Modified Equivalent Compression Stress Block for Normal-Strength Concrete Flexural Design using Energy Modeling

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ABSTRACT

The equivalent stress block is recommended for use in the design of reinforced concrete sections to simplify the analysis of the composite behavior of concrete and steel reinforcement. In most current codes, a rectangular equivalent stress block is provided. The design parameters of the equivalent block were recommended many years ago. Due to the importance of the equivalent stress block concept, numerous investigations have been performed to increase its accuracy. In the current paper, an exploration of the rectangular equivalent stress block has been carried out using the energy modeling approach. Energy modeling is a new general approach for studying the behavior of concrete elements. In this method, the energy consumed (work done) can be determined by integrating the force-displacement diagram (in the current study this will be the concrete stress-strain curve in compression). Schematic and equivalent stress-strain curves for concrete in uniaxial compression provided in most current codes and relevant textbooks were considered in this research. The codes taken into account in the current study are ACI-318-19, Canadian Code CSA A23.3-04, Eurocode EC-2, and Chinese standard GB 500 10 – 2002. The energy consumed by these curves for different values of concrete strength has been compared with numerous experimental results. This comparison shows that the results of the equivalent stress block provided in most of the considered current codes are conservative. Applying the energy modeling for the considered experimental stress-strain curves a modified equivalent stress block is recommended for practical use. The results of the proposed equivalent stress block are in good agreement with the experimental ones. The ratio between the predicted total energy engaging the proposed model and the total energy calculated for the experimental results ranges between 0.95 and 1.08 with a mean value equal to unity.

Keywords-concrete; stress-strain curve; energy modeling; equivalent stress block

I. INTRODUCTION

The equivalent stress block is a concept used in concrete design to simplify the actual stress distribution in the compressive zone of concrete into a rectangular shape in most current codes. Different international codes provide different stress block parameters, which can be reduced to two basic factors: the strength reduction factor and the factor for the depth of the resultant compressive force. These two parameters are different in most current codes. This concept allows for simple accurate prediction of the flexural strength of concrete elements with or without axial force. The design parameters of the equivalent block were recommended many years ago [1, 2]. The design parameters of the equivalent stress block have been verified in many times [3-7]. Authors in [3] developed an equivalent stress block for high strength concrete in revision of the Indian Code, following the same procedure followed in [1]. Authors in [4, 5], modified the equivalent stress block considering the strain gradient effect, again following the classical procedure deployed in [1]. Authors in [7] utilized Artificial Neural Networks (ANNs) to study the compressive strength of concrete. Author in [8] carried out a comparative analysis of schematic and equivalent stress blocks implementing energy modeling, without consideration of experimental results. In addition, no practical results were recommended. In the current paper, the energy modeling approach will be adopted to investigate the schematic concrete stress-strain curves and the equivalent stress block in many current codes against experimental results from the literature. The energy approach has been recently applied to investigate the behavior of concrete elements and to determine design parameters [8, 9].

II. SCHEMATIC STRESS-STRAIN CURVE

The stress-strain curve of concrete can vary based on many factors, such as mix design, curing conditions, and aggregate type. The model of the concrete schematic stress-strain curve in most current codes is approximately similar with minor differences. In all models, the curve consists of two portions, first is a parabolic curve up to the apex and then a decay branch down to the failure point. The schematic models provided in different current codes will be explained in the following sections.

A. ACI-318-19 [10]

ACI-318-19 provides a schematic stress-strain curve for concrete. However, in textbooks [11], references are only given to the Hognestad diagram [1, 2]. Description of the Hognestad curve is presented in [11] for concrete with strength up to about 42 MPa. It consists of a second-degree parabola with apex at a strain of ε_0, where $f_c'' = 0.9\ f_c'$, followed by a downward-sloping line terminating at a stress of 0.85 f_c'' and a limiting strain of $\varepsilon_{cu} = 0.0038$.

$$f_c = f_c''\left[\frac{2\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2\right] \quad (1)$$

$$\varepsilon_0 = \frac{1.8 f_c''}{E_c} \quad (2)$$

The schematic stress-strain curve used for the design of the concrete elements has the same configuration as the Hognestad diagram but with the apex at strain $\varepsilon_0 = 0.002$, and stress f_c', followed by a downward-sloping line terminating at a stress of 0.85 $f_c'$and ultimate strain $\varepsilon_{cu} = 0.003$ [11]. The ultimate strain is limited to 0.003 to avoid cover spalling at lower strains and to provide a margin of safety against unforeseen variations in material properties, construction quality, and loading conditions.

