Mathematical Modeling of Dynamic Supply Chains Subject to Demand Fluctuations

Yasser A. Davizon
Instituto Tecnologico de Ciudad Juarez, Chihuahua, Division de Estudios de Posgrado e Investigacion, Mexico
ydavizon@upmys.edu.mx (corresponding author)

Jesus Mateo Amillano-Cisneros
Universidad Politecnica del Mar y la Sierra, Mexico
jamillano@upmys.edu.mx

Jose Belisario Leyva-Morales
Centro de Investigación en Recursos Naturales y Sustentabilidad (CIRENYS), Universidad Bernardo O'Higgins, Santiago de Chile 8370993, Chile
bleyva@upmys.edu.mx

Eric D. Smith
Industrial Manufacturing and Systems Engineering, University of Texas at El Paso, TX, USA
esmith2@utep.edu

Jaime Sanchez-Leal
Industrial Manufacturing and Systems Engineering, University of Texas at El Paso, TX, USA
jsanchez21@utep.edu

Neale R. Smith
Escuela de Ingenieria y Ciencias, Tecnologico de Monterrey, Mexico
nsmith@tec.mx

Received: 9 October 2023 | Revised: 30 October 2023 | Accepted: 7 November 2023

 DOI: https://doi.org/10.48084/etasr.6491

ABSTRACT

This research work aims to develop the mathematical modeling for a class of dynamic supply chains. Demand fluctuation corresponds to product demand volatility, which increases or decreases over a given time frame. Industrial engineering practitioners should consider the function that applied mathematical modeling plays in providing approximations of solutions that may be used in simulations and technical implementations at the strategic, tactical, and operational levels of an organization. In order to achieve proper results, two mathematical models are presented in this paper: In addition to a finite-dimensional system of Ordinary Differential Equations (ODEs) for coupled dynamic pricing, production rate, and inventory level, which properly integrates Lyapunov stability analysis of the dynamical system and simulations, there is an infinite-dimensional Partial Differential Equation (PDE) production level modeling system available. Infinite and finite-dimensional systems incorporate a dynamic pricing approach in the mathematical modeling. The main research goal of this work is to explore the dynamic nature of supply chains applying PDE and ODE methods, with proper analytical analysis and simulations for both systems.

Keywords-dynamic supply chains; Lyapunov stability; infinite dimensional system; finite dimensional system

I. INTRODUCTION

Nowadays, in industrial engineering, mathematical modeling is crucial, especially in dynamic Supply Chains (SCs), which are high volume production systems with high levels of complexity and low variability. The use of PDEs and ODEs is important given the increased volume of data and commercial judgments that decision-makers must make. Therefore, real-time information can be used to enhance, assist, and validate business decision-making, and this interest is expanding [1], while taking into account the influence of decision-makers on the strategic, tactical, and operational levels of enterprises, along the SC. Enterprises today strive for rapid, dependable, and adaptable business processes, as well as entrepreneurship practices [2] which increase economic growth. As a result, they must put systematic decision-making processes in place [3]. Supply Chain Management (SCM) and Industry 4.0 network unpredictability, feedback loops, and system dynamics are concerns in production and logistical systems [4]. This research work aims to explore the dynamic nature for a class of SCs, via the development of infinite dimensional systems (PDEs) and finite dimensional systems, subject to demand fluctuations. The operations and management sciences rely on Inventory Management (IM). SCM is the cooperative process management of material and information flows [5]. An SC is a network of locations and businesses involved in distribution, including suppliers, manufacturers, distributors, and retailers [6]. Recent technologies such as blockchain [7] and Artificial Intelligence (AI) [8] have gained special attention between SCM practitioners.

In general, a SC is characterized by three flows: materials and cash (forward flow) and information (backward flow). IM is a component of the SCM that efficiently and effectively coordinates the forward and reverse movement of goods and services between the point of origin and the point of consumption in order to satisfy customer needs [6, 9]. Inherently, inventories are present along the entire SC and inventory control is a critical task for the management of a company [10]. Maintaining proper inventory levels is an important responsibility for a business [11] given that quick and effective customer service is associated with large inventory levels (which raise costs), while low inventory levels could result in scarcity. Supply, transportation, production, and demand variation are all disrupted in SCM systems [12]. In production-inventory systems some works analyzes supply disruption [13], with multi-echelon production-inventory system supply disruption [14]. In relation to production disruptions in [15] presents deteriorating items, while random disruptions are presented in [16-17] and a system dynamics approach is developed for production process disruption in [18]. SCM dynamics build the strategic orientation of SC disruption orientation [19]. In a typical SC network, the efficient collaboration between all parts is vital, in times of uncertainty and unpredictable disruption [20]. Table I summarizes a literature review in demand fluctuation with proper application sector.

