The Efficiency of Surface Impedance Technique in the Transverse Wave Approach for the EM-Modeling of Fractal-Like Tree Structure used in 5G Applications

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ABSTRACT

Fractal antenna technology is a promising approach for 5G applications because its complex nature offers optimization potential in terms of time and space trade-offs. However, the computational effort required to analyze such antennas is significant. This paper investigates the Advanced Transverse Wave Approach (ATWA), which utilizes the surface impedance technique to improve simulation efficiency. This study introduces and analyzes a fractal-like 5G tree structure, displaying improved computational accuracy and efficacy, as well as peak memory utilization compared to current works. The proposed approach demonstrates significant effectiveness in enhancing the performance of complex fractal antennas for 5G technology and shows promise for integration with cloud, fog, and edge computing environments. This integration could potentially optimize data processing and network efficiency in these advanced computing landscapes.

Keywords-5G fractal-like tree; advanced transverse wave approach; surface impedance technique; computational efficiency; cloud computing; fog computing; edge computing

I. INTRODUCTION

In the dynamic landscape of wireless communications, the perpetual quest for innovation drives researchers and engineers to continually improve the efficiency of antennas. Fractal antennas are a breakthrough that has captured considerable attention. Drawing inspiration from the captivating realm of fractal geometry, these antennas have ushered in a revolution in wireless communications. Fractal antennas emerge from the fusion of fractal geometry with antenna design. The concept revolves around creating antenna structures that replicate themselves on diminishing scales, reflecting the self-replicating nature of fractal patterns [1-3]. This innovative approach enables fractal antennas to exhibit extraordinary characteristics, including compact size, broadband frequency coverage, and enhanced performance. Fractal antennas offer several key advantages. First, their compact size stands out in contrast to traditional antennas that require specific lengths for resonance. This compactness is particularly valuable in devices with limited space, such as smartphones and laptops [4]. Second, their broadband capability allows them to cover a wide range of frequencies simultaneously, which is essential in modern communication systems using multiple bands, such as Wi-Fi, GPS, and cellular networks [5]. Third, fractal antennas are designed for efficient operation in multiple frequency bands, demonstrating crucial versatility in supporting various wireless communication standards [6]. Lastly, their intricate and self-similar structures contribute to enhanced radiation patterns and reduced interference, leading to superior signal quality and reception [7].

Fractal antennas find diverse applications in different industries. They are widely used in wireless communications, ensuring efficient data transmission. In the field of radio astronomy, fractal antennas serve as valuable tools capable of capturing signals from celestial sources by covering multiple frequencies [8]. In military and defense, these antennas are employed in communication systems and radar applications, offering adaptability to different frequencies in a compact form factor. The medical field benefits from fractal antennas, as they play crucial roles in wireless telemetry devices, remote monitoring systems, and even medical imaging equipment [9]. In addition, the automotive industry has adopted fractal antennas for applications, like GPS, satellite radio, and keyless entry systems [10]. Recent advances in fractal antenna research showcase the potential for transformative solutions in the ever-progressing field of wireless communication. Notable studies focus on optimizing fractal-based antennas for 5G networks, addressing challenges related to accommodating multiple frequency bands and the need for compact and efficient designs [11]. Miniaturization efforts, particularly for portable and IoT devices, have been explored, revealing that fractal geometries allow the creation of compact antennas without compromising performance [12]. The versatility of fractal antennas in covering a wide range of frequencies was emphasized in multiband and wideband operation, addressing the demands of various wireless communication standards [13]. In the realm of satellite communication, fractal antennas contribute to enhancing the efficiency and reliability of satellite links, supporting global connectivity [14]. Furthermore, the integration of 3D printing technology into fractal antenna design has been investigated, demonstrating cost-effective and customizable solutions for various wireless communication applications [15]. These recent studies underscore the evolving landscape of fractal antennas, affirming their potential to address contemporary challenges and shape the future of wireless communications.

