

The Effect of Opening Size and Expansion Ratio on the Flexural Behavior of Hot Rolled Wide Flange Steel Beams with Expanded Web

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ABSTRACT

In some design cases, such as in castellated beams, cellular beams, or steel beams with expanded web, it is advantageous to strengthen and enhance the performance of steel beams by increasing the web depth. However, expanding the web of a steel beam is a unique engineering strategy. This modification not only enhances its ability to bear heavy loads but also reduces any bending or flexing while enabling longer distances between supports. The expanded steel beams can be achieved by making a horizontal cutting in the steel beam web and then adding an increment plate (with the same web thickness), called spacer plate, between the two halves of the web sections, thus improving stiffness and strength. To evaluate the behavior of such beams, nine specimens of HEA steel beams with expanded web ratios of 150%, 200%, and 250%, and one specimen as a reference beam, were fabricated. The specimens were evaluated under two-point load over a clear span length of 280 cm. From the experimental work it was found that for steel beams with expanded web that have 18 openings with 80 mm, the beams with an expansion ratio of 150% had the best performance according to load-deflection behavior with a reduction in load capacity by only 11%. Additionally, the beams with expansion ratios of 200% and 250% had no economic viability according to the analysis of load-deflection behavior with a reduction in load capacity by 49% and 62%, compared with the solid expanded steel beam with the same expansion ratio. For the second type of steel beam with expanding web with 9 openings of 160 mm width, the beams with expansion ratios of 200% and 250% were observed to perform better than the first type with a reduction in load capacity by 28% and 50%. Using larger opening width and a smaller opening number is considered the best option for beams with expansion web ratios of 200% and 250%, while on the contrary, using smaller opening width and higher numbers seems to be a better option for beams with 150% expansion web ratio. Expanding the depth of a steel beam provides design flexibility, reduces deflections, and allows for longer spans. These benefits enhance material efficiency and open up new architectural possibilities.

Keywords-expansion ratio; Vierendeel mechanisms; web post-buckling; web opening ratio

I. INTRODUCTION

Large-scale steel structural projects like parking structures, supermarkets, and storage facilities have frequently used beams with expanded web depth. Expanding the web of a steel beam is a unique engineering strategy to enhance the strength and overall performance of structures. Not only does this modification ameliorate its ability to bear heavy loads, but also reduces bending or flexing while enabling longer distances between supports compared to the original section [1, 2]. In some design cases, it is advantageous to strengthen the steel beams by increasing the web depth, e.g. in castellated beams, cellular beams, or expanded steel beams [2]. However, the presence of an opening in steel beams affects beam stiffness

and causes an increase in the deflection, decreases the beam capacity, and increase the stress at the corners of the opening [3]. Castellated beams and cellular beams are steel (I or H) section members with square, rectangular, circular, or hexagonal shape openings, positioned at regular intervals along the web [5]. Compared to solid beams, beams with openings offer advantages such as weight reduction and functional versatility but may experience reduced strength and stiffness, as well as potential serviceability challenges. Selecting the proper beam type depends on specific application requirements and design considerations. The utilization of steel beams with web openings presents numerous advantages when compared to classical I-section elements in modern construction. These benefits encompass improved architectural appearance, a

superior strength-to-weight ratio, and the seamless integration of utilities, such as pipes, ducts, or electrical conduits without any compromise to the beam's structural integrity. This innovative design facilitates efficient space utilization and simplifies the installation process, making it a favored option for constructing large span structures, including multi-story buildings, bridges, and stadiums [4]. Nevertheless, it is important to note that the presence of openings in expanded steel beams leads to a reduction in shear capacity and altering failure modes. In contrast, the steel beam openings decrease the stiffness of beams and hence result in larger deflection, which introduces more complex design considerations. The fabrication costs of expanded steel girders are higher compared to regular plain webbed beams. However, these additional costs can be offset using materials that are more economical. Moreover, residual stresses are introduced within steel beams with openings, which can affect their structural behavior. These stresses contribute to a decrease in the critical buckling load and must be considered during the design process. Many studies investigated the effect of multiple web openings along the span length of RC beams [5-8].