In the future, our interest is to apply System Dynamics (SD) which is founded upon the integration of multiple theoretical frameworks, encompassing operating theory, system theory, control theory, information feedback theory, decision-making theory, mechanical systems theory, and computer science [30]. The research goals of this paper are summarized as follows:

- To present a mathematical modeling for infinite dimensional systems which explores the dynamic nature of production-inventory systems with some assumptions in capacity level, inventory level, and demand, with the decision variables being price level and time.
- To analyze the mathematical modeling of an ODE coupled set which summarizes the behavior of a dynamic SCs, with a Lyapunov stability analysis for the dynamical system.
- To present proper simulations for infinite and finite systems that represent the analytical solution form of PDE and the numerical solution for the ODE systems.

TABLE I. LITERATURE REVIEW SUMMARY

Ref.	Contribution	Application
[21]	Considers a supplier–retailer system, with an imperfect production process and a possibility of having demand fluctuation.	SC
[22]	Explores sales and increased income in a SC, in which suppliers frequently choose to grant their retailers a payment delay time.	SC
[23]	Presents a fuzzy inventory model to counteract the demand fluctuation in supply demand networks.	Supply demand networks
[24]	Presents two types of flexibility investment, which are flexible technology and flexible capacity, under demand fluctuations.	Flexible manufacturing systems
[25]	Demand fluctuations can be caused by both predictable oscillations like seasonality and unpredictable disasters like pandemics..	Assembly line balancing
[26]	The ready-mix concrete business experiences significant variations in demand as a result of the volatility observed in local construction markets. The oscillations in question presents impacts on the mix, size, and investment level within the industry.	Construction industry
[27]	Investigates the challenges associated with dynamic pricing in ride-hailing systems, specifically focusing on the issue of supply-demand imbalance resulting from demand fluctuations.	Transportation
[28]	Travel time variability can be attributed to the fluctuations in demand and the deterioration of the capacity within a stochastic network.	Transportation
[29]	Introduces the shuttle bus rerouting and rescheduling strategy, where the operator can change the visited stops and can operate backup buses to handle the passenger demand fluctuations.	Transportation

II. INFINITE DIMENSIONAL PRODUCT LEVEL MODELING

In general, a production system combines humans, machinery, and equipment that are next to common material and information flow [31]. In [32], a PDE for the production level subject to dynamic pricing and time was presented with the form:

$$v_p \frac{\partial^2 \varphi(p,t)}{\partial p \partial t} + \left(\beta + a_p\right) \frac{\partial \varphi(p,t)}{\partial p} + \alpha\beta \frac{\partial \varphi(p,t)}{\partial t} + K\varphi(p,t) = D(p,t) \quad (1)$$

where $\varphi(p,t)$ is the production level subject to price and time, $D(p,t)$ is the demand level, v_p is the price velocity, a_p is the price acceleration, β is the damping coefficient, α is the scaling production factor, and K is the production system resilience, with $\beta + a_p > 0$. Our aim is to solve (1) analytically, subject to similar conditions for which a production system approaches to demand levels near zero.

Theorem 1. Consider the PDE in (1) assumming that: $D(p,t) = wP'$, (where w is a scaling factor and P' the price rate), $\beta + a_p = 1$ and $\alpha\beta = 1$, we get the following solution:

$$\varphi(p,t) = e^{-\alpha p}\left(e^{-\left(\frac{K-\alpha}{1-\alpha v_p}\right)t} + \frac{\alpha w}{\alpha - K}\right)$$

Proof:

$$v_p \frac{\partial^2 \varphi(p,t)}{\partial p \partial t} + \frac{\partial \varphi(p,t)}{\partial p} + \frac{\partial \varphi(p,t)}{\partial t} + K\varphi(p,t) = wP' \quad (2)$$