B. Canadian Code CSA A23.3-04 [12]

The schematic stress-strain curve for concrete in compression is provided in design manuals and textbooks [13, 14]. The strain in concrete at the apex ε_c^{pic}, corresponding to the concrete strength f_c', increases with f_c', is not less than 0.002, and can be estimated as a function of f_c' by:

$$\varepsilon_c^{pic} = \frac{140 + f_c'}{80000} \geq 0.002 \quad (3)$$

The ultimate concrete strain in compression generally varies between 0.003 and 0.004. However, its value is limited to $\varepsilon_{cu} = 0.0035$ and $\varepsilon_c^{pic} = 0.002$ [12].

C. EuroCode [15]

EC-2 provides a schematic stress-strain curve as a second-degree parabola [16, 17] with the following form:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\cdot\eta - \eta^2}{1 + (k-2)\cdot\eta} \quad (4)$$

where:

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \quad (5)$$

and, according to EC-2 [15], ε_{c1} is the strain at peak stress, and:

$$k = 1.05\ E_{cm} \times |\varepsilon_{c1}|/f_{cm},$$

$$f_{cm} = 8 + f_{ck},$$

$$E_{cm} = 22\left[\frac{f_{cm}}{10}\right]^{0.3},$$

$$\varepsilon_{c1} = 0.7\ (f_{cm})^{0.31},$$

$$\varepsilon_{cu1} = 0.0035.$$

D. Chinese Standard GB 500 10 – 2002 [17]

The stress-strain curve of concrete under uniaxial compression is determined by:

$$\sigma = (1 - d_c)E_c\ \varepsilon \quad (6)$$

$$d_c = \begin{cases} 1 - \frac{\rho_c\ n}{n - 1 + x^n} & x \leq 1 \\ 1 - \frac{\rho_c}{a_c(x-1)^2 + x} & x > 1 \end{cases} \quad (7)$$

$$\rho_c = \frac{f_{c,r}}{E_c\ \varepsilon_{c,r}} \quad (8)$$

$$n = \frac{E_c\ \varepsilon_{c,r}}{E_c\ \varepsilon_{c,r} - f_{c,r}} \quad (9)$$

$$x = \frac{\varepsilon}{\varepsilon_{c,r}} \quad (10)$$

where:

$$\sigma_c = f_c\left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_0}\right)^n\right] \text{ for } \varepsilon_c \leq \varepsilon_0,$$

$$\sigma_c = f_c \text{ for } \varepsilon_0 \leq \varepsilon_c \leq \varepsilon_{cu},$$

$$n = 2 - \frac{1}{60}\left(f_{cu,k} - 50\right),$$

where ε_0 is the value of the relative compression deformation at stress equal to f_c, not more than 0.002, ε_{cu} is the limit value of the relative compression deformation, which is no more than 0.0033 and for axial compression is equal to ε_0,and f_c is the calculated value of the resistance of concrete to axial compression [17], $f_{cu,k}$ is the characteristic (normative) value of concrete's resistance to compression of cubes, and *n* is a coefficient assumed to be no more than 2.

III. COMPARISON WITH EXPERIMENTAL RESULTS

Experimental results collected from [18-21] were compared with the schematic curves provided in different current codes and design manuals. For example, some results for different values of concrete compressive strength are presented in Figure 1.

In most cases, the Canadian schematic curve is very close to the experimental ones, while the Chinese curve is more deviated than others. The total energy (work done) for all selected experimental results has been determined for the experimental and schematic curves of the considered codes. The results are displayed in Figure 6. It can be observed that for concrete strength lower than 30 MPa, the results of EC-2 and Canadian Code are close to the experimental ones. For concrete strength larger than 30 MPa, the Chinese code results agree with the experimental results.

In Figure 7, a comparison of the relative centroid of the stress-strain curve of the schematic curve of different codes with the experimental curves is shown. It can be observed that the Canadian code is closer to the experimental results.