The use of expanded steel beams has become more popular in the last two decades and it is controlled by well-established design rules and practice codes, which are updated on a regular basis to reflect new results and advancements. [9-11]. Unlike castellated beams or cellular beams, the expanded steel beams are constructed by making a horizontal cutting in the steel beam web and then adding an increment plate called spacer plate (with the same web thickness) between the two halves of the web sections. The new member can be created with or without openings. This method improves the stiffness and strength of the steel beam [12]. It is important to note that the design and installation of increment plates should be done in accordance with engineering standards and guidelines. Consulting with a structural engineer or a professional in the field of steel construction is recommended to ensure proper design and implementation for a safe and effective expansion of the steel web [13 12]. It has been stated that, beams with openings can be used to replace solid beams with the same requirements if stiffeners are provided in the right number, size, and location. Furthermore, it has been noted that the existence of web openings in steel beams can lead to reduced weight and increased structural efficiency [15, 16]. Yet, there is a lack in literature regarding the behavior of steel beams with expanded web with horizontal cutting patterns. Accordingly, in this paper, the behavior of expanded steel beams with horizontal cutting patterns is examined. Different expanding ratios and opening details will be investigated.

II. THE EXPERIMENTAL PROGRAM

A. HEA Steel Beam

Hot rolled standard steel (HE160A) was used in this study to fabricate the steel beams with expanded web. Its characteristics can be seen in Tables I-II.

B. Plates

To implement the strengthening technique, plates were employed as stiffeners. Their characteristics can be observed in Table III.

TABLE I. HEA BEAM (160A) SECTION CHARACTERISTICS

Parameter	G	h	tf	tw	A	r	I	Sx	Zx
Unit	kg/m	mm	mm	mm	mm ²	mm	mm ⁴	mm ³	mm ³
Value	30.4	152	9	6	38.8	15	167	220	245

TABLE II. HEA BEAM MECHANICAL CHARACTERISTICS

Property	Ultimate yield strength	Ultimate tensile strength	Modulus of elasticity	Elongation percentage
Unit	MPa	MPa	MPa	%
Value	322	459	210000	22

TABLE III. PLATE MECHANICAL AND SECTION CHARACTERISTICS

Property	Thickness	Yield stress	Ultimate stress	Elongation
Unit	mm	MPa	MPa	%
Value	5.9	253	482	30

C. Tested Specimens

Nine specimens were fabricated as HEA steel beams with an expanded web and one specimen as a reference beam. The specimens were tested under two-point load over a clear span length of 280 cm and a total length of 300 cm. The fabricated samples' details are explained in Figures 1-5 and Table IV.

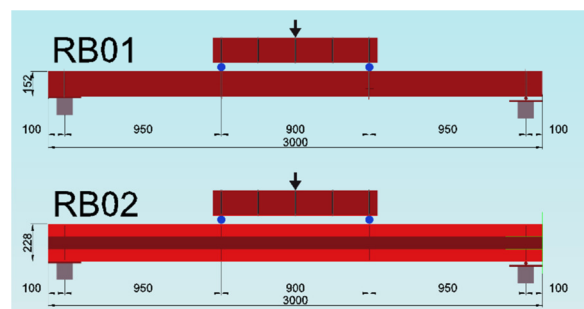


Fig. 1. Dimensions of beams RB01 and RB02.

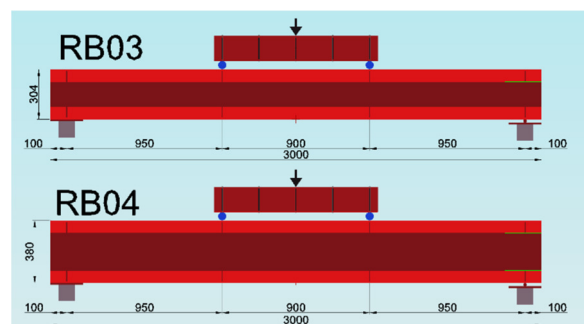


Fig. 2. Dimensions of beams RB03 and RB04.