Simulating and analyzing the performance of fractal antennas is a vital step in their design and optimization, with computational time standing out as a crucial aspect impacting the efficiency of the design process. The complexity of fractal geometry, especially with intricate self-replicating patterns, increases computational demands, particularly for designs such as the Koch or Sierpinski curves [16]. The wide frequency range and bandwidth coverage of fractal antennas require comprehensive simulations, which affect computational time, especially for wideband scenarios or multiband operations. The choice of simulation software and algorithms, along with considerations for mesh density and accuracy, further influences computational time. Hardware resources, including high-performance computing clusters and GPUs, play a significant role in reducing simulation time for complex fractal antenna designs. Specific simulation goals, iterative optimization, and sensitivity analysis add layers to computational time considerations, emphasizing the need for a balanced approach between result accuracy and available resources to harness the potential of fractal antennas for wireless communications. Numerical electromagnetic (EM) methods play a pivotal role in the realm of 5G planar structures, serving as the foundational framework. These methods encompass various techniques, among which are the Method Of Moments (MOM) [17] with its alternative Fast Multipole Method (FMM) and the extensive Partial Element Equivalent Circuit (PEEC) method specifically designed to tackle integral equations related to EM waves. In addition, methods, such as the Finite Element Method (FEM), the Finite Difference Time Domain (FDTD) method, and the Discontinuous Galerkin Method (DGM) [18-21] serve as differential equation solvers. These numerical approaches require a lot of memory and processing power, combined with the requirement of high accuracy in the functions and design. This is particularly important for performing EM-modeling of planar structures used in wireless systems.

This study investigates the Advanced Transverse Wave Approach (ATWA) [22-24], which stands out from previous numerical EM methods due to its notable advantages in terms of speed, compactness, and memory efficiency, introducing the surface impedance technique to tailor ATWA for fractal 5G antenna simulations and enhance its adaptability. Thorough tests were performed to assess the efficiency and stability of the adapted ATWA approach within this specific domain. Drawing inspiration from the aforementioned studies, a Fractal-Like Tree Structure was used as a suitable prototype, to validate the proposed approach in the context of 5G applications. This prototype provides a foundation for investigating more intricate antennas, contributing to the advancements in ongoing wireless technology.

II. THEORETICAL FRAMEWORK OF ATWA

This section addresses challenges in dealing with EM radiation and scattering by acknowledging the difficulty of obtaining analytic solutions for integral equations. As a solution, computational techniques are employed, leading to a concise exploration of the mathematical foundation and theoretical framework of the ATWA. ATWA, a numerical EM method, is characterized by its efficiency and strengths. The method, specifically designed for full and millimeter-wave analysis of planar structures, stands out for its avoidance of matrix inversion, freedom from constraints on component forms, prevention of numerical instabilities linked to large matrices, and assured convergence irrespective of planar structure interfaces. The fundamental integral relation connecting the electric field and current density is expressed as:

$$E(p,q) = \int_S G(p,q,p',q')J(p',q')dp'dq' \qquad (1)$$

where $G(p, q, p', q')$ is the dyadic Green's function. Derived from the wave concept, the equation linking transverse electric field E_T to transverse magnetic field H_T is:

$$\begin{vmatrix} \vec{A}_r \\ \vec{B}_r \end{vmatrix} = \widehat{K} \begin{vmatrix} \vec{E}_T^r \\ \vec{H}^r{}_T \times \vec{n}_r \end{vmatrix} = \widehat{K} \begin{vmatrix} \vec{E}_T^r \\ \vec{J}_T^r \end{vmatrix} \qquad (2)$$

$\widehat{K}$ guarantees the transition from integral EM field to algebraic EM waves.

$$\widehat{K} = \frac{1}{2} \begin{vmatrix} Z_{0r}^{-\frac{1}{2}} & Z_{0r}^{\frac{1}{2}} \\ Z_{0r}^{-\frac{1}{2}} & -Z_{0r}^{\frac{1}{2}} \end{vmatrix} \qquad (3)$$

The wave impedance Z_{0r} for homogeneous isotropic region r is:

$$Z_{0r} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r_r}}} \qquad (4)$$

where μ_0 is vacuum permeability, ε_0 is vacuum permittivity, and ε_{r_r} is the dielectric constant from region r. The incident (A) and reflected (B) waves are related by:

$$\begin{cases} A = \hat{\Gamma} B & \text{(modal domain)} \\ B = \hat{S} A + \mathrm{B}_{(0)} & \text{(spatial domain)} \end{cases} \tag{6}$$

where $B_{(0)}$ denotes the overall excitation wave at the source.

The shift from spatial to modal spaces entails implementing the two-Dimensional Fast Fourier Transform (2D-FFT) or the two-Dimensional Non-Uniform Fast Fourier Transform (2D-NUFFT) at the discontinuity interface for accelerated iterative processes. Transforming from spectral to modal spaces includes specific rotation as defined in [22, 25]. This iterative process continues until system convergence. The 2D-TWA is expedited by an anisotropic mesh technique, ensuring both memory and computational complexity for the Forward and Backward of the ATWA process at O(Nlog$N$), N being the meshing density, which is faster than conventional computational EM methods such as MOM or FEM.

