Behavior of Reinforced Concrete Beams with Vertically Penetrated Holes across the Cross -Sectional Depth

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ABSTRACT

In Iraq and other countries, RC beams are frequently used as alternative small slab penetrations in lowrise constructions. The structure is capable of transferring stresses effectively and the existence of minor gaps normally has little impact on the structural performance. However, special consideration must be given to the analysis and design of such beams depending on the effect of the openings. Hence, the objective of this paper is to address this research gap by evaluating and enhancing the behavior of RC beams with a vertical opening that penetrates the entire depth of the beams and obstructs the reinforcement bars since all the building design codes and guidelines introduce very limited recommendations for beams with vertical openings. To achieve that, five specimens were fabricated as RC beams with vertical openings and one specimen as a reference beam. The six specimens were tested under two-point loads over a clear span length of 180 cm and a total length of 200 cm. It was found that by adopting a vertical opening into the RC beam without any strengthening methods, the ultimate carrying load capacity decreased and the mid-span deflection increased compared with the reference solid beam.

Keywords-vertically penetrated holes; reinforced concrete beam; vertical opening

I. INTRODUCTION

In practice, a network of ducts and pipes is needed to accommodate utilities like sewage, power, air conditioning, telephone, and water supply [1]. Accordingly, most of the Reinforced Concrete (RC) structures in Iraq contain holes in RC beams, commonly known as penetrations or openings, which are created to accommodate the passage of piping or other utilities [2]. These openings are strategically placed in the slab or beams to allow the installation, maintenance, or repair of pipes without compromising structural strength [2]. Particularly in low-rise structures, RC slab penetrations with small gaps frequently have little impact on the structural behavior because the structure can transfer stresses effectively. Vertical openings in RC slabs are therefore preferable since they offer a practical solution without affecting the building's overall structural integrity [2]. However, it is crucial to recognize that it may take up precious space and stand out aesthetically, perhaps detracting from the building's aesthetic attractiveness. To overcome this issue, it could be essential to employ a suspended ceiling or unique ornamental components to obtain a pleasant aesthetic result. On the other hand, vertical openings in RC beams have evolved to be a standard practice

because they enable ducts and pipes to be hidden until they reach their intended destinations behind walls, as shown in Figure 1. The effect of openings on a beam's strength and deformation typically relies on their size and position. Nevertheless, the use of openings in beams introduces limitations such as reduced structural capacity, increased vulnerability to shear, compromised flexural strength, and potential seismic concerns [3]. Hence, it is crucial to ensure the provision of sufficient special reinforcement at the opening area. This reinforcement serves the purpose of controlling the width of cracks and mitigating the risk of any potential premature failure of the beam. Furthermore, strengthening RC beams with vertical openings can be achieved through methods like bonding steel plates, external post-tensioning, Fiber-Reinforced Polymer (FRP) wraps, steel sleeves, steel reinforcement insertion, and external prestressing. Combining these methods may be necessary for optimal results, with the choice depending on specific project requirements and constraints [4-6]. There is limited literature related to RC beams with vertical openings because beams with vertical openings are less common in practice compared to regular solid beams [7].



Fig. 1. Various service pipes passing through RC beams in commercial buildings.

Authors in [8], emphasize how crucial it is to take into account the existence and placement of openings in T-beams when determining their shear capability. To maintain the structural integrity and safety of reinforced concrete structures, these elements must be taken into consideration during the design and construction process [8]. In [4], the authors reported a similar study with the use of CFRP sheets. Authors in [9] found that beams with openings located at the mid-span failed due to shear, which was unexpected. On the other hand, beams with openings located at the support failed due to flexural failure without any significant impact on the ultimate load [9]. It was reported that beams with openings reinforced with a steel sleeve exhibit higher bearing capacity and lower deformation capabilities compared to circular beams without reinforcement [10]. However, the effectiveness of this approach is limited due to the significant difference in stiffness between the concrete and steel sleeve, leading to concentrated stress and uncoordinated deformations in the surrounding concrete. Consequently, cracks may occur [10]. It has been shown that the influence of the openings on the load-bearing and seismic performance of the RC beams can be reduced by adding extra reinforcements around the openings. Openings' configuration was investigated in and an increase in ultimate load capacity when circular openings were used was reported [11]. These codes typically cover aspects such as load material properties, structural analysis, combinations, reinforcement detailing, and construction practices relevant to RC beams. While they may not specifically address beams with openings, engineers can apply the principles and provisions outlined in these codes to design such elements safely and efficiently [12].