D. Manufacturing of Steel Beams with Expanded Web

In the experimental work and in order to produce the steel beams with expanded web specimens, the web section was linearly cut in a horizontal pattern along their centerline using a numerical control (CNC) plasma-cutting machine. Then the two halves were separated as shown in Figure 6. In order to produce a solid expanded web, an increment plate was added between the two web halves that have been cut horizontally

(the plate had the same beam length and thickness with the beam web), then welding was used to join the plate and the two web halves together to create a new expanded beam. In addition, for producing steel beam with expanded web specimen contain openings, increment plates with specific dimensions in regular intervals were adopted. Vertical bearing stiffeners were welded to the web at concentrated point loads, and at ends (above supports) as illustrated in Figure 7. The plates above the supports are 200 mm wide and the plates under loads are 100 mm wide in all specimens with openings. In all beam specimens, proper sequence of welding was adopted since distortion would take place with two welders employed and the welding was conducted from the center outwards to the ends [17].

TABLE IV. DETAILS OF STEEL BEAM SPECIMENS (mm)

Se. No.	<i>h</i> (mm)	Expansion ratio %	Opening ratio %	Opening height (mm)	Opening width (mm)	No. of openings
RB01	152	NA	0	NA	NA	0
RB02	228	150	0	NA	NA	0
RB03	304	200	0	NA	NA	0
RB04	380	250	0	NA	NA	0
G2B1	228	150	48	76	80	18
G2B2	228	150	48	76	160	9
G3B1	304	200	48	152	80	18
G3B2	305	200	48	152	160	9
G4B1	380	250	48	228	80	18
G4B2	380	250	48	228	160	9

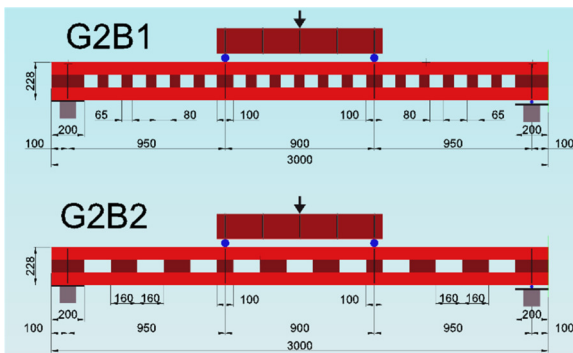


Fig. 3. Dimensions of beams G2B1 and G2B2.

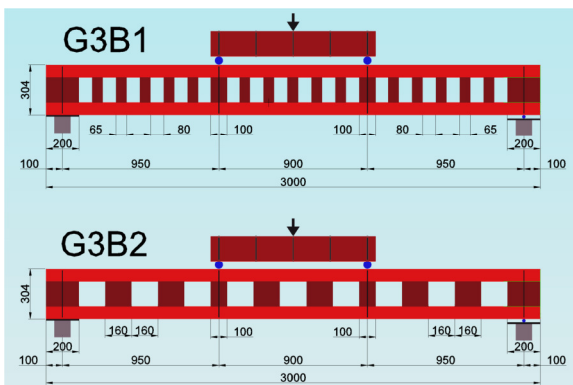


Fig. 4. Dimensions of beams G3B1 and G3B2.

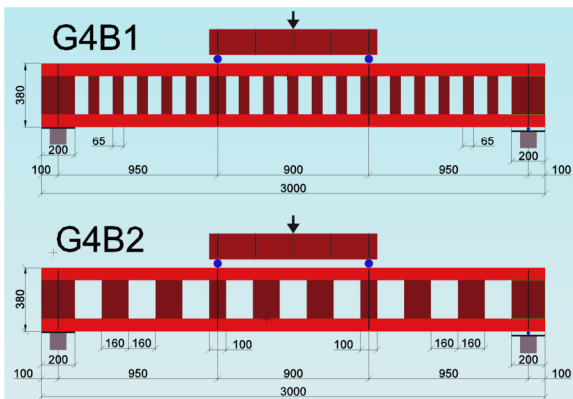


Fig. 5. Dimensions of beams G4B1 and G4B2.



Fig. 6. Fabrication of expanded steel beam web with openings.

E. Test Setup

The beams were subjected to testing in the Structure Laboratory of the Civil Engineering Department of Baghdad University. The testing procedure consisted of applying static top-loading utilizing a hydraulic jack and a load cell with a capacity of 200 tons. The load was incrementally increased, with approximately 500 kg added each time, until the beams reached the point of failure. During the testing process, multiple measurements were taken, including deflections at the center of the beams and strains. Each beam was loaded with two concentrated loads, each load located 450mm away from the beam span center [17, 18].