Applying the separation of variables method, we have:

$$\varphi(p,t) = P(p)T(t) \quad (3)$$

Substituting (3) in the PDE (2):

$$v_p P'T' + P'T - wP' + PT' + KPT = 0 \quad (4)$$

where: $P' = \frac{dP}{dp}$ and $T' = \frac{dT}{dt}$

Grouping the terms from (4) we have:

$$P'\left(v_p T' + T - w\right) = -P(T' + KT). \quad (5)$$

Separating terms in (5) we get:

$$-\frac{P'}{P} = \frac{(T' + KT)}{v_p T' + T - w} = \alpha. \quad (6)$$

For separate ODE, in time domain:

$$\frac{P'}{P} = -\alpha. \quad (7)$$

Therefore:

$$\frac{dP}{P} = -\alpha dp. \quad (8)$$

Solving (8) we have:

$$P(p) = e^{-\alpha p}. \quad (9)$$

For the ODE, in time domain:

$$T' + KT = \alpha\left(v_p T' + T - w\right). \quad (10)$$

After some algebraic manipulation, (10) becomes:

$$T' + \left(\frac{K-\alpha}{1-\alpha v_p}\right)T = -\left(\frac{\alpha w}{1-\alpha v_p}\right). \quad (11)$$

Solving (11), we have:

$$T(t) = Ce^{-\left(\frac{K-\alpha}{1-\alpha v_p}\right)t} + \frac{\alpha w}{K-\alpha} \quad (12)$$

Finally, the solution for the PDE production level subject to price and time is:

$$\varphi_1(p,t) = e^{-\alpha p}\left(e^{-\left(\frac{K-\alpha}{1-\alpha v_p}\right)t} + \frac{\alpha w}{\alpha - K}\right) \quad (13)$$

Corollary 1. Considering the PDE in (1) and assumming that demand fluctuates towards: $D(p,t) = 0$, we get the following solution:

$$\varphi_2(p,t) = e^{(\gamma p - \alpha t)}$$

In Figure 1, a PDE production level normalized solution in function of price and time is shown, considering a tendency to zero production level conforming price and time tending to infinity.

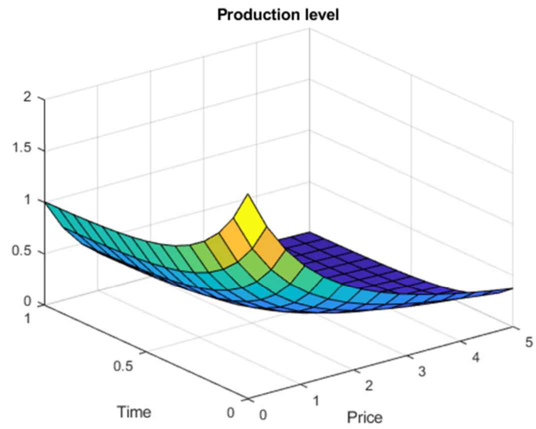

Fig. 1. PDE production level solution in function of price and time.

In general, production systems are required to effectively manage the challenges that arise from the presence of variabilities and complexities resulting from globalization and technological improvements, which impacts in demand fluctuation along the supply chain. The following section explores the dynamic SC mathematical modeling with a proper stability analysis and simulations.

III. FINITE DIMENSIONAL SC MODELING

In [33], a dynamic production-inventory system via optimal control theory is presented. In general, along the supply chains there are three flows: material, cash, and information. IM presents a crucial role for the SC operation by considering the synchronic and diachronic evolution of the SC to achieve equilibrium and coordination between supply and demand. This paper presents an expansion of the previous work, focusing on the incorporation of dynamic pricing and the inclusion of time delay effects for the purpose of mathematical modeling of a class of SC.

A. *Mathematical Modeling*

A time-delayed production inventory system is proposed via a set of coupled ODEs which are described as follows: To incorporate a dynamic pricing perspective in the dynamic SC, we propose:

$$\frac{dp}{dt} = u_1(t) - p(t) + k\frac{I(t)}{C} \quad (14)$$

where u_1 denotes the purchase price level, p denotes the sales price level, I is the inventory level, k is the associated cost for the inventory, and C is the capacity level.

