

A Study of an Empirical Sequent Depths Equation of the Hydraulic Jump in a Horizontal Trapezoidal Channel

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ABSTRACT

This study used existing studies and incorporated Pi theory to establish the connection between the sequent depth ratio (y_2/y_1) and the influencing factors (Fr_1 and M_1) on the hydraulic jump on a smooth horizontal trapezoidal channel, using a physical model with a side slope of 1:1. The study proposed four equations, from which the empirical equation (Y_3) was used to calculate the y_2/y_1 ratio of the steady jump ($4.0 \leq Fr_1 \leq 9.0$). An analysis of statistical indicators for (Y_3) showed that the maximum error was 7.4%, R^2 was 0.98, and other statistical indicators were close to the ideal point at zero (MSE = 0.027, RMSE = 0.163, MEA = 0.129, and MAPE \approx 2.2%). Furthermore, statistical analysis for test data also provided good results ($R^2 = 0.94$, MSE = 0.107, RMSE = 0.327, MEA = 0.25, and MAPE \approx 3.7%), and the maximum error reached 8.8%. Therefore, the proposed equation ensures that the calculated values can be used in practice.

Keywords-hydraulic jump; sequent depth; trapezoidal channel; empirical equation; Bélanger

I. INTRODUCTION

The hydraulic jump, discovered by Leonardo da Vinci in the 16th century, is used to design energy-efficient structures on the toe of spillways and other applications [1-3]. The hydraulic jump zone is a disturbance between water and air, creating complex rollers and causing energy loss. The hydraulic structure of the jump can be simulated by the Navier-Stokes equation and solved by simplifying the hydraulic factors [4-5]. Bélanger was the first to propose an equation to calculate the sequent depth of the jump in a rectangular channel [6]. Later, scientists improved this equation by including other influencing factors, such as shear stress [7] and the kinetic energy correction factor $\alpha = 1.045$ [8]. In [9], an empirical coefficient μ was introduced that depends on the inflow Froude number (Fr_1). As there is currently no adequate theoretical equation for the trapezoidal channel, most studies are experimental or semi-empirical. Although there has been relatively little study on the hydraulic jump in trapezoidal channels and there has been minimal appraisal of its applications, this phenomenon does occur frequently and has some significance in the design of dissipative energy structures.

The equation of the sequent depth ratio (y_2/y_1) of the jump depends on many factors. The critical depth of flow (y_c) can be

mentioned in the empirical sequent depth equation [5]. In [10], the equation of momentum was used to develop a quadratic phenomenon equation to determine y_2/y_1 based on the Fr_1 and M_1 coefficients. Based on the experimental values of the physical model, a graph of the relationship between y_2/y_1 and Fr_1 was built, according to M_1 , and tested with experimental data with theoretical curves in case of a jump in a trapezoidal channel with a 1:1 side slope. In [11], the momentum balance equation was used to determine the upstream and downstream cross-sections of the jump, forming an equation to determine the jump's y_2/y_1 for a smooth horizontal trapezoidal channel disregarding friction and solving it to determine the jump's subsequent depths using diagrams or tables. However, this study only examined theoretical data. In [12], the correlation between y_2/y_1 and Fr_1 of the hydraulic jump was examined in a trapezoidal channel with a side slope angle of 76.2° and a channel slope ranging from -0.005 to 0.02. This study constructed a linear empirical equation between y_2/y_1 and Fr_1 without considering the value of M_1 . In [13], experimental research was conducted on the sequent depth of the hydraulic jump in a trapezoidal channel with dimensions of 10 m in length and 0.2 m in bed width, and the results were similar to [12] but with variations in the experimental coefficients. In [14], experimental research was carried out on the jump in the trapezoidal channel with a trapezoidal cross-section of 0.2 m in

6 m bed width and a side angle of 72.68° . This study determined seven empirical equations for each case of y_1 and Fr_1 . However, as it was widely dispersed and poorly organized, it is challenging to translate these results into a practical equation. In [15], the hydraulic jump was investigated in a channel system that abruptly changed from a trapezoidal channel with a side angle of 73° , 4 m length, and 20 cm bed width to a rectangular channel with 0.6 m width and 6 m length, showing that the subsequent depths of the jump will depend on where the jump toe first appears in each channel. In [16], a horizontal trapezoidal channel with side slopes 1:1 and 1:1.5 and a bed width of 0.2 m was examined, creating empirical linear and nonlinear equations to gauge Fr_1 according to the y_2/y_1 and the M_1 coefficient. The limitation of this study was that the R^2 coefficient was not high, with values of 0.814 and 0.802, respectively, the relationships did not have enough data (there were 8 values for the case of the side slope 1:1.5 and 9 values for the case of the side slope 1:1), and Fr_1 had very little change (from 4.0 to 5.5). In [17], experimental research was carried out on 3 types of side slopes (0.26, 0.58, and 1.0) with bed width 0.2 of the horizontal trapezoidal channel. The discharge range was from 30 to 90 l/s and Fr_1 ranged from 1.2 to 8.67. This study indicated that the relationship between Fr_1 and the y_2/y_1 was close and applied numerical models for experimental tests.