III. TEST RESULTS AND DISCUSSION

All beams were tested engaging a universal testing machine. All specimens were tested under the same loading conditions. The tested specimens were compared with the control specimen. The deflection Serviceability Limit $L/240$ with 12 mm value was used.

A. Load-Deflection Curves

1) Expansion Web Ratio Effect on Solid HEA Beams

The behavior of steel beams is generally assessed based on their strength and their elastic deformations [19]. The influences of increasing web depth by 150%, 200%, and 250% of the original HEA beam web on the load-deflection behavior of solid steel beams with expanded web are shown in Figure 8. The yield point determining procedure is explained in detail in [20].

2) Ultimate Load

It was observed that with the adoption of a steel plate to increase the web depth, the ultimate capacity of carrying loads of beams RB02, RB03, and RB04 increased by 61%, 124%, and 207% with an expanded ratio of 150%, 200%, and 250%, respectively, compared to the reference solid beam RB01. At the deflection service point (12 mm), the carrying loads of beams RB02, RB03, and RB04 increased by 108%, 201%, and 310%, respectively.

3) Deflection

The mid-span deflection at ultimate load of beams RB02, RB03, and RB04 compared with the reference beam RB01 decreased by 16.67%, 57.25%, and 66.67%, respectively. The mid-span deflection at the yield point decreased by 43.75%, 51.25%, and 61.25%. It was thus concluded that as the web expanding ratio increases, the ultimate load increases and the deflection decreases. It should be noted that there is greater influence of expanding web at the deflection service point (12 mm), compared to the ultimate carrying load, and there is greater influence on the deflection at the yield point compared to the mid-deflection at ultimate load. However, it is worth mentioning that increasing the web height can also lead to an increase in weight, because a deeper web requires more material, which can impact the overall weight and cost of the beam. Engineers and designers need to carefully consider the trade-off between increased load-carrying capacity and stiffness versus the potential increase in weight and cost when selecting the expanding web height ratio of a steel beam.

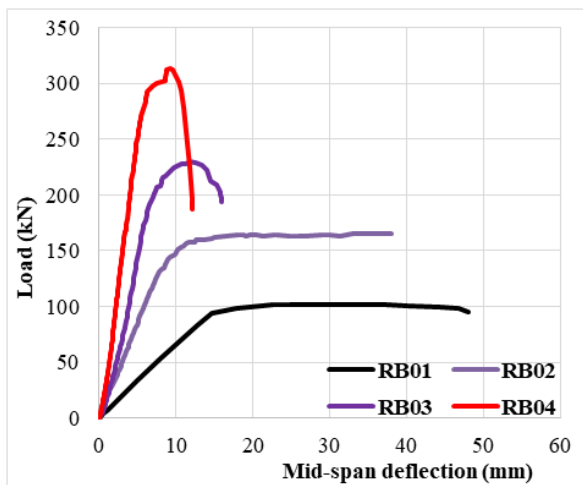


Fig. 7. Load-mid-span deflection curves of beams RB01-RB04.

4) Expanded Web Ratio Effect on Steel Beams with Openings

The influence of the increased (by 150%, 200%, and 250%) web depth of steel beams with the adoption of web openings on the load-deflection behavior is shown in Figures 8-9. The related beams are divided into two categories according to steel web opening width and number. The first type (B1) had a smaller opening width and higher hole numbers whereas the second type (B2) had larger opening width and smaller hole number.

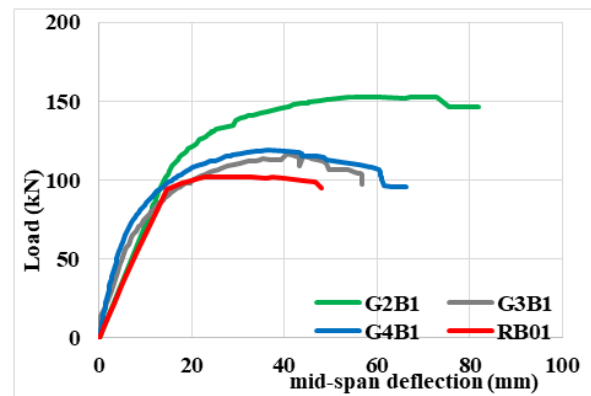


Fig. 8. Load-mid-span deflection curves of beams RB01, G2B1, G3B1, and G4B1.