This study aims to evaluate and enhance the behavior of RC beams with a vertical opening that spans the entire depth of the

beams and obstructs the reinforcement bars since all the building design codes and guidelines introduce very limited recommendations for beams with vertical openings [13-16].

II. EXPERIMENTAL PROGRAM

The performance of an RC beam with vertical openings can be influenced by various parameters that may impact both serviceability and ultimate limit states. The three major parameters considered in the experimental program were: the use of steel tubes to create the vertical opening, and using different two special types of reinforcement configuration around the vertical opening. All other parameters were kept constant.

A. Concrete

In this experimental work, concrete with compressive strength of 45 MPa was utilized for the beam specimens. The concrete mix manufacturing procedure took into consideration the balance between economy and the requirements for strength, density, durability, and workability central mixing.

B. Steel Tubes

Xinyue's steel tubes manufactured at Tianlin Xinyue industry with inner diameter of 63 mm were used. The steel tube specifications are shown in Table I.

Specimen outer diameter (mm)	73
Thickness (mm)	3
Length (mm)	6000
Ultimate strength (MPa)	451
Yield strength (MPa)	300
Elongation (%)	25

CHARACTERISTICS

PLATE MECHANICAL AND SECTION

C. Reinforcement Bars

TABLE I.

In this work, 10 and 16 mm diameter deformed steel reinforcement bars were used, as shown in Figures 2 and 3. Additional 10 mm reinforcement was applied in some specimens to strengthen the steel tube as illustrated in Figure 4. A tensile tests for reinforcement steel bars were performed according to ASTM A615, 2016 and ASTM A496, 2007. The test results are shown in Table II.

TABLE II. MECHANICAL CHARACTERISTICS OF STEEL BARS

Spec. nominal diameter (mm)	10	16
Diameter (mm)	10.2	16.2
Ult. strength (MPa)	743	847
Yield strength (MPa)	529	585
Elongation (%)	18.23	19.4

D. Tested Specimens

Five specimens were fabricated as RC beams with vertical openings and one specimen as the reference beam. The six specimens were tested under two point loads over a clear span length of 180 cm and a total length of 200 cm. Descriptions of fabricated samples adopted for this work are explained in Table III.





Fig. 3. Dimensions of the beams BC 3,4,5.



Fig. 4. Steel tube with additional reinforcement.

TABLE III. DETAILS OF STEEL BEAM SPECIMEN VARIANTS (mm)

Beam	Strengthen methods
B-SOLID	Reference
BC-1	non
BC-2	Additional horizontal top reinforcement Ø10mm on both sides
BC-3	Steel tubes used to strengthen vertical openings
BC-4	Steel tubes used to strengthen the vertical openings with additional diagonal reinforcement 1ø10mm on both sides
BC-5	Steel tubes used to strengthen the vertical openings with additional top reinforcement 2ø10mm on both sides

E. Test Setup

Using a load sell hydraulic testing machine with a 500 kN capacity, two-point load was applied to each specimen until the point of failure.

The load was applied gradually with an incremental rate of 10 kN. The mid-span deflection was recorded by using LVDT placed at the mid-span. The supports were constructed to provide a simple support reaction. A dial-gauge with precision up to 0.01 mm and a maximum capacity of 80 mm were adopted. Figure 5 presents the dial gauge's location during the test of the investigation beams. Three cameras were set up to track the cracking formation and capture data from the three dial gauges.



Fig. 5. Specimens under two-point load tests.

III. TEST RESULTS AND DISCUSSION

As previously mentioned, this paper aims to study the influence of existing vertical openings in RC beams throughout the overall beam depth under two-point load and their performance after using different strengthening techniques. Two-point load (two identical concentrated loads), each load located 200 mm away from the beam span center, were applied until failure. In all specimens except the reference beam, a vertical opening was inserted as shown in Figures 2 and 3.

