conference on Computer Engineering and Intelligent Control (ICCEIC)*, Chongqing, China, Nov. 2021, pp. 35–40, https://doi.org/10.1109/ICCEIC54227.2021.00015.

[7] M. S. Kiran, "TSA: Tree-seed algorithm for continuous optimization," *Expert Systems with Applications*, vol. 42, no. 19, pp. 6686–6698, Nov. 2015, https://doi.org/10.1016/j.eswa.2015.04.055.

[8] Y.-C. Yeh, Y.-H. Kuo, and D.-S. Hsu, "Building an expert system for debugging FEM input data with artificial neural networks," *Expert Systems with Applications*, vol. 5, no. 1, pp. 59–70, Jan. 1992, https://doi.org/10.1016/0957-4174(92)90095-A.

[9] J. Ghaboussi, J. H. Garrett, and X. Wu, "Knowledge-Based Modeling of Material Behavior with Neural Networks," *Journal of Engineering Mechanics*, vol. 117, no. 1, pp. 132–153, Jan. 1991, https://doi.org/10.1061/(ASCE)0733-9399(1991)117:1(132).

[10] S. J. Alghamdi, "Prediction of Concrete's Compressive Strength via Artificial Neural Network Trained on Synthetic Data," *Engineering, Technology & Applied Science Research*, vol. 13, no. 6, pp. 12404–12408, Dec. 2023, https://doi.org/10.48084/etasr.6560.

[11] L. Arabet, M. Hidjeb, and F. Belaabed, "A Comparative Study of Reinforced Soil Shear Strength Prediction by the Analytical Approach and Artificial Neural Networks," *Engineering, Technology & Applied Science Research*, vol. 12, no. 6, pp. 9795–9801, Dec. 2022, https://doi.org/10.48084/etasr.5394.

[12] L. Arabet, M. Hidjeb, and F. Belaabed, "A Comparative Study of Reinforced Soil Shear Strength Prediction by the Analytical Approach and Artificial Neural Networks," *Engineering, Technology & Applied Science Research*, vol. 12, no. 6, pp. 9795–9801, Dec. 2022, https://doi.org/10.48084/etasr.5394.

[13] M. A. Shahin, "Load–settlement modeling of axially loaded steel driven piles using CPT-based recurrent neural networks," *Soils and Foundations*, vol. 54, no. 3, pp. 515–522, Jun. 2014, https://doi.org/10.1016/j.sandf.2014.04.015.

[14] A. Kordjazi, F. Pooya Nejad, and M. B. Jaksa, "Prediction of ultimate axial load-carrying capacity of piles using a support vector machine based on CPT data," *Computers and Geotechnics*, vol. 55, pp. 91–102, Jan. 2014, https://doi.org/10.1016/j.compgeo.2013.08.001.

[15] A. Ismail, D.-S. Jeng, and L. L. Zhang, "An optimised product-unit neural network with a novel PSO–BP hybrid training algorithm: Applications to load–deformation analysis of axially loaded piles," *Engineering Applications of Artificial Intelligence*, vol. 26, no. 10, pp. 2305–2314, Nov. 2013, https://doi.org/10.1016/j.engappai.2013.04.007.

[16] A. Ahangar-Asr, A. A. Javadi, A. Johari, and Y. Chen, "Lateral load bearing capacity modelling of piles in cohesive soils in undrained conditions: An intelligent evolutionary approach," *Applied Soft Computing*, vol. 24, pp. 822–828, Nov. 2014, https://doi.org/10.1016/j.asoc.2014.07.027.

[17] R. Eslami, S. H. H. Sadeghi, and H. A. Abyaneh, "A Probabilistic Approach for the Evaluation of Fault Detection Schemes in Microgrids," *Engineering, Technology & Applied Science Research*, vol. 7, no. 5, pp. 1967–1973, Oct. 2017, https://doi.org/10.48084/etasr.1472.

[18] B. H. Fellenius, "Unified Design of Piled Foundations with Emphasis on Settlement Analysis," in *Current Practices and Future Trends in Deep Foundations*, Apr. 2012, pp. 253–275, https://doi.org/10.1061/40743(142)15.

[19] C. V. Hoa, "Pile design with consideration of down drag," in *Geotechnics for Sustainable Infrastructure Development*, Singapore, 2020, pp. 145–151, https://doi.org/10.1007/978-981-15-2184-3_18.

[20] H. V. Cao and T. A. Nguyen, "Verification and Validation of the Pile Design Method with Consideration of down Drag," *International Journal of GEOMATE*, vol. 23, no. 96, pp. 145–152, Aug. 2022, https://doi.org/10.21660/2022.96.3433.

[21] H. C. Van and T. A. Nguyen, "Numerical Simulation of Pile Design Method that Considers Negative Friction," *Civil Engineering and Architecture*, vol. 11, no. 5, pp. 2285–2292, Sep. 2023, https://doi.org/10.13189/cea.2023.110503.

[22] C. V. Hoa, "Rational Pile Design using Computer-based Program Coding in Matlab: A Case Study," *Engineering, Technology & Applied Science Research*, vol. 14, no. 2, pp. 13160–13166, Apr. 2024, https://doi.org/10.48084/etasr.6867.

[23] P. Anbazhagan, M. Neaz Sheikh, K. Bajaj, P. J. Mariya Dayana, H. Madhura, and G. R. Reddy, "Empirical models for the prediction of ground motion duration for intraplate earthquakes," *Journal of Seismology*, vol. 21, no. 4, pp. 1001–1021, Jul. 2017, https://doi.org/10.1007/s10950-017-9648-2.