A. Surface Impedance Technique

Considering (1) and (2), the Green operator, in the case of homogeneous media, can be viewed as an impedance operator. The complex surface impedance establishes a connection between the electric field and the current density within any subdomain delineating the discontinuity plane. The behavior of the surface impedance subdomain, denoted by Z_S, resembles that of a metallic domain when Z_S is zero and functions akin to a dielectric domain when Z_S approaches infinity. Within this surface impedance subdomain, the continuity and boundary conditions of transverse EM fields in LIM media necessitate the equality of electric fields on both sides of the interface, linking the transverse electric field to the total tangential current density in the following manner:

$$\begin{cases} E_T = E_T^{r=1} = E_T^{r=2} \\ J_{T_{total}} = J_T^{r=1} + J_T^{r=2} \\ E_T = Z_s \cdot J_{T_{total}} \end{cases} \tag{7}$$

In terms of waves:

$$\begin{cases} Z_{01}^{\frac{1}{2}} \times (A_1 + B_1) = Z_{02}^{\frac{1}{2}} \times (A_2 + B_2) \\ \left(Z_{01}^{\frac{-1}{2}} \times (A_1 - B_1) + Z_{02}^{-\frac{1}{2}} \times (A_2 - B_2) \right) \cdot Z_s = Z_{01}^{\frac{1}{2}} \times (A_1 + B_1) \end{cases} \tag{8}$$

From (6), the expression of $S_{\Omega_{SUB}}$ on the surface impedance subdomain $\Omega_{SUB=I_{Surf}}$ becomes:

$$S_{\Omega_{SUB=I_{Surf}}} = \begin{bmatrix} \frac{Z_S \times (1-\rho^2) - Z_{01}}{Z_S \times (1+\rho^2) + Z_{01}} & \frac{2\rho \times Z_S}{Z_S \times (1+\rho^2) + Z_{01}} \\ \frac{2\rho \times Z_S}{Z_S \times (1+\rho^2) + Z_{01}} & \frac{Z_S \times (\rho^2 - 1) - Z_{01}}{Z_S \times (1+\rho^2) + Z_{01}} \end{bmatrix} \tag{9}$$

where:

$$\rho = \sqrt{\left(\frac{\mu_{r_1} \varepsilon_{r_2}}{\mu_{r_2} \varepsilon_{r_1}} \right)^{\frac{1}{2}}} \tag{10}$$

III. FRACTAL-LIKE TREE STRUCTURE FOR 5G

The fractal-like tree structure for 5G is a specialized antenna design crafted to meet the specific requirements of 5G wireless communication systems. Drawing inspiration from fractal geometry, it integrates self-replicating patterns to optimize antenna performance. With a tree-like configuration, it offers distinct advantages, including compactness, multiband capability, and efficient operation across a wide frequency spectrum. Tailored to address the challenges of modern wireless networks, this antenna holds significant promise in enhancing data rates, expanding network coverage, and overall efficiency within the rapidly evolving landscape of 5G technology. This antenna was chosen to validate the proposed approach and demonstrate its effectiveness. The evolution of fractal-like trees is achieved through recursive processes. Every terminal segment endeavors to spawn fresh branches extending forward, at a 45° angle to the left, or a 45° angle to the right. The integration of a segment into the structure is contingent upon its noncollision with any existing or prospective segments. Different colors serve to delineate different stages in the generation process, as shown in Figure 1.

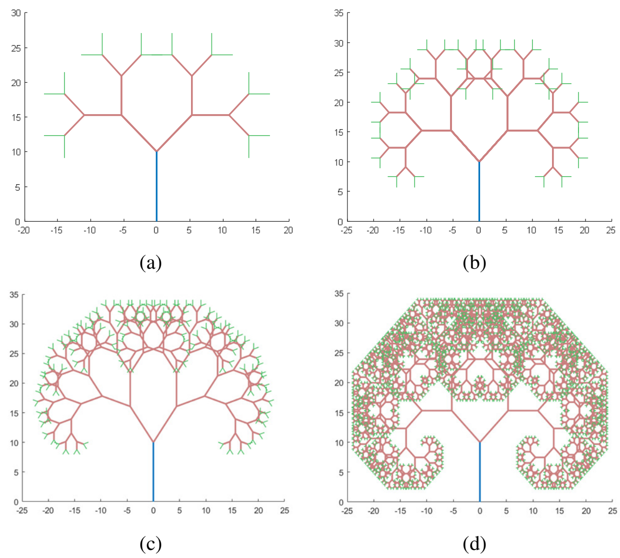


Fig. 1. Fractal-like antennas with scale factor 0.75 and angular vector $\pi/4$ at different iterations: (a) n = 4, (b) n = 6, (c) n = 8, (d) n = 12.