To develop the production-inventory system dynamics for production level and production rates, the following equation is proposed:

$$C\frac{d^2Q}{dt^2} + I(t)\frac{dQ}{dt} + \gamma Q(t-\theta) = d(t) \quad (15)$$

where Q represents the production level, dQ/dt is the production rate, d is the demand level, θ represents the lead time, and γ is the production resilience factor.

$$\frac{dI}{dt} = \mu I(t-\theta) + \gamma Q(t) - d(t) \quad (16)$$

In order to provide a linear approximation, via Taylor series, for the time delay terms as in [34], and neglecting high order terms:

$$x(t-\tau) \approx x(t) - \tau\dot{x}(t) \quad (17)$$

Developing a state space formulation for the time delayed production inventory system from (14)-(16) and after applying (17), our new state space system is:

$$\dot{x_1}(t) = -x_1(t) + k_1 x_4(t) \quad (18)$$

$$\dot{x_2}(t) = x_3(t) \quad (19)$$

$$\dot{x_3}(t) = -k_2 x_3(t) x_4(t) - k_3 x_2(t) + k_4 x_3(t) \quad (20$$

$$\dot{x_4}(t) = k_5 x_2(t) + k_6 x_4(t) \quad (21)$$

where $x_1(t) = p(t)$, $x_2(t)=Q(t)$, $x_3(t)=dQ/dt$, $x_4(t) = I(t)$ and constants $k_1 = k/C$, $k_2 = 1/C$, $k_3 = \gamma/C$, $k_4 = \gamma\theta/C$, $k_5 = \gamma/(1+\mu\theta)$, $k_6 = \mu/(1+\mu\theta)$.

B. Stability Analysis

Theorem 2. In order to have a stability analysis, for the equation system (18)-(21), the following conditions must be satisfied, $\vartheta > 0$, $\mu > 0$, and $C > 1 + \mu\theta$.

Proof. In order to proof Theorem 2, the following Lyapunov candidate function is proposed:

$$V(x_1, x_2, x_3, x_4) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}(x_3 - x_4)^2 \quad (22)$$

Applying the derivative with respect to time, to the Lyapunov candidate function:

$$\dot{V}(x_1, x_2, x_3, x_4) = x_1\dot{x}_1 + x_2\dot{x}_2 + (x_3 - x_4)(\dot{x}_3 - \dot{x}_4) \quad (23)$$

After some algebraic manipulation, and applying the LaSalle Theorem in the equilibrium point, we get:

$$\dot{V}(x_1, x_2, x_3, x_4) < -x_1^2 + k_4 x_3^2 + k_6 x_4^2 - (k_3 - k_5 - 1)|Q|^2 - (k_4 + k_6)|O|^2 - (k_5 - k_3)|P|^2 \quad (24)$$

Equation (24) can be simplified to:

$$\dot{V}(x_1, x_2, x_3, x_4) < -x_1^2 - (k_4 + k_6)|O|^2 - (k_5 - k_3)|P|^2 \quad (25)$$

In order to achieve stability, the following conditions must be satisfied:

$$k_4 + k_6 > 0$$

$$k_5 - k_3 > 0$$

Therefore: $k_4 > 0$ and $k_6 > 0$

Considering that: $k_4 = \frac{\gamma\theta}{C}$, $k_6 = \frac{\mu}{1+\mu\theta}$, and $k_5 > k_3$.

From which we can conclude that:

$\gamma\vartheta > 0$, $\mu > 0$, and $C > 1 + \mu\theta$.

C. Simulations

Simulations were conducted in MATLAB for (18)-(21). Figure 2 presents the decaying performance for the price level of the finite dimensional production system considering (14), which presents the dynamic pricing analysis for production-inventory system. It is confirmed that as time increases, the price level tends to zero.

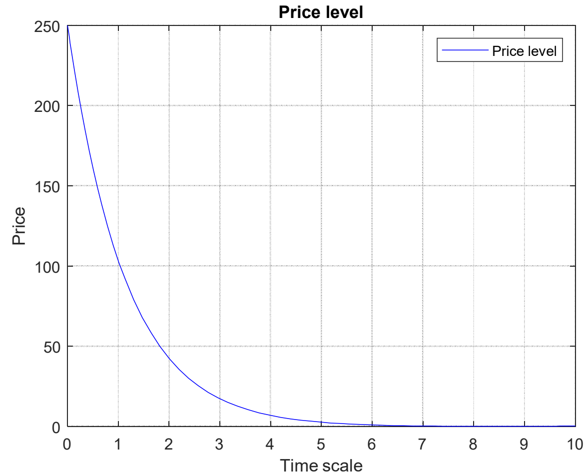

Fig. 2. Price level for the finite dimensional production system.