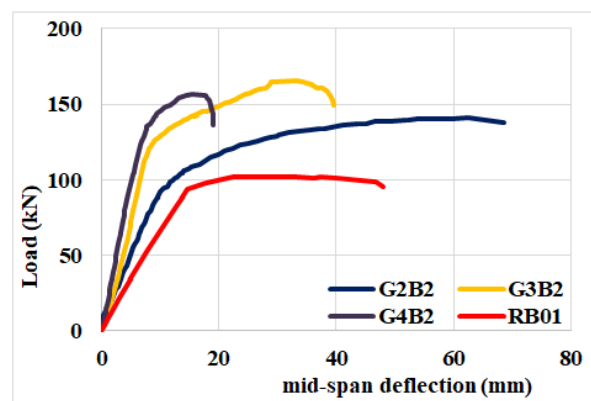


Fig. 9. Load-mid-span deflection curves of beams (RB01, G2B2, G3B2, and G4B2).

5) Ultimate Load

For the first type (B1): It was noticed that with the adoption of a steel plate to increase the web depth of HEA beams with openings, the ultimate capacity of carrying loads increased by 43%, 13%, and 16% for beams G2B1, G3B1, and G4B1 compared to the reference solid beam RB01. At the deflection service point, the carrying loads of beams G2B1, G3B1, and G4B1 increased by 13.84%, 8.60%, and 17.76%.

For the second type (B2): It was observed that with the adoption of a steel plate to increase the web depth of HEA beams with openings, the ultimate capacity of carrying loads increased by 37%, 60%, and 52% for beams G2B2, G3B2, and G4B2 compared to the reference solid beam RB01. At the deflection service point, the carrying loads of beams G2B2, G3B2, and G4B2 were increased by 30.84%, 77.94%, and 89.72%.

6) Deflection

For the first type (B1): the mid-span deflection at ultimate load of beams G2B1, G3B1, and G4B1 was increased by 112.32%, 47.10%, and 31.52% compared to the reference beam (RB01). The mid-span deflection at the yield point was increased by 43.75%, and 12.50%, respectively.

For the second type (B2): The mid-span deflection at ultimate load of G2B2, and G3B2 was increased by 126.81%,

and 21.01% and that of beam G4B2 was decreased by 44.20% compared to the reference beam (RB01). Additionally, the mid-span deflection at the yield point for beam G2B2 increased by 37.50% and decreased by 25.00% and 46.88% for beams G3B2 and G4B2).

From the above, it was concluded that the expansion of the web with openings can cause an increase in the ultimate load. This improvement in flexural capacity is related to the enhancement in the section stiffness after the expansion process. However, the expansion ratio effect for a solid steel beam with expanded web is noted to be higher than its effect on steel beams with expanded web that has openings, due to the presence of the holes in the web that weaken the overall structural integrity of the steel beam. The web is responsible for carrying shear forces, and any holes can reduce its ability to resist these forces, potentially leading to structural failure. The presence of holes creates stress concentrations, which can result in localized failure and a decrease in the overall strength of the beam, so the presence of holes weakens the web, making it more prone to buckling under compressive loads. It can be stated that, in contrast to solid steel beams with expanded web, the web openings with expanding web influence the carrying load at the deflection service point and yield deflection. Opening width and hole number can have a great impact on the expansion web ratio, influencing the performance of steel beams with openings.

B. General Observations

For the first type, with 80 mm opening width and 18 holes, the beam with an expansion ratio of 150%, had the best performance according to the load-deflection behavior, demonstrating a reduction in load capacity by only 11% compared to a solid expanded steel beam with the same expansion ratio. The beams with expansion ratios of 200%, and 250%, had no economic viability according to the analysis of load-deflection behavior with a reduction in load capacity of 49% and 62%.

For the second type, with 160 mm opening width and 9 holes, the beams with expansion ratios of 200% and 250%, performed better than the first type according to the load-deflection behavior, showcasing a reduction in load capacity of 28%, and 50% compared to the solid expanded steel beam with the same expansion ratio. The beam with expansion ratio of 150% had a reduction in load capacity of 15% compared to the solid expanded steel beam with the same expansion ratio.