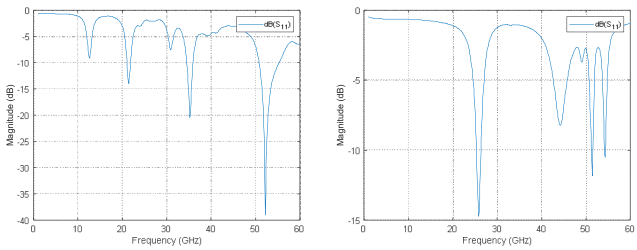
IV. SIMULATION RESULTS AND DISCUSSION

Two methods were used to examine and enhance the fractal-like tree structure of the antenna across varying iterations (n = 4, 6, 8, 12) within the domain of 5G technology: the direct ATWA-based solver and the ATWA method incorporating the Surface Impedance Technique (SIT). In the initial phase, the computation of the complex value of surface impedance (Z_s) was executed for each pixel using (3) and [27]. The mean average square of Z_s was subsequently employed to characterize the equivalent fractal zone on each pixel. As an illustrative example, for the antenna-like tree with n = 8 iterations, the computed Z_s manifested as 0.737i.

A discernible correlation emerged when comparing the direct and SIT methods. The reflection coefficient (S11) demonstrated multifrequency bands, notably at [21, 23.1] and [50.5, 55.8] GHz for the direct method, and [24.9, 28] and [50.8, 51.5] GHz for the SIT technique. The resonance frequencies were closely aligned at 22 and 52.2 GHz for the direct method and 26 and 51 GHz for the SIT approach. The corresponding return losses were determined at -14.3 and -38.7 dB for the direct method, and -14.83 and -12.2 dB for the SIT approach. The relative error between the two methods was approximately 1.7%, a deemed acceptable margin considering the computational and memory demands intrinsic to fractal systems. The occurrence of double resonances was ascribed to the inherent fractal nature of the antenna, a validation supported by simulation results based on 2D-ATWA. The Voltage Standing Wave Ratio (VSWR) at 1.027 dB within the 51 GHz band showcased favorable impedance characteristics, particularly pertinent to 5G wireless systems. An in-depth review of the simulation results, encompassing the impedance evolution across the [1, 60] GHz frequency range, as well as azimuthal, elevational, and 3D patterns at 51 GHz, accentuated the validation of the antenna through the chosen approach. When combined, the simulation results robustly confirm the efficacy of the SIT technique in the EM analysis of fractal-like tree structures within the domain of 5G technology.

TABLE I. FRACTAL-LIKE TREE DESIGN AND MODELING PARAMETERS

Design Parameters	**Description**
Length	7.3185 mm
Height	0.12227 mm
Number of Stages (Iteration)	4
Ground plane length	8.1297 mm
Ground plane width	8.945 mm
Feed diameter	0.16795mm
Substrate (dielectric)	RO4725JXR $\varepsilon_{r_1} = 1$ $\varepsilon_{r_2} = 2.55$ Loss tangent= 0.0022 Thickness= 0.12227 mm
Meshing	512×512
Modeling Parameters	**Description**
Nature of box	Periodic walls
Type of polarization	Bilateral in x-direction
Number of iterations	$N_{iter} = 200$
Value of surface impedance	Z_S=0.737i
Waveband	F_{min} = 1 GHz F_{max} = 60GHz $Step_{F_{rq}}$ = 0.1 GHz



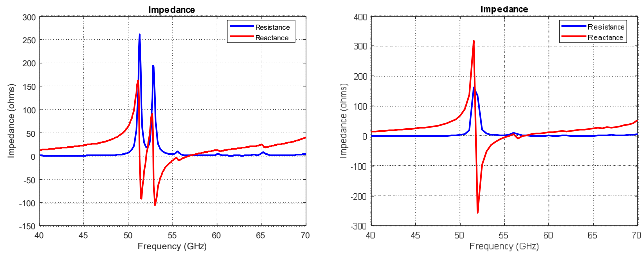
(a) (b)

Fig. 2. Evolution of insertion parameter S11 (dB) with a comparison between (a) direct method and (b) SIT.

Table II compares the performance of the selected antenna with two benchmark antennas, focusing on the multiband functionality, which is essential for Ultra-Wideband (UBW) and 5G networks. This comparison goes beyond basic efficiency metrics, showcasing the proposed antenna's superior bandwidth and gain. The results highlight the significant advantage of the proposed antenna in handling the diverse frequency demands of modern UBW and 5G applications, demonstrating its potential as a robust solution in the evolving landscape of wireless communication. Furthermore, considering the effectiveness of both the SIT and the AMT techniques [28] introduced by the ATWA approach, a promising avenue is created for exploring compact and responsive fractal structures applicable to emerging 5G wireless technologies.

