

# Rational Pile Design using Computer-based Program Coding in Matlab: A Case Study

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## ABSTRACT

Scientific approaches to pile design have made significant progress in recent years. However, despite these advancements, estimating the axial resistance and settlement of piles still heavily relies on empirical correlations. The design of resource-efficient and environmentally friendly piles is a pressing need. Yet, there is no explicit theoretical or practical experience to guide pile design rationally. Typically, determining a pile's resistance and settlement is treated as separate problems without considering the interactions between the pile and the soil. Additionally, soil data are inconsistent due to the heterogeneous and isotropic character of the soil in the half-space under the foundation. In this study, the modified Fellenius Unified method was coded in Matlab and applied to analyze pile behavior, considering the resistance and settlement of each pile, as well as interactions between piles and the soil simultaneously. The results showed that this approach is promising for practical applications. Moreover, its implementation in the evaluation of pile design for an apartment project in Binh Duong, Vietnam, suggests that the pile's length can be reduced even further than it currently is.

*Keywords-pile design; computer-based program; rational design; resistance; pile settlement*

## I. INTRODUCTION

To rationally design piles, it is often necessary to determine the accuracy and representativeness of soil parameters through a large number of tests and the best selection of testing methods, including sampling methods, laboratory test types, and onsite tests. A pile design method should simultaneously estimate the resistance and settlement at the head, toe, and along the pile body. In [1], 63 static pile load tests were examined to validate and refine well-known pile design methods based on published soil properties. Many studies utilized statistical approaches in safety selection to suggest improvements to the existing standards for pile design [2-3]. This approach distinguishes between the variability of pile resistance within individual sites and the global variability on which model correlations are based.

Currently, there are many methods available to estimate the resistance of a pile [4]. This article uses the resistance calculation methods proposed by the Vietnamese National Standard (VNS) [5] to increase practical applicability. The VNS considers the effect of negative friction using an impact factor on the resistance at the pile head. However, the unified method approach for negative friction estimation is also utilized. In [6], it was shown that when dealing with negative skin friction, pile design codes treat the drag force as an unfavorable design load that should be imposed on the pile. These codes increase the value of the drag force while ignoring the shaft resistance above the neutral plane and decreasing the positive shaft resistance below the neutral plane. This means

that pile design in deep, soft clays, such as those found in Singapore and Asian coastal plains, will result in excessively long piles to meet the code requirements. In [7], the level of confidence that can be placed (a) in the conceptual and analytical frameworks to estimate pile resistance and (b) in the quantitative parameters required for design were discussed. From a practical point of view, it is necessary to design approaches that are less sensitive to the estimated pile resistance. In general, resistance is defined by shaft friction resistance and toe resistance. However, several studies considered both resistance and load along the entire length of the pile, from the pile toe to the pile head, to overcome these limitations [4, 8-10].

For settlement estimation, design codes typically estimate pile settlement by considering the settlement of the equivalent raft for the entire pile group [5, 11-12]. This is then assumed to be the settlement of the foundation at the bottom of the raft, which is also at the head of the piles. However, various studies [4, 8-10, 13-14] indicated that pile displacement and deformation can differ from pile head to toe. Traditional settlement calculation methods do not consider the load-settlement behavior at the pile toe or soil-pile and pile-pile interactions. In [15], the philosophy of using piles as settlement reducers was discussed along with the conditions under which such an approach can be successful. In [16], designs for the undrained behavior of a piled raft system criteria were developed based on examining the average and differential settlements, the raft bending moment, and the load ratio to be born by the pile.

In [17-18], an optimal design method was presented to determine the pile length for piled raft foundations. The method considered raft, piles, and soil interaction to evaluate foundation settlement. Settlement analysis was simplified using Steinbrenner's equation, and the total pile length was minimized while considering the settlement constraint. On the other hand, in [19], it was argued that a conventional piled foundation is designed to provide sufficient resistance and limit overall settlement, including controlling differential settlement within acceptable limits. As such, piles are often the same length and size. In [20], the design parameters for the CFG pile compound foundation were empirically investigated using PLAXIS software to evaluate different pile lengths and analyze their relationship with various factors. The pile length was optimized and found that it could be 2 m shorter than the conventional design, reducing costs. Construction practice did not show any problems during operation.