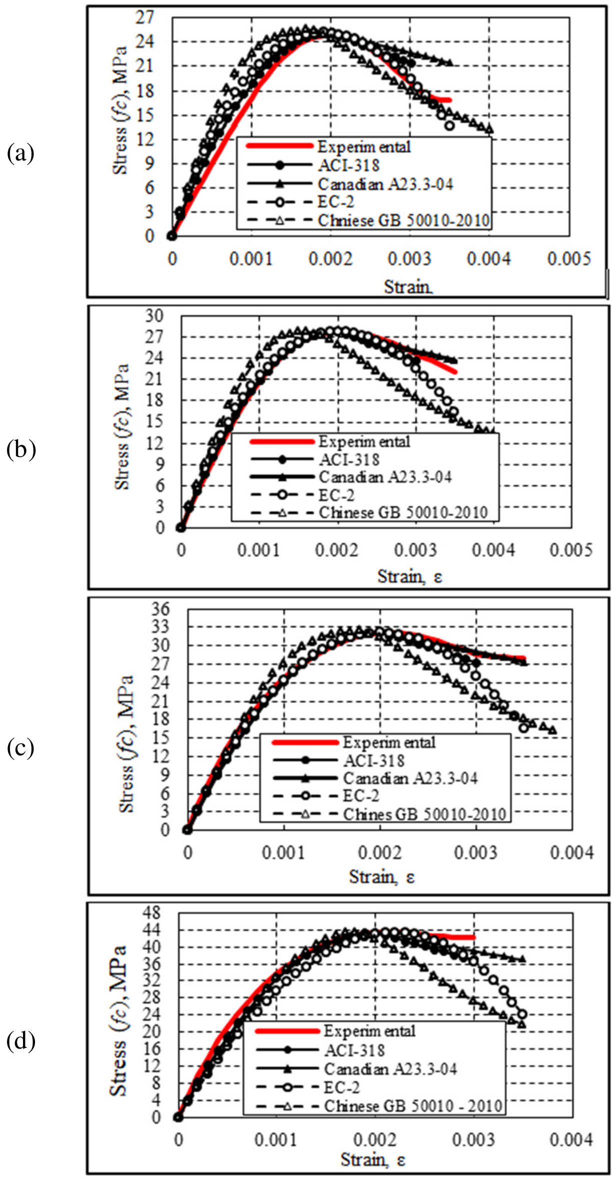

Fig. 1. Comparison of schematic curves with the experimental results, for concrete strength f_c' of: (a) 25.2 MPa, (b) 27.9 MPa, (c) 32.2 MPa, and (d) 43.4 MPa.

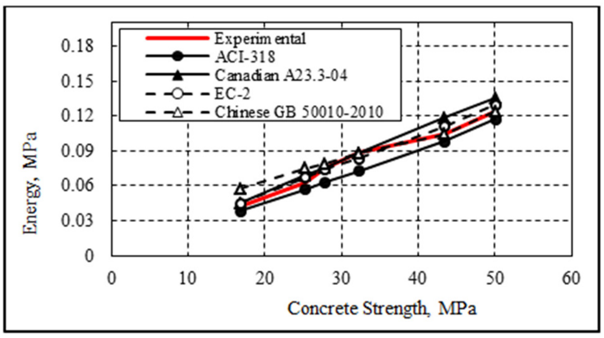

Fig. 2. Comparison of the total energy of the experimental results and the considered codes.

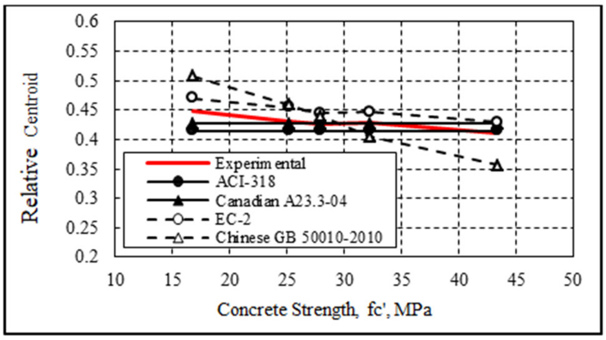

Fig. 3. Centroid.

IV. EQUIVALENT STRESS BLOCK

The concept of an equivalent stress block is implemented in the design of reinforced concrete sections to simplify the analysis of the composite behavior of concrete and steel reinforcement. This approach is adopted by various design codes and standards around the world. The popular shape of the equivalent stress block is rectangular with two main coefficients (α_1 and β_1) as shown in Figure 4. The rectangular stress block has an average stress of $\alpha_1 f_c'$ and a height of $a = \beta_1 c$. The values of these two coefficients in the considered codes are presented in Table. 1. The ultimate strain ε_{cu} in most current codes and recent investigations is recommended to be 0.0035 [3].