A. The Effect of Verticle Opening on Load-Deflection Curves

Three loading stages were chosen to study the behavior of the beams with vertical openings, i.e. the elastic uncrack stage (40 kN), the service crack stage (100 kN), and the ultimate load stage. Accordingly, by creating a vertical opening into the RC beam (B-C1) as shown in Figure 6, the following can be observed:

The ultimate capacity of the carrying load of B-C1 beam decreased by 8.8% compared with the reference solid beam (B-Solid). The mid-span deflection of B-C1 beam increased by 21.2% at the ultimate load. This behavior was equivalent for all loading stages, represented by increasing deflection at the uncracked (40 kN) and service cracked (100 kN) loads with 58% and 46%, respectively compared with the reference solid beam.

In contrast to the reduction in load-carrying capacity, the presence of vertical openings in RC beams causes an increase in mid-span deflection. This can be explained by the weakening of the cross-sectional area caused by the openings, which lowers the stiffness and resistance of the beam to the applied loads.

Furthermore, the presence of openings can cause stress concentrations around the opening edges. This distortion of stresses can lead to localized areas of higher stress, potentially affecting the overall structural integrity and durability of the beam.

Finally, openings in RC beams can increase the chance of cracking, especially around the edges of the openings. These cracks can propagate and affect the overall structural performance, including the beam's ability to resist bending and shear stresses.



Fig. 6. Load-mid-span deflection curves of beams B-SOLID, and B-C1.

B. The Effect on Load-Deflection Curves of Additional top Reinforcement to Strengthen the Vertical Openings of RC Beams

The influence of creating a vertical opening in the RC beam with additional top (compression) reinforcement on both sides of the opening is an attempt to strengthen and compensate the top compression zone, as shown in Figure 7. As a result, the RC beam's load-deflection behavior shows improvement over the B-C1 beam, although it is still lower than the reference beam. Appropriately, the following observation can be noted:

The ultimate capacity of carrying loads of B-C2 beam increases by 5.12% when compared with the B-C1 beam. But in comparison to the reference beam (B-Solid), a reduction of 4.14% was noticed.



Fig. 7. Load-mid-span deflection curves of beams B-SOLID, B-C1, and, B-C2.

In relation to the B-Solid, the mid-span deflection of beam B-C2 increased with each loading step: at 40 kN, 100 kN, and ultimate loads, it was 16%, 26.3%, and 7.6%, respectively.

However, as compared to the B-C1, the addition of compression reinforcement results in a decrease in beam deflection of 26.6%, 13.5%, and 11.2% for the same loading stages, indicating an increase in beam stiffness.

When an additional top reinforcement and vertical opening are added to an RC beam, the load-carrying capacity increases and the mid-span deflection decreases, suggesting that the additional compression reinforcement has balanced out the beam compression zone. Also, the adoption of additional top reinforcement around the opening helps distribute the stresses more evenly, reduces the risk of localized failure around the opening, increases beam stiffness, reduces deflections, and enhances its overall structural performance. Also, the additional reinforcement can control cracking in the vicinity of the opening, and provides additional ductility to the beam.

C. The Effect on Load-Deflection Curves of Steel Tube to Strengthen the Vertical Openings of RC Beams

The influences of the adoption of vertical opening in the RC beam using steel tube (B-C3) to create and strengthen the vertical opening of the RC beam on the load-deflection behavior is shown in Figure 8. Therefore, the following observations can be noted:

The final carrying capacity is 15.1% higher than that of the B-C1 beam. The same observation was made regarding the reference beam (B-Solid), showing a 4.94% improvement and full-strength recovery on par with the reference beam.

On the other hand, the mid-span deflection of beam B-C3 decreased with each loading step in relation to the B-Solid: it was 14%, 0%, and 6.1%, respectively, at 40 kN, 100 kN, and ultimate load. In contrast, as compared to the B-C1, the insertion of steel tube results in a decrease in beam deflection of 27.8%, 32.4%, and 12.5% for the identical loading stages. This indicates an increase in beam stiffness.



Fig. 8. Load-mid-span deflection curves of beams B-SOLID, B-C1, and, B-C3.