In Figure 3, the production level achieves a maximum for the finite dimensional system which is based on the context that this is the level of the production rate.

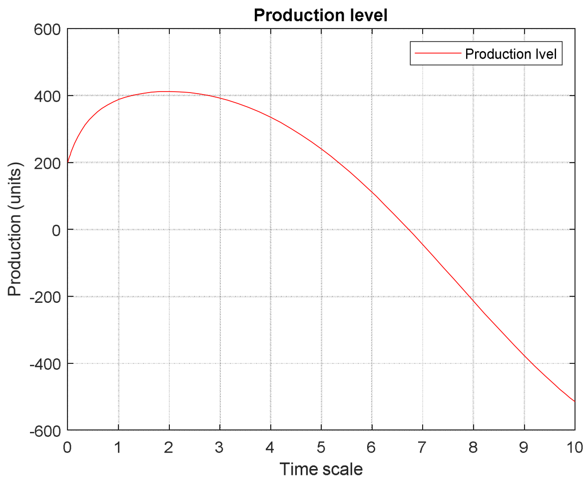

Fig. 3. Production level for the finite dimensional production system.

Unproduction is related with a negative production rate (Figure 4). This effect is present around $t = 2$, and when t tends to higher values, the production rate decreases.

Figure 5 presents the decaying inventory level, confirming the production rate tending to a negative value, which is an effect of the relation of maximum production level with the lower levels for inventory. This is apparent from (16), which presents a dynamic inventory level with time delay.

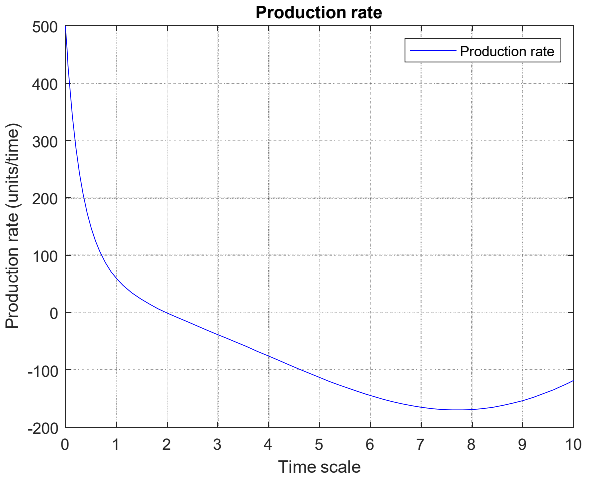

Fig. 4. Production rate of the finite dimensional production system.

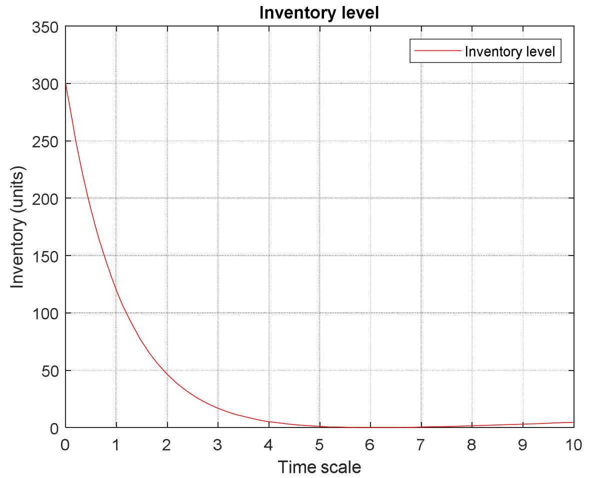

Fig. 5. Inventory level for the finite dimensional production system.

IV. CONCLUSION

Applied modeling plays an important role for industrial engineering practitioners. This research paper addressed two problems in the context of high-volume supply chain, by applying infinite and finite dimensional mathematical approaches. Several research works have presented the use of PDEs for production systems [35-36], whereas the use of ODEs in the mathematical modeling for production-inventory systems was more common [37-38]. In this work, infinite and finite dimensional systems incorporate dynamic pricing into the description of a dynamical system while taking demand fluctuations into account.