In [21], the optimal conceptual design of pile foundations was studied during the initial design stage, developing a minimum-cost optimization model that considered multiple design constraints based on the Chinese code and a cardinality constraint. This model aimed to achieve the concurrent optimization of pile size and layout. In [22], an optimization problem was proposed to achieve the most economical design of the pile foundation layout. This study compared the performance of two different design procedures in assessing each candidate design obtained during the optimization process. The proposed formulation was validated on a real-world structure. Furthermore, in [23], a user guide was presented for a computer program called PILEOPT, which can help to analyze and design pile foundation layouts. In [24], an automated optimal design method was introduced that used a hybrid genetic algorithm for pile group foundation design. This method aimed to optimize the size and pile arrangement and minimize the volume of the material utilized in the foundation. To achieve this, the piles' configuration, number, and cross-sectional dimensions, along with the thickness of the pile cap, were considered as design variables. The proposed hybrid genetic algorithm successfully minimized the volume of material consumption, and the result aligned with the engineering expectations. In [25], a study on dynamic soil-pile interaction was performed, proposing a model that considered the strong nonlinearity near the pile shaft during dynamic loading. The model engaged the Winkler's hypothesis and paid particular attention to the gap that forms at the soil-pile interface. The former successfully predicted the dynamic pile response observed in field dynamic pile load tests. On the other hand, in [26], different objective procedures were evaluated to reduce uncertainty in the design process. This study focused on three factors: adopting a pile resistance model, selecting a soil strength and soil profile for an ultimate limit state check, and estimating pile head settlement for a serviceability limit state check.

Numerous studies have investigated rational pile design with the primary objective of reducing the amount of materials used. Several constraints, such as resistance, pile settlement, pile arrangement in the foundation, and soil deformation, were considered. Many studies simulated pile behavior using the Finite Element Method (FEM) or elastic solutions combined

with Winkler springs, considering both pile-soil and pile-pile interactions. This study employs the modified unified pile design method that considers down drag - Pile Design with Consideration of Down-Drag (PDWDD) [8-10]. The resistance of the pile was determined using the formula guided by VNS [5], while the settlement of the soil around the pile and the settlement of the pile were determined by the hybrid method, which combines FEM and theory of elasticity.

## II. RESEARCH SIGNIFICANCE

This study uses the pile design of Connect 2 Apartment, Binh Duong Province, Vietnam, to verify four objective functions: the load function that acts on each pile element considering negative friction, the resistance function of the pile, the allowable displacement function of the pile, and the settlement function of the soil causing negative friction. To obtain high applicability for design practice, the resistance function of the pile was determined using the VNS-recommended formulas. This study determines the displacement of each pile instead of calculating the settlement of the entire pile group according to the equivalent raft method, which has no relation to the pile's resistance calculation. The displacement of each pile was determined through the elastic solution pile-soil interaction, in which the pile is simulated by bar elements and the soil by springs attached to pile nodes.

## III. MATERIALS AND METHODS

### A. Load, Resistance, Pile Displacement, and Soil Settlement

The function below defines the load acting along pile length, including down-drag load:

$$f_1(z_i) = P_i + U_{i,n} \Delta \sum_0^{n_{zi}} (6.25N_{i,n}C + 3.33N_{i,n}(1 - C)) \quad (1)$$

The function below defines the resistance of a pile along its length.

$$f_2(z_i) = Q_{ult,i} - U_{i,n} \Delta \sum_0^{n_{zi}} (6.25N_{i,n}C + 3.33N_{i,n}(1 - C)) \quad (2)$$