The difference between the first type (B1) and the second type (B2) is the web post area divergence that influences web post-buckling. Finally, it can be concluded that using larger opening width and smaller hole number is regarded the best option for beams with expansion web ratios of 200% and 250%. On the contrary, using smaller opening width and higher hole number is the best option for beams with expansion web ratio of 150%. Increasing the web depth in a steel beam improves structural performance, offering design flexibility, reduced deflections, and longer spans, thus enhancing material efficiency and architectural possibilities. However, challenges like construction logistics and potential cost implications should be carefully managed.

C. Modes of Failure

In specimens RB01, RB02, RB03, and RB04 made from solid steel beams (reference non-expanded beam and expanded web), plastic flexural deformation is observed, as depicted in Figures 10-13.

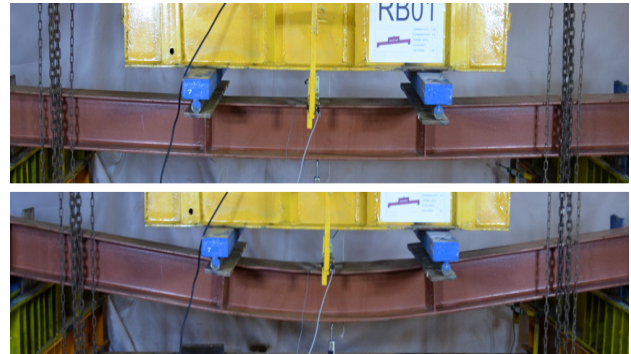


Fig. 10. Mode of failure for beam RB01.

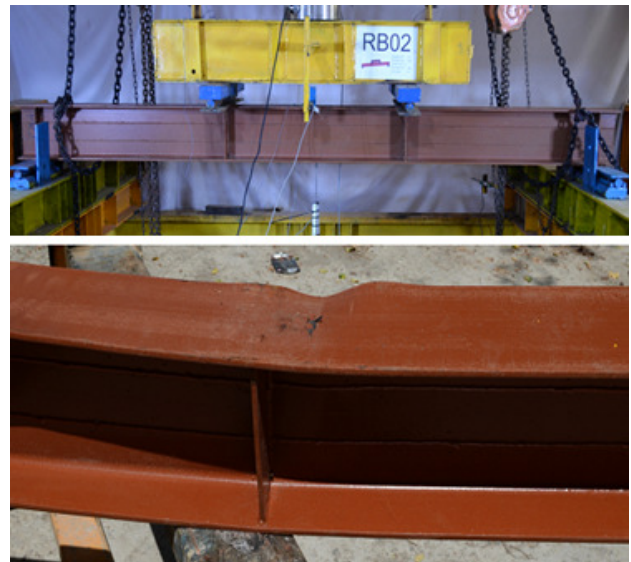


Fig. 11. Mode of failure for beam RB02.



Fig. 12. Mode of failure for beam RB03.

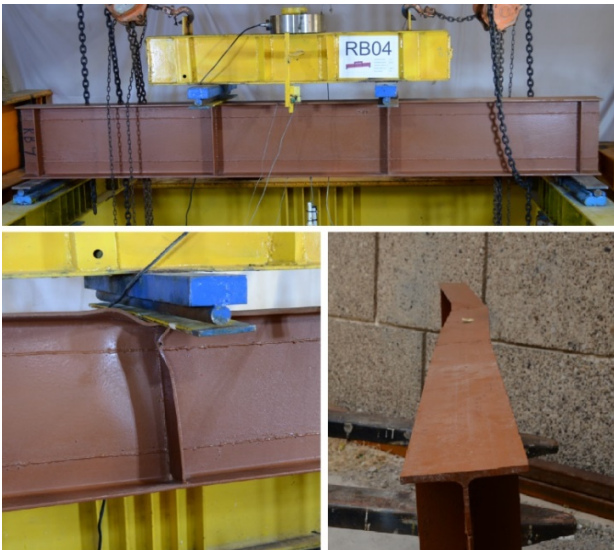


Fig. 13. Mode of failure for beam RB04.