So, the addition of a steel tube within the opening provides additional strength and stiffness, which is particularly noticeable in the compression zone because the steel tube acts as a reinforcement element. This explains the increase in loadcarrying capacity and decrease in mid-span deflection caused by the adoption of the steel tube. Additionally, the steel tube's presence aids in more effectively distributing the applied stresses across the beam, hence lowering the concentration of stress. This can lead to increased overall structural integrity and load-carrying capability, particularly in cases where the debonding between the concrete and the steel tube has not occurred. The steel tube can significantly influence the flexural behavior of the beam by acting as external confinement for the opening, preventing the formation and propagation of cracks. This confinement helps maintain the structural integrity of the beams and reduces the risk of premature failure. It enhances the beam's resistance to bending and can help control deflections and deformations under load.

D. The Effect on load-deflection curves of a Steel Tube with Additional Diagonal Reinforcement to Strengthen the vertical Openings of RC Beams

The influences of the adoption of vertical opening in the RC beam while using a steel tube with additional diagonal reinforcement on both tube sides on the load-deflection behavior of RC beam is demonstrated in Figure 9. By creating a vertical opening while using the steel tube with additional diagonal reinforcement as revealed in Figures 3 and 4, the following observation can be summarized:

The ultimate load carrying capability is 17.9% higher than that of the B-C1 beam. On the other hand, the enhancement was 7.48% relative to the reference beam (B-Solid).

The behavior of B-C4 at earlier loading stages, as exhibited in Figure 9 converges to the (B-Solid) reference beam, and then deviation can be observed with decrease in deflection as loading increases, whereas, in comparison to B-C1 the behavior diverges along all loading stages. B-C4 exhibits higher stiffness. It was 35.4%, 36.1%, and 12.9% at 40 kN, 100 kN, and ultimate loading stages, respectively.

The observable increment in load-carrying capacity and reduction in mid-span deflection due to the addition of steel tube with diagonal reinforcement enhances the load-carrying capacity of the RC beams. The diagonal reinforcement provides additional strength and stiffness, allowing the beams to withstand higher loads without failure.



Fig. 9. Load-mid-span deflection curves of beams B-SOLID, B-C1, and B-C4.

The steel tube acts as external confinement for the RC beam and helps to maintain the structural integrity of the beams, reducing the risk of premature failure. The combination of steel tubes and diagonal reinforcement in the RC beams improves the load-carrying capacity and ductility, reducing deflection, reducing the risk of premature failure, and preventing the formation and propagation of cracks. By enhancing ductility, the beam can undergo larger deformations without collapse, providing a margin of safety during extreme loading conditions. Also, vertical openings in RC beams can weaken their resistance to shear forces. The use of steel tube with diagonal reinforcement helps to mitigate this issue by providing additional shear strength. The diagonal reinforcement acts as a bracing system, effectively transferring shear forces and reducing the vulnerability of the beams to shear failure.

E. The Effect on load-deflection curves of Steel Tubes with Additional Horizontal Eeinforcement to Strengthen the Vertical Openings of the RC Beams

The influence of using steel tubes with additional horizontal reinforcement around vertical openings with adequate development length on the load-deflection behavior of RC beams is illustrated in Figure 10. The following observations can be listed:

When comparing B-C5 to B-C1, the ultimate carrying capacity improved by 17.9%. However, this improvement was 7.5% compared to the solid reference beam (B-Solid).

In relation to the B-Solid, the mid-span deflection of B-C5 beam decreased with each loading step: at 40 kN and 100 kN, it was 2%, and 4.5 %, respectively and increased at ultimate loads only by 6%. However, as compared to the B-C1, the adoption of steel tubes to strengthen the openings results in a decrease in beam deflection of 37.9%, 34.6%, and 12.6% for the same loading stages, indicating an increase in beam stiffness.

The horizontal reinforcement provides resistance against bending moments and balances the compression zone efficiently and by incorporating steel tubes with horizontal reinforcement makes the beams more capable of carrying increased loads without failure. The beams are able to control and limit the formation and propagation of cracks. This helps the beams to maintain their overall strength and durability. The presence of steel tubes with horizontal reinforcement increases the stiffness of the RC beams. This improved stiffness reduces deflections and deformations, ensuring that the beams maintain their shape and structural integrity under load. Beams B-C4 and B-C5 converge and exhibit behavior enhancements that are roughly identical to one another.