The ultimate resistance can be calculated by:

$$Q_{ult,i} = \alpha N_{iL} A_{i,n} + U_{i,n} \Delta \sum_0^{n_{Li}} (6.25N_{i,n}C + 3.33N_{i,n}(1 - C)) \quad (3)$$

where,  $i$  is the pile number,  $n$  is the number of soil layers in the range from pile top to pile toe (pile segment),  $n_{zi}$  is the number of soil layers to depth  $z_i$ ,  $n_{Li}$  is the number of soil layers to pile toe of pile  $i$  (to the depth of pile length of  $L$ ),  $\alpha$  is the coefficient depending on pile type (30 for driven, press-in piles, or 15 for bore piles),  $N_{iL}$  is the SPT value at the toe of the pile,  $N_{i,n}$  is the SPT value of  $n$ -th soil layer at pile  $i$ ,  $C = 1$  if it is a clayey layer and  $C = 0$  if it is a sandy layer,  $A_i$  is the cross-section of pile  $i$ , and  $U_i$  is the perimeter of pile  $i$ . Cumulative pile settlement along its length can be estimated by:

$$f_3(Z_i) = S_{allow} - 2\Delta \sum_0^{n_{zi}} (P_{i,n} n / 3E_{i,n} A_{i,n}) \quad (4)$$

Soil settlement along pile length is estimated by:

$$f_4(z_i) = \sum_0^{z_i} w_i \quad (5)$$

where settlement at any point in the subsoil is a sum of settlement caused by load acting at all pile's nodes.

$$w_i = \sum_1^j a_{ij} P_j \tag{6}$$

where  $j$  is the node number,  $S_{allow}$  is the allowable settlement depending on national codes,  $P_{i,n}$  is the external load acting at the pile node,  $E_{i,n}$  is Young's modulus of pile material,  $\bar{G}$  is the average shear modulus, and  $r_{ij}$ ,  $c_j$ ,  $R_{1,ij}$ , and  $R_{2,ij}$  are defined in the Mindlin formula.

$$a_{ij} = \frac{1}{16\pi\bar{G}(1-\nu)} \left[ \frac{\frac{3-4\nu}{R_{1,ij}} + \frac{8(1-\nu)^2 - (3-4\nu)}{R_{2,ij}}}{(z_i - c_j)^2} + \frac{R_{1,ij}^3}{(3-4\nu)(z_i + c_j)^2 - 2c_j z_k} + \frac{R_{2,ij}^3}{6c_j z_i (z_i + c_j)^2} + \frac{R_{2,ij}^5}{R_{2,ij}^5} \right] \tag{7}$$

Functions (1), (2), (4), and (5) are used to simulate pile behavior, both settlement and resistance.

**B. Mathematical Model**

The goal is to determine the length  $L$  of the piles such that functions (1) and (2) and functions (3) and (4) are equal.

$$f_1(z_i) = f_2(z_i) \tag{8}$$

$$f_3(z_i) = f_4(z_i) \tag{9}$$

Equation (8) can be solved to find  $z_{(1,2)}$ , and (9) can be solved to find  $z_{(3,4)}$ . Adjust so that  $z_{(1,2)} \geq z_{(3,4)}$ . In case  $z_{(1,2)} = z_{(3,4)}$ , the pile is considered to have the optimal length.

**C. Matlab Modeling**

The Matlab model was based on the one described in [10]. The computation of functions (1), (2), (4), and (5) were coded in Matlab. For simplification, the soil was characterized by a single parameter, N-SPT. The N-SPT value can be calibrated by the results of static loading and strain tests conducted on two test piles, TP01 and TP02, and should be compared and adjusted with the settlement monitoring findings during the calculation. The input data required for this analysis include the coordinates of the pile, its length and diameter, the number of piles and pile elements, the N-SPT number of each soil layer, the N-SPT value under the pile tip, Poisson's index of the soil, allowable settlement of the pile, and elastic modulus of the pile material. The pile head load can be calculated using the SAFE software. This study used the load in the designer calculation sheets for comparison. The friction resistance (negative and positive) was calculated using the VNS formula, which is similar to that proposed by the Japanese Architectural Institute. The load and down-drag force due to negative friction were calculated for each pile node according to (1). The pile resistance at each node was determined using (2). The displacement at the pile head was taken as the allowable displacement according to VNS. The displacement at the pile tip was calculated from the pile head displacement minus the elastic deformation, according to (4). The soil settlement at any node was computed using the first Mindlin solution. The soil

settlement at any analyzed node along the pile equals the sum of the settlement caused by loads acting at all other nodes in the pile group through the Mindlin solution. The cumulative settlement of the analyzed node was calculated using (5), counting from the pile tip.