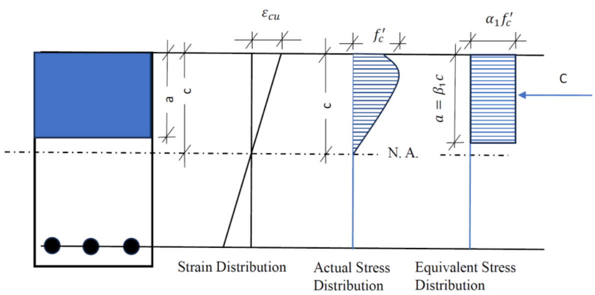

Fig. 4. Equivalent stress block.

TABLE I. EQUIVALENT STRESS BLOCK COEFFICIENTS

Code	α_1	β_1
ACI-318 (19)	0.85	$0.85 - 0.007(f_c' - 28) \geq 0.65$
Canadian Code	$0.85 - 0.0015\, f_c' \geq 0.67$	$0.97 - 0.0025 f_c' \geq 0.67$
Eurocode 2	0.85	0.8
Chinese Code	1.0	0.8

The total energy obtained from the equivalent stress block of the considered different codes is compared with the values acquired from the experimental curves as presented in Figure 5. The results of the Chinese code are very close to the experimental ones for concrete strength less than 30 MPa. For concrete strength of more than 30, the EC-2 and Canadian codes' results are closer to the calculated experimental total energy. This is due to the difference of the ultimate strain in the schematic and equivalent stress-strain of the Chinese code.

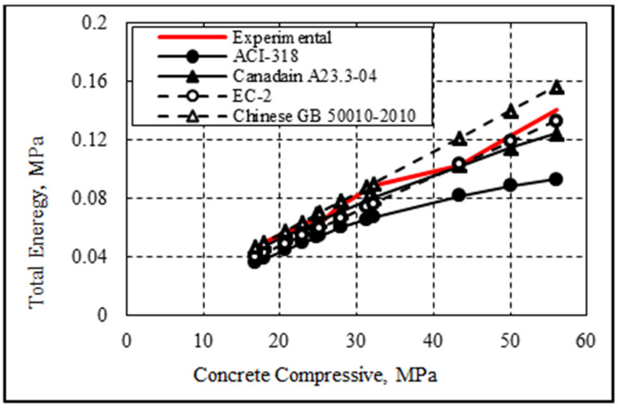

Fig. 5. Total energy of the experimental and equivalent stress block.

V. PROPOSED EQUATIONS

Based on the energy modeling employed in the current study, the experimental values of the equivalent stress block coefficients α_1 and β_1 were determined as presented in Figures 6 and 7. Curve fitting for the experimental results has been performed and the following two equations were obtained:

$$\alpha_1 = 0.95 - 0.0009 \times f_c' \tag{11}$$

$$\beta_1 = 0.86 - 0.0013 \times f_c' \tag{12}$$

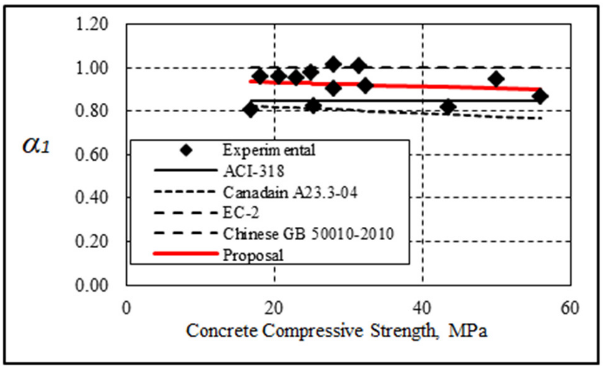

Fig. 6. Coefficient α_1.