Fig. 10. Load-mid-span deflection curves of beams B-SOLID, B-C1, and B-C5.

F. First Crack

Cracks were recorded and measured using marker pen and a video camera. It was observed, and is detailed in Table IV, that at first, cracks appear, (vertical or diagonal) sooner and at lower loads for the B-C1 beam with a decrease of 13% in comparison with the reference (B-Solid) beam. Additionally, by using strengthening methods first vertical or diagonal cracks were increases by 4.5%, 11.9%, 11.9%, and 9.0% for beams (B-C2, B-C3, B-C4, and B-C5), respectively, compared with the unstrengthening beam (B-C1). As the load increased, a vertical crack started forming in the concrete face at the mid of the beams in the flexure failure zone from the bottom into the upper side. Diagonal cracks also appeared at the surface of concrete on both sides of the beam for all specimens.

TABLE IV. LOADS AND DEFLECTION VALUES AT FIRST CRACK AND WHEN CRACKS LENGTH \geq (14CM).

Beam	Load (kN) at first crack	% decrease *	% decrease **	Load (kN) at crack length ≥14 cm
B-SOILD	77	-	-	128
B-C1	67	13	-	120
B-C2	70	9.1	4.5	127
B-C3	75	2.6	11.9	130
B-C4	75	2.6	11.9	131
B-C5	73	5.2	9.0	128

*% decrease with respect to B-Solid

**% decrease with respect to B-C1

G. Mode of Failure

Figures 11-21, exhibit all beams' specimens including reference beam and RC beams with vertical movement experienced flexural failure as the main mode of failure. Furthermore, the concrete crushing at the upper middle of the beams was also noted.

It was seen that using steel tube in the vertical opening can prevent splitting resulting more strength in the area surrounding the openings. However, using horizontals and diagonal bars as additional reinforcement around the openings have not seen significant effect regarding the mode of failure.



Fig. 11. Mode of failure of B-SOLID beam.



Fig. 12. Mode of failure of B-C1 beam.



Fig. 13. Mode of failure of B-C2 beam.



Fig. 14. Mode of failure of B-C3 beam.



Fig. 15. Mode of failure of B-C4 beam.



Fig. 16. Mode of failure of beam B-C5.

IV. CONCLUSIONS

Based on the outcomes of the experimental research, the following conclusions can be drawn:

It was found that by adopting a vertical opening into the RC beam without any strengthening methods, the ultimate carrying load capacity decreased by 8.8% and the mid-span deflection increased by (21%%) compared with the reference solid beam. This behavior was equivalent for all loading stages, represented by increasing deflection at the uncracked stage (40 kN) and service cracked (100kN) loads with (58% and, 46%), respectively.

With additional top (compression) reinforcement on both sides of the opening in the RC beam, the ultimate capacity of carrying load increased by 5.12%, the mid-span deflection increased with each loading step: at 40 kN, 100 kN, and ultimate loads, it was 16%, 26.3%, and 7.6%, respectively.

By using steel tubes to strengthen vertical openings of RC beams, the ultimate capacity of carrying loads increases by 15.1%, the mid-span deflection increased with each loading

step: at 40 kN, 100 kN, and ultimate loads, was 14%, 0%, and 6.1%, respectively.

By using steel tubes with additional diagonal reinforcement around the vertical openings, the ultimate capacity of carrying loads increases by 17.9%, the mid-span deflection increased with each loading step: at 40 kN, 100 kN, and ultimate loads, it was 2%, 0.65%, and 6.07%, respectively.

By using steel tubes with additional horizontal reinforcement around the vertical openings, the ultimate capacity of carrying loads increases by 17.9%. The mid-span deflection decreased with each loading step: at 40 kN, 100 kN, it was 2%, and 4.5% respectively, and increased at ultimate loads by 6%.

All beams' specimens including the reference beam and RC beams with vertical movement, experienced flexural failure as the main mode of failure. Furthermore, the concrete crushing at the upper middle of the beams was also noted.

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