**IV. THE APARTMENT BUILDING CONNECT 2**

**A. The Connect 2**

Connect 2 consists of two blocks. Drilling of boreholes of test piles started in May 2020, followed by basement excavation and foundation construction in October 2020. The superstructure construction began in November 2021, and all construction work ended in April 2023. During construction, no accidents occurred, and to this day, the apartment building is in regular operation. During the design period, after reviewing the design, the owner requested to minimize the pile's cost. The building is a 30-story tower that is 97.25 m high. Its foundation was designed as a pile group consisting of 177 piles. Among these piles, 5 are 1 m in diameter and 45 m long, 122 are 1 m in diameter and 52 m long, and 50 are 1.2 m in diameter and 57 m long. The piles with a length of 45 m have their toes located in coarse sand, which is dense to very dense, with an N-SPT of about 36. The piles of 50 or 57 m long have their toes in medium-sized sand, which is yellow to white-grey and dense, with an N-SPT value of about 37. The underground water level is at a depth of 11 m. Figure 1 shows the arrangement plan for the piles. The red marked piles, which are 57 m long and have a diameter of 1.2 m, were placed under the stairs and elevator case. The green and blue marked piles, which were 50 m long and had a diameter of 1 m, were arranged under the column raft and the shear wall. Five uncolored piles were placed under the swimming pool, measuring 45 m long and 1 m in diameter. Table I displays the N-SPT value by depth.

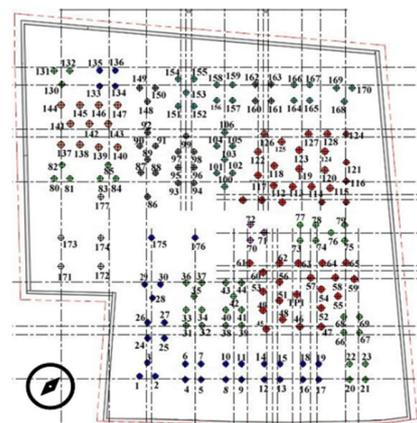


Fig. 1. Pile arrangement.

Static loading and strain tests were performed on two piles, TP01 and TP02, by the Vibrobis company, which has its headquarters in Ho Chi Minh City. The tests started from the end of May to the beginning of June 2020. The load was applied to the pile in two cycles, 100%, and 200% design resistance, complying with VNS. The strain gauges were mounted on the test piles and conducted in parallel with the

static loading test. There were 21 strain gauges mounted at seven points along the pile body, with a distance of 7-10 m between each point. The test results are summarized as follows:

- Test pile TP01 had a diameter of 1.2 m and a length of 62.3 m. Static loading testing, combined with strain measurement, showed that when the pile was loaded with 27,000 kN (equivalent to 200% design load), the load distributed for the pile tip was only 1,010 kN, about 3.7% of the pile head load. During the test, the pile head moved 34.12 millimeters. After comparing the test results with the calculations, the design engineer concluded that the resistance of piles in groups should range between 12,500 and 13,000 kN.
- Test pile TP02 had a diameter of 1.0 m and a length of 53.3 m. The static loading and strain measurement test results showed that with a test load of 18,000 kN (equivalent to 200% design load), the distributed load at the pile tip was only about 680 kN. The displacement at the pile head was 27.7 mm. The designer evaluated and determined the pile's resistance in groups ranging from 9,000 to 9,500 kN.