A supply chain refers to a collection of organizations, which collaborate in the process of bringing a product to its final consumer [39]. Discrete event simulation has been applied to increase productivity of enterprises [40]. This mathematical modeling considers low variability and cause-effect relation for a deterministic approach in high volume supply chains. In general, demand fluctuation corresponds to product demand variation, which is increased or decreased in a time horizon period. This is the main contribution of the current work.

REFERENCES

[1] K. Sourirajan, B. Ramachandran, and L. An, "Application of control theoretic principles to manage inventory replenishment in a supply chain," *International Journal of Production Research*, vol. 46, no. 21, pp. 6163–6188, Nov. 2008, https://doi.org/10.1080/00207540601178151.

[2] S. Tripopsakul, T. Mokkhamakkul, and W. Puriwat, "The Development of the Entrepreneurial Spirit Index: An Application of the Entrepreneurial Cognition Approach," *Emerging Science Journal*, vol. 6, no. 3, pp. 493–504, Apr. 2022, https://doi.org/10.28991/ESJ-2022-06-03-05.

[3] E. Perea, I. Grossmann, E. Ydstie, and T. Tahmassebi, "Dynamic modeling and classical control theory for supply chain management," *Computers & Chemical Engineering*, vol. 24, no. 2, pp. 1143–1149, Jul. 2000, https://doi.org/10.1016/S0098-1354(00)00495-6.

[4] A. Dolgui, D. Ivanov, S. Sethi, and B. Sokolov, "Control theory applications to operations systems, supply chain management and Industry 4.0 networks," *IFAC-PapersOnLine*, vol. 51, no. 11, pp. 1536–1541, Jan. 2018, https://doi.org/10.1016/j.ifacol.2018.08.279.

[5] E. Sucky, "Inventory management in supply chains: A bargaining problem," *International Journal of Production Economics*, vol. 93–94, pp. 253–262, Jan. 2005, https://doi.org/10.1016/j.ijpe.2004.06.025.

[6] H. Sarimveis, P. Patrinos, C. D. Tarantilis, and C. T. Kiranoudis, "Dynamic modeling and control of supply chain systems: A review," *Computers & Operations Research*, vol. 35, no. 11, pp. 3530–3561, Nov. 2008, https://doi.org/10.1016/j.cor.2007.01.017.

[7] A. Ruangkanjanases, T. Hariguna, A. M. Adiandari, and K. M. Alfawaz, "Assessing Blockchain Adoption in Supply Chain Management, Antecedent of Technology Readiness, Knowledge Sharing and Trading Need," *Emerging Science Journal*, vol. 6, no. 5, pp. 921–937, Jun. 2022, https://doi.org/10.28991/ESJ-2022-06-05-01.

[8] A. Deif and T. Vivek, "Understanding AI Application Dynamics in Oil and Gas Supply Chain Management and Development: A Location Perspective," *HighTech and Innovation Journal*, vol. 3, pp. 1–14, Mar. 2022, https://doi.org/10.28991/HIJ-SP2022-03-01.

[9] D. Singh and A. Verma, "Inventory Management in Supply Chain," *Materials Today: Proceedings*, vol. 5, no. 2, Part 1, pp. 3867–3872, Jan. 2018, https://doi.org/10.1016/j.matpr.2017.11.641.

[10] M. Bieniek, "The ubiquitous nature of inventory: Vendor Managed Consignment Inventory in adverse market conditions," *European Journal of Operational Research*, vol. 291, no. 2, pp. 411–420, Jun. 2021, https://doi.org/10.1016/j.ejor.2019.07.070.

[11] L. Duan and J. A. Ventura, "A Dynamic Supplier Selection and Inventory Management Model for a Serial Supply Chain with a Novel Supplier Price Break Scheme and Flexible Time Periods," *European Journal of Operational Research*, vol. 272, no. 3, pp. 979–998, Feb. 2019, https://doi.org/10.1016/j.ejor.2018.07.031.

[12] S. K. Paul, R. Sarker, and D. Essam, "Managing risk and disruption in production-inventory and supply chain systems: A review," *Journal of Industrial and Management Optimization*, vol. 12, no. 3, pp. 1009–1029, Sep. 2015, https://doi.org/10.3934/jimo.2016.12.1009.