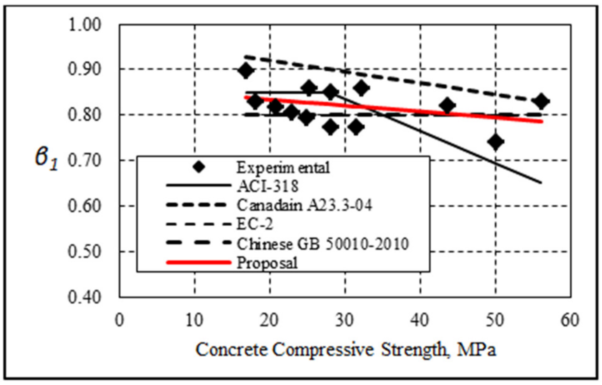

Fig. 7. Coefficient β_1.

A verification of the proposed equation findings is displayed in Tables II and III. In Table II, the total energy determined from the experimental curves is compared with the results of equivalent stress blocks of different codes. The proposed stress block provides results with better agreement with the experimental values in comparison with the different codes, with an average value equal to unity. Also, the height of the proposed equivalent stress block agrees more with the experimental one than those of the different codes.

The results portrayed in Table II indicate that the total energy obtained using the proposed equations is closer to the experimental results with deviation ranges from +8% to -5% and a mean value equal to unity. Chinese code gives good results for some cases with mean value 1.06, whereas ACI-318, Canadian, and EC-2 codes give results lower than the experimental ones with mean values equal to 0.78, 0.95, and 0.9, respectively. The comparison of the equivalent stress block height (parameter β_1) is showcased in Table III. The mean value obtained from the proposed equation (12) is equal to 1, whereas it differs from 1 for code results. It is also noticed that the ACI-318 code gives more accurate results for the parameter β_1 than the other codes.

TABLE II. CALCULATED AND EXPERIMENTAL TOTAL ENERGY RATIO COMPARISON

f_c'	**Proposed /Exp.**	**ACI-318 /Exp.**	**Canadian /Exp.**	**EC-2 /Exp.**	**Chinese /Exp.**
16.8	1.08	0.85	1.06	0.94	1.1
17.99	0.98	0.78	0.96	0.85	1
20.6	0.98	0.78	0.95	0.86	1.01
22.9	1	0.8	0.97	0.88	1.04
24.8	0.98	0.79	0.94	0.87	1.02
25.2	1.08	0.87	1.03	0.95	1.12
27.88	0.97	0.79	0.93	0.87	1.02
27.9	0.99	0.8	0.94	0.88	1.04
31.3	0.97	0.77	0.92	0.87	1.02
32.2	0.95	0.76	0.9	0.86	1.01
43.4	1.08	0.8	1	1.01	1.18
50	1.02	0.72	0.93	0.96	1.13
56	0.98	0.66	0.88	0.94	1.11
Av.	1	0.78	0.95	0.9	1.06

TABLE III. COMPARISON OF THE EXPERIMENTAL VALUE OF β_1 WITH DIFFERENT CODES AND PROPOSED EQUIVALENT STRESS BLOCK

f_c'	**Proposal /Exp.**	**ACI-318 /Exp.**	**Canadian /Exp.**	**EC-2 & Chinese /Exp.**
16.8	0.93	0.95	1.03	0.89
17.99	1.01	1.02	1.11	0.96
20.6	1.02	1.04	1.12	0.98
22.9	1.03	1.05	1.13	0.99
24.8	1.04	1.07	1.14	1.01
25.2	0.96	0.99	1.05	0.93
27.88	1.07	1.1	1.16	1.03
27.9	0.97	1	1.06	0.94
31.3	1.06	1.07	1.15	1.03
32.2	0.95	0.95	1.03	0.93
43.4	0.98	0.9	1.05	0.97
50	1.07	0.93	1.14	1.08
56	0.95	0.78	1	0.96
Av.	1	0.99	1.09	0.98

VI. CONCLUSIONS

In this paper, the energy modeling approach is deployed to investigate both the schematic and equivalent concrete stress-strain of different codes. Experimental results were compared with the results of different schematic and equivalent stress blocks of the current codes. Based on the findings of the energy modeling, a modified equivalent stress block is proposed for the design of normal-strength concrete elements. The results of the proposed equivalent stress block show good agreement with the experimental ones. The mean value of the ratio between the predicted and the experimental values of the total energy is equal to 1 for the proposed equations, while it varies from 0.78 for ACI-318 Code to 1.06 for Chinese Code. The same can be concluded for the results of the β_1 parameter. It is also observed that the results of the considered codes for the total energy and the β_1 parameter are closer in some cases to the experimental results.

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