TABLE I. SOIL PROPERTIES

No.	Soil layer	Depth (m)	N-SPT
1	Sandy clay, medium stiff	10-16	13.5
2	Fine sand, medium-dense	17-32	16.1
3	Clay, stiff	33-36	20.25
4	Fine sand, medium-dense	37-39	17.5
5	Clay, red-brown, stiff	40-50	32.7
6	Sandy clay, medium stiff	51-53	32
7	Coarse sand, very dense	54-56	41.3
8	Clay, stiff	57-58	64
9	Medium sand, dense	59-67	40

B. Results Analyzed by Matlab

The analysis using Matlab was conducted on a total of five piles. It was found that piles No. 1, No. 21, No. 131, and No. 170, located at the corners of the building, had displacements of 3.4 mm to approximately four mm. Figures 2, 3, 4, 5, and 6 visually represent the relationship between load, resistance, pile displacement, and soil settlement of the piles. A pile is considered capable of supporting the load and meeting the allowable settlement requirement when the curve of allowable settlement (y3) of the pile intersects with the curve of soil settlement l (y4) at a depth smaller than the intersection of the load curve (y1) and pile resistance curve (y2).

Figure 2 shows the correlation between load, resistance, pile displacement, and soil settlement for pile No. 1. The load and resistance equilibrium plane is located at an elevation of -38.1 m, while the pile displacement and soil settlement equilibrium plane is at an elevation of -36.8 m. The results show that the pile is capable of bearing the load and that the settlement of the pile is approximately 3.4 mm. Figure 3 shows the correlation between load, resistance, pile displacement, and soil settlement for pile No. 21. The results indicate that the pile is capable of bearing the load (the load and resistance curves intersect at -38.3 m deeper than the intersection of the pile and soil settlement curves at -25.7 m). The soil settlement is approximately 3.6 mm.

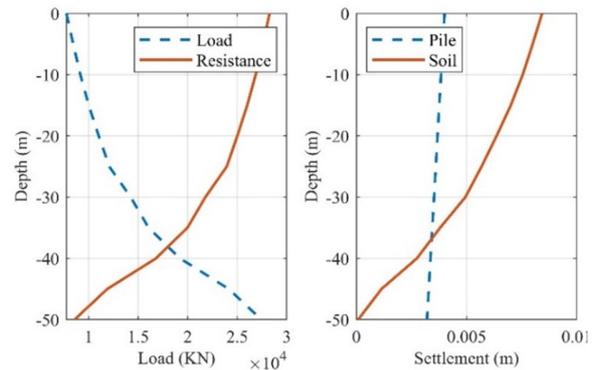


Fig. 2. Pile No. 1: load-resistant and pile-soil settlement relationship.

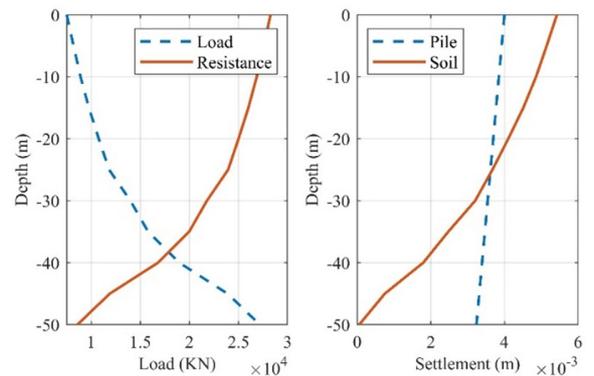


Fig. 3. Pile No.2: load-resistant and pile-soil settlement relationship.

Figures 4 and 5 show that the piles located at the remaining two corners, No. 131 and 170, respectively, are also capable of supporting the load (38.7 vs. 39.7 m and -39.4 vs. -30.1 m), with settlements of 3.5 and 3.7 mm, respectively. For pile No. 131, if the allowable settlement is approximately 4.0 mm, the pile resistance will be greater than the pile load (the load and resistance curves intersect at -38.8 m at a lower elevation than the intersection of pile and soil settlement curves at -37.7 m), indicating it is capable of bearing load. Figure 6 displays the behavior of pile No. 57, which is located beneath the raft of the elevator core. The pile can bear the load because the load and resistance curves intersect at -38.3 m deeper than the intersection of the pile and soil settlement curves at -25.7 m. The settlement of this pile is only 3.5 mm.