[13] S. Li, Y. He, and L. Chen, "Dynamic strategies for supply disruptions in production-inventory systems," *International Journal of Production Economics*, vol. 194, pp. 88–101, Dec. 2017, https://doi.org/10.1016/j.ijpe.2017.04.003.

[14] B. Pal, S. S. Sana, and K. Chaudhuri, "A multi-echelon production–inventory system with supply disruption," *Journal of Manufacturing Systems*, vol. 33, no. 2, pp. 262–276, Apr. 2014, https://doi.org/10.1016/j.jmsy.2013.12.010.

[15] Y. He and J. He, "A Production Model for Deteriorating Inventory Items with Production Disruptions," *Discrete Dynamics in Nature and Society*, vol. 2010, Aug. 2010, Art. no. e189017, https://doi.org/10.1155/2010/189017.

[16] R. Karim and K. Nakade, "A stochastic model of a production-inventory system with consideration of production disruption," *International Journal of Advanced Operations Management*, vol. 11, no. 4, pp. 287–316, Jan. 2019, https://doi.org/10.1504/IJAOM.2019.103152.

[17] K. Moinzadeh and P. Aggarwal, "Analysis of a Production/Inventory System Subject to Random Disruptions," *Management Science*, vol. 43, no. 11, pp. 1577–1588, Nov. 1997, https://doi.org/10.1287/mnsc.43.11.1577.

[18] C. Sanchez-Ramirez, R. Ramos-Hernandez, J. R. Mendoza Fong, G. Alor-Hernandez, and J. L. Garcia-Alcaraz, "A System Dynamics Model to Evaluate the Impact of Production Process Disruption on Order Shipping," *Applied Sciences*, vol. 10, no. 1, Jan. 2020, Art. no. 208, https://doi.org/10.3390/app10010208.

[19] A. R. Stephens, M. Kang, and C. A. Robb, "Linking Supply Chain Disruption Orientation to Supply Chain Resilience and Market Performance with the Stimulus–Organism–Response Model," *Journal of Risk and Financial Management*, vol. 15, no. 5, May 2022, Art. no. 227, https://doi.org/10.3390/jrfm15050227.

[20] M. Nunes, A. Abreu, J. Bagnjuk, E. Nunes, and C. Saraiva, "A Strategic Process to Manage Collaborative Risks in Supply Chain Networks (SCN) to Improve Resilience and Sustainability," *Sustainability*, vol. 14, no. 9, Jan. 2022, Art. no. 5237, https://doi.org/10.3390/su14095237.

[21] S. K. Paul, R. Sarker, and D. Essam, "Managing real-time demand fluctuation under a supplier–retailer coordinated system," *International Journal of Production Economics*, vol. 158, pp. 231–243, Dec. 2014, https://doi.org/10.1016/j.ijpe.2014.08.007.

[22] H.-L. Yang, "An optimal replenishment cycle and order quantity inventory model for deteriorating items with fluctuating demand," *Supply Chain Analytics*, vol. 3, Sep. 2023, Art. no. 100021, https://doi.org/10.1016/j.sca.2023.100021.

[23] G. Xiong and P. Helo, "An application of cost-effective fuzzy inventory controller to counteract demand fluctuation caused by bullwhip effect," *International Journal of Production Research*, vol. 44, no. 24, pp. 5261–5277, Dec. 2006, https://doi.org/10.1080/00207540600600114.

[24] L. Yang, C. T. Ng, and T. C. E. Cheng, "Optimal production strategy under demand fluctuations: Technology versus capacity," *European Journal of Operational Research*, vol. 214, no. 2, pp. 393–402, Oct. 2011, https://doi.org/10.1016/j.ejor.2011.04.028.

[25] T. C. Lopes, A. S. Michels, C. G. S. Sikora, N. Brauner, and L. Magatão, "Assembly line balancing for two cycle times: Anticipating demand fluctuations," *Computers & Industrial Engineering*, vol. 162, Dec. 2021, Art. no. 107685, https://doi.org/10.1016/j.cie.2021.107685.

[26] A. Collard-Wexler, "Demand Fluctuations in the Ready-Mix Concrete Industry," *Econometrica*, vol. 81, no. 3, pp. 1003–1037, 2013, https://doi.org/10.3982/ECTA6877.