Furthermore, for solid steel beams with expanded web ratios of 0% and 150%, only flexural failure was observed. Yet, for solid steel beams with expanded web ratios of 200 and 250%, both flexure failure and local failure at the stiffeners were spotted.

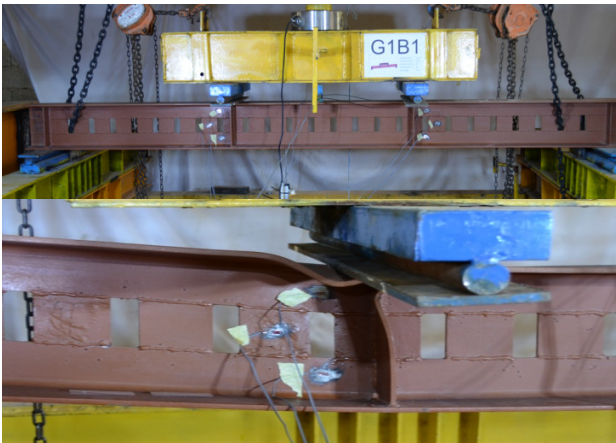


Fig. 14. Mode of failure for beam G1B1.

In specimens G1B1 and G1B2, made from steel beams with an expanded web ratio of 150%, opening ratio of 30%, opening ratio widths of 30, and 60 mm, and 24 and 12 holes, flexure failure and local failure at the stiffeners were observed, as shown in Figures 14-15.

In specimens G2B1, and G2B2 made from a steel beam with an expanded web ratio of 150%, opening ratio of 60%, opening width of 80 mm, and 18 openings, Vierendeel mechanism was observed. However, with an opening width of 160 mm and 9 holes, Vierendeel mechanism and web weld fracture were detected while a tear in the corner of the openings was also noted, as presented in Figures 16-17.



Fig. 15. Mode of failure for beam G1B2.



Fig. 16. Mode of failure for beam G2B1.



Fig. 17. Mode of failure for beam G2B2.

In specimens G3B1 and G3B2 made from steel beams with an expanded web ratio of 200%, opening ratio of 60%, opening width of 80 mm, and 18 openings, web post-buckling and rapture at the weld line were pinpointed. For the beam with an opening width of 160 mm and 9 holes, web post-buckling was detected, as displayed in Figures 18-19.

In specimens G4B1 and G4B2 made from steel beams with an expanded web ratio of 250%, opening ratio of 60%, opening widths of 80 and 160 mm and 18 and 9 holes, web post buckling was observed, as illustrated in Figures 21 and 22.



Fig. 18. Mode of failure for beam G3B1.



Fig. 19. Mode of failure for beam G3B2.



Fig. 20. Mode of failure for beam G4B1.



Fig. 21. Mode of failure for beam G4B2.

IV. CONCLUSIONS

Based on the outcomes of the experimental research, the following conclusions were drawn:

- There is greater influence when expanding the web of steel beams at the deflection service point (12 mm), compared to the ultimate carrying load, and there is greater influence on deflection at the yield point compared to the mid-deflection at the ultimate load.
- For steel beams with expanded web that have 18 openings of 80 mm, the beam with an expansion ratio of 150%, has the best performance according to the load-deflection behavior with a reduction in load capacity by only 11%, whereas the beams with expansion ratios of 200% and 250% have no economic viability.
- For the second type with 9 openings of 160 mm width, the beams with expansion ratios of 200%, and 250%, perform better than the first type according to the load-deflection behavior with a reduction in load capacity of 28% and 50%, and the beam with the expansion ratio of 150% presents a reduction in load capacity of 15%.
- As a conclusion, using larger opening width and a smaller hole number is considered the best option for beams with expansion web ratios of 200% and 250, while utilizing smaller opening width and higher hole number is the best option for beams with expansion web ratio of 150%.
- Expanding the depth of a steel beam has many advantages considering its structural performance. It provides design flexibility, reduces deflections and allows for longer spans. These benefits enhance material efficiency and open up new architectural possibilities including higher material and manufacturing costs, increased weight, connection challenges, space constraints, buckling concerns, serviceability issues, fabrication complexities, and potential unavailability of specialized sections. Careful consideration of project requirements and trade-offs is crucial for optimal design.

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