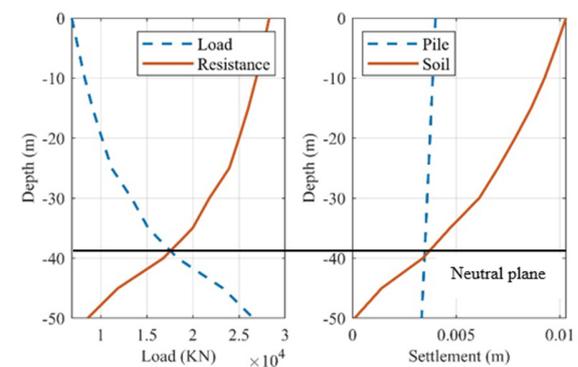


Fig. 4. Pile No. 131: load-resistant and pile-soil settlement relationship.

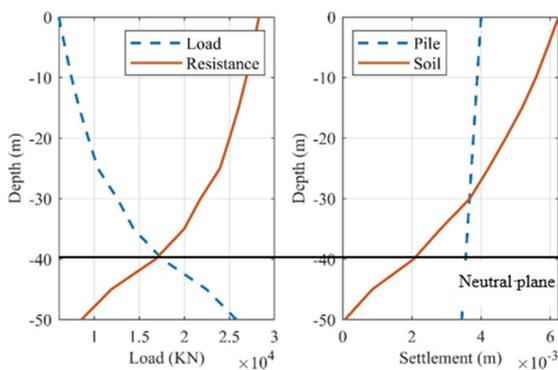


Fig. 5. Pile No. 170: load-resistant, pile-soil settlement relationship.

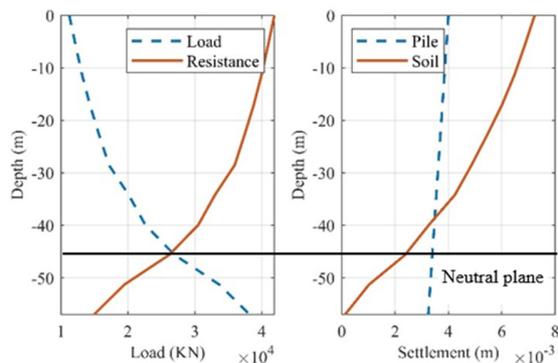


Fig. 6. Pile No. 57: load-resistant and pile-soil settlement relationship.

The results of analyzing 177 piles in the studied pile group using Matlab showed that the average settlement value was determined to be 3.56 mm, and the differential settlement was approximately 1 mm. The settlement monitored during the 16th measurement period on April 19, 2023, was recorded between 1.49 and 2.39 mm. It should also be noted that this is the monitoring result right after the project was completed and handed over for use, so the settlement is still not completed. Over time, the settlement of the soil can become much more. In addition, the settlement calculated from the program is elastic so that the actual soil settlement may reach a more significant value in the long run. However, it can be said that the pile group was designed safely and met all VNS requirements. Comparing the computed results of 3.6 mm with the static loading test results of about 10 mm at 100% design load and the settlement at the monitoring period at the end of construction of about 2 mm, it can be concluded that they are relatively consistent. Although the final monitoring results are expected to be delivered six months and one year later, it is evident that this research direction shows promise.

### C. Discussion

This study used the statistical N-SPT value, as suggested by experimental engineers, to determine the behavior of the soil. However, it should be noted that the calculated N-SPT value may vary significantly due to factors, such as energy loss caused by friction between the penetrating rod and the soil, the experimenter's experience, and friction between the falling hammer and the shaft. The graphs in Figures 2-6 show that the curves of functions (1), (2), (4), and (5) do not intersect in a

single neutral plane. Therefore, an optimal pile design analysis should be further studied.