[27] Z. Sun, Q. Xu, and B. Shi, "Dynamic Pricing of Ride-Hailing Platforms considering Service Quality and Supply Capacity under Demand Fluctuation," *Mathematical Problems in Engineering*, vol. 2020, Jul. 2020, Art. no. e5620834, https://doi.org/10.1155/2020/5620834.

[28] J. Zhang, M. Liu, and B. Zhou, "Analytical Model for Travel Time-Based BPR Function with Demand Fluctuation and Capacity Degradation," *Mathematical Problems in Engineering*, vol. 2019, Nov. 2019, Art. no. e5916479, https://doi.org/10.1155/2019/5916479.

[29] Y. Wang and P. Shang, "Shuttle Bus Rerouting and Rescheduling Problem considering Daily Demand Fluctuation," *Mathematical Problems in Engineering*, vol. 2022, Oct. 2022, Art. no. e2917240, https://doi.org/10.1155/2022/2917240.

[30] M. T. M. Espino and L. M. Bellotindos, "A System Dynamics Modeling and Computer-based Simulation in Forecasting Long-term Sufficiency: A Philippine Chicken Meat Sector Case Study," *Engineering, Technology & Applied Science Research*, vol. 10, no. 2, pp. 5406–5411, Apr. 2020, https://doi.org/10.48084/etasr.3301.

[31] K. Alexopoulos, N. Papakostas, D. Mourtzis, P. Gogos, and G. Chryssolouris, "Oscillator analogy for modelling the manufacturing systems dynamics," *International Journal of Production Research*, vol. 46, no. 10, pp. 2547–2563, May 2008, https://doi.org/10.1080/00207540601103118.

[32] Y. A. Davizon, R. Soto, J. de J. Rodriguez, E. Rodriguez-Leal, C. Martinez-Olvera, and C. Hinojosa, "Demand Management Based on Model Predictive Control Techniques," *Mathematical Problems in Engineering*, vol. 2014, Jun. 2014, Art. no. e702642, https://doi.org/10.1155/2014/702642.

[33] Y. A. Davizon, C. Martinez-Olvera, R. Soto, C. Hinojosa, and P. Espino-Roman, "Optimal Control Approaches to the Aggregate Production Planning Problem," *Sustainability*, vol. 7, no. 12, pp. 16324–16339, Dec. 2015, https://doi.org/10.3390/su71215819.

[34] T. Insperger, "On the Approximation of Delayed Systems by Taylor Series Expansion," *Journal of Computational and Nonlinear Dynamics*, vol. 10, no. 2, Mar. 2015, Art. no. 024503, https://doi.org/10.1115/1.4027180.

[35] D. Armbruster and K. G. Kempf, *Decision Policies for Production Networks*. New York, NY, USA: Springer, 2012.

[36] M. La Marca, D. Armbruster, M. Herty, and C. Ringhofer, "Control of Continuum Models of Production Systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 11, pp. 2511–2526, Aug. 2010, https://doi.org/10.1109/TAC.2010.2046925.

[37] W. Wang, D. E. Rivera, and K. G. Kempf, "Model predictive control strategies for supply chain management in semiconductor manufacturing," *International Journal of Production Economics*, vol. 107, no. 1, pp. 56–77, May 2007, https://doi.org/10.1016/j.ijpe.2006.05.013.

[38] J. D. Schwartz and D. E. Rivera, "A process control approach to tactical inventory management in production-inventory systems," *International Journal of Production Economics*, vol. 125, no. 1, pp. 111–124, May 2010, https://doi.org/10.1016/j.ijpe.2010.01.011.

[39] M. L. C. Hernandez, L. V. Rosas, R. F. R. Mantilla, G. J. E. Martinez, and V. V. Romero, "Supply Chain Cooperation by Agreed Reduction of Behavior Variability: A Simulation-based Study," *Engineering, Technology & Applied Science Research*, vol. 7, no. 2, pp. 1546–1551, Apr. 2017, https://doi.org/10.48084/etasr.1039.

[40] B. O. Odedairo and N. Nwabuokei, "Framework for Operational Performance Measurements in Small and Medium Scale Industries Using Discrete Event Simulation Approach," *Engineering, Technology & Applied Science Research*, vol. 8, no. 4, pp. 3103–3107, Aug. 2018, https://doi.org/10.48084/etasr.2106.