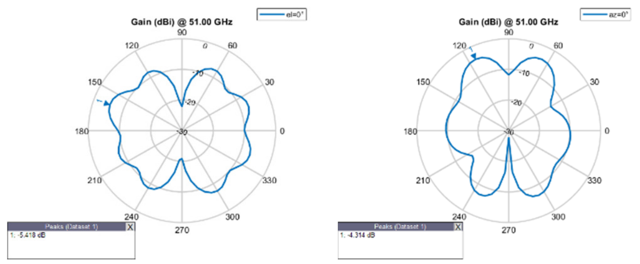
TABLE II. COMPARISON BETWEEN THE PROPOSED FRACTAL-LIKE TREE STRUCTURE AND REFERENCE ANTENNAS

Antenna/ Parameter	**Proposed antenna**	**[29]**	**[30]**
Type	fractal-like tree	fractal-like tree	fractal-like tree
Feeding Technique	CPW	CPW	CPW
Application	UWB, 5G	UWB, 5G	WLAN, Wi-Fi
Angular Vector	π/4	π/2	π/3
Scale Factor	0.75	0.7	0.7
Iteration level	4	2	2
Substrate	RT/duroid 5880	RT/duroid 5880	*FR4_epoxy*
Size (mm)	13.3×7.43×0.58	90×110×16	13×8×1.58
Simulator	Proposed simulator	HFSS	HFSS
Numerical EM method	A-TWA	FEM	FEM
Mode of Operation	multiband	multiband	multiband
Impedance bandwidth	$S_{11} \leq -10\ dB$	$S_{11} \leq -10\ dB$	$S_{11} \leq -10\ dB$
Ranges (GHz)	(21–23.1), (34–36.2), (50.5–55.8)	(1.6–7.5), (8–10), (10.5–12.2), (12.7–16.1)	(1.12–2.3), (2.4–3.6)
VSWR	< 2	< 2	< 2



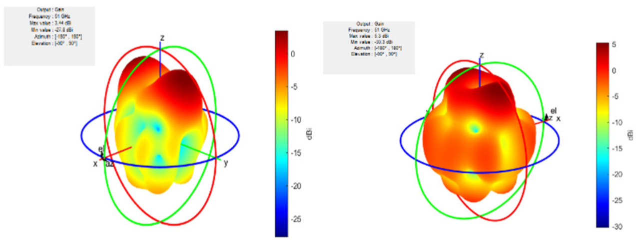
(a) (b)

Fig. 3. Evolution of the impedance as a function of frequency with a comparison between (a) direct method and (b) SIT.



(a) (b)

Fig. 4. Azimuth pattern at 51GHz with a comparison between (a) direct method and (b) SIT.



(a) (b)

Fig. 5. 3D-pattern at 51GHz with a comparison between (a) direct method and (b) SIT.

V. CONCLUSIONS

This study presented a succinct theoretical framework and mathematical underpinnings for the proposed 2D-ATWA, incorporating the SIT for comprehensive full and mm-wave applications. Design and analysis of a 5G fractal-like tree antenna were executed precisely, and rigorous validation, evaluation, and comparison of simulation results were performed against both the direct and SIT-based ATWA approaches. The results emphasized the EM stability and the efficiency of the SIT technique when applied to the analysis of fractal-like tree structures within the 5G technology landscape. The integration of enhanced ATWA with an anisotropic mesh technique holds promise for providing effective solutions for stabilizing and selecting optimal parameters, especially in the intricate domain of 5G fractal antennas. The improved efficacy of the ATWA method in modeling complex structures may be crucial to developing antennas for cloud, fog, and edge computing environments. These computing paradigms often require advanced wireless communication technologies, such as 5G, to ensure high-speed and reliable connectivity. The fractal-like tree structures modeled in this study can support dense, distributed networks characteristic of cloud, fog, and edge computing, offering optimized coverage and bandwidth capabilities. Moreover, the emphasis of ATWA on computational efficiency aligns well with the resource-constrained environments of fog and edge computing, where processing power and memory are at a premium. This fact will be investigated in future work. Furthermore, in future work, the goal is to use cloud computing resources to model larger problem sizes. This enables the exploration of more complex fractal-like structures and their applications in advanced 5G networks. Utilizing cloud computing's vast processing and storage capabilities will allow for more detailed and comprehensive simulations, further enhancing the practicality and applicability of the ATWA method in real-world scenarios.

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