This analysis estimates soil settlement based on the elastic pile-soil interaction. The two factors that affect soil settlement are the load applied at the pile nodes and the elastic modulus of the soil. The load distributed to pile nodes was determined according to Coyle and Reese (1966). The value of the load acting at the pile tip was calculated from tip settlement and soil modulus, which is calibrated with the results of loading tests and strain tests at two test piles, TP01 and TP02, for problem simulation. The settlement of soil under the raft that caused negative friction was estimated based on Mindlin's first solution. Although the estimation of soil settlement under the raft was based on elastic modulus, it does not reflect the actual settlement of the soil. However, according to many studies, the elastic simulation of soil by springs fully represents the pile-soil interaction. Based on a preliminary analysis of pile No. 57, it was observed that the elastic Young modulus variation resulted in different soil settlement values. If the value of Young's modulus equaled the value of the modulus of deformation, the estimated soil settlement at ground level was approximately 82 mm and the pile displacement was 4.4 mm. However, if the elastic modulus used is calculated by  $E_{elas.} = 1000 \times N$ , the pile displacement is only 3.7 mm, and the ground surface settlement is 7.3 mm.

A pilot study was carried out for a single pile cap, PC09, which consists of 11 piles with a diameter of 1.0 m and a length of 52 m, of the same project currently under study. The analysis conducted in pile No. 35 showed that the load equilibrium plane was at an elevation of -45 m, the settlement equilibrium plane was at a level of -15 m, and the settlement was found only 2 mm. However, this analysis only considered 7 of the total 177 piles in the foundation, leading to incorrect results. When considering the interaction of all 177 piles, the load equilibrium plane was at an elevation of -45 m, the settlement equilibrium plane was at a level of -38 m, and the settlement was 3.9 mm. This indicates that the pile should consider the entire pile-soil interaction of all piles to produce results closer to reality. Therefore, it is essential to consider the radius of influence of pile-soil interaction when designing piles for large foundations, which will be incorporated into the computer program being developed.

A project verification company suggested shortening the piles to reduce the total length from 9,669 m (according to the initial design) to 8,765 m, saving 904 meters. After considering the proposal, the design company decided on a total pile length of 9,175 m, offering significant cost savings. It is important to note that there is no established theoretical basis for reducing pile length before construction, even though actual construction later proves the feasibility. The computed results showed that the load and settlement equilibrium planes of the 177 piles were at various elevations. Therefore, it is necessary to match the equilibrium planes to optimize the pile lengths. Using the pile design method, considering PDWDD combined with Matlab, engineers can design piles more reasonably, as it can analyze pile resistance and settlement simultaneously and provide a theoretical basis for designing piles. This method can also help optimize pile lengths.

## V. CONCLUSION

Using the Matlab-coded PDWDD method to analyze the resistance and settlement of piles simultaneously while also considering long-term negative friction provides engineers with a sense of security when designing piles. This study investigated the behavior of the 177 piles in the foundation system of the selected construction project, drawing the relationships between the resistance and load and between the pile's allowable displacement and soil settlement. The loads at one pile node cause the settlement of the soil around other nodes in the pile group, which in turn causes negative friction to the nearby piles. This negative friction is the main reason for the long-term displacement of the pile. Based on the initial findings, the settlement calculated by the program (~3.56 mm) matches well with the monitoring results (~2.1 mm). The computing time to calculate the resistance and settlement of the project's pile group was approximately two minutes. Therefore, the proposed computer-based program promises to have practical applications.

According to the findings, the total length of the pile system used in the project can save more than 494 m compared to the initial design that followed VNS. The Matlab technique demonstrates that the length of piles in the pile group can be reduced even more. Most importantly, the technique has provided engineers with a theoretical basis for pile behavior to select suitable economical design options. Shortening the pile length caused the elevation of the load and the settlement equilibrium planes to change. For these two planes to coincide, the settlement of the pile must be increased within the requirements of the codes. Therefore, accepting a significant settlement can result in a reasonable pile length. The elastic settlement of the soil is usually small, so engineers use their experience to predict long-term settlement. This study provides a theoretical foundation for designing appropriate piles and predicting long-term settlement. In future studies, research will be carried out to enhance the Matlab program, calibrate soil characteristic parameters, and verify the program through actual projects.

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