The sequent depth (y_2/y_1) of the jump in a trapezoidal channel was studied mainly in the form of an empirical equation that depends on Fr_1 , but, for channels with different slopes, the equation has a significant change in the experimental coefficients. More research is needed to improve the general equations for calculating the conjugate depth of the trapezoidal channel. The jump is divided into 4 types based on Fr_1 : weak jump, oscillating jump, steady jump, and choppy jump [6]. The steady jump is commonly used, and this case is handled to adjust and design works that use the hydraulic jump in practice. This study focused on the factors influencing y_2/y_1 by examining existing studies on hydraulic jumps in both rectangular and trapezoidal channels. Buckingham's Pi theory was employed to assess and construct an objective function. In addition, experiments were carried out using a physical model to collect relevant data for steady jumps within the range of $4.0 \leq Fr_1 \leq 9.0$. By combining the experimental data, the objective function, and the structure of the existing equations, new empirical equations were constructed to determine the sequent depth ratio for steady hydraulic jumps, which were tested under the same experimental data and conditions as [10].

II. ANALYZING THE FACTORS AFFECTING THE SEQUENT DEPTHS

It is important to identify the variables that affect the hydraulic jump phenomena to choose the direction of the investigation and set a foundation for developing empirical equations based on those variables. Based on investigations of the hydraulic jump's subsequent depth in trapezoidal and rectangular channels with stilling basins, slopes, and smooth or uneven beds, an analysis of the influencing factors follows.

- Fr_1 is a factor that has the greatest influence on the conjugate depth of the jump and plays an important role in

the formation of the equation to determine y_2/y_1 . This factor appears in most sequent depth studies, such as the studies of the jump in a rectangular channel with a smooth or rough bed [7-9, 19-24]. Authors in [10-16] studied the trapezoidal channel. The higher the value of Fr_1 , the greater the y_2/y_1 , and the greater the energy dissipation in the jump, as shown in the hydraulic jump classification. However, the increase in the y_2/y_1 versus the upstream Froude number varies according to the geometrical characteristics of the cross-section.

- Some studies also created the relationship between y_2/y_1 and the inflow Froude number by Fr_1-1 , such as [25-27].
- In fundamental studies, the influence of bed friction is frequently ignored, resulting in theoretical equations that often require the inclusion of experimental coefficients. Nevertheless, certain investigations on hydraulic jumps in rectangular channels took into account the impact of this factor. For example, the equation can incorporate the shear stress coefficient [7] or the diameter of the bed roughness [24].
- Bélanger's equation had a value of 8 but was increased to 10.4 by the effects of the velocity distribution, as demonstrated in [8] in which the kinetic energy correction factor to obtain the equation for the sequent depth of the jump in the rectangular channel was 1.045.
- The slope of the channel affects the conjugate depth of the jump. A negative or positive slope will result in the establishment of various equations to measure the sequent depth ratio. For example, in [20] the bottom slope ($S^{1/2}$) was included in the equation to calculate the y_2/y_1 of the jump in the rectangular channel, which corresponds to studies of the jump in bed slopes with roughness or wide openings. For trapezoidal channels, some studies investigated the influence of bed slope on the y_2/y_1 equation [12-15].
- The ratio coefficient between the side slope, the inflow depth of the jump, and the bed width of the channel (M_1) is a characteristic coefficient that represents the geometric shape ratio of the trapezoidal channel. The value of M_1 can be found in divergent studies [10, 16, 18].
- Geometrical features of the critical depth (y_c)[5].

General consideration shows that the sequent depth ratio in a smooth horizontal prismatic channel has a general function as follows:

$$Y = \frac{y_2}{y_1} = f(Fr_1, M_1) \quad (1)$$

III. APPLYING THE PI THEORY IN STUDYING THE SEQUENT DEPTH

Using a momentum equation to study a geometrical feature (y_2/y_1) of the jump over the smooth bed (the friction force can be ignored), gives:

$$F_1 - F_2 = P_2 - P_1 \quad (2)$$

Equation (2) has been incorporated in several studies on the y_2/y_1 of the jump [28-31]. From the theory of dimensional analysis, the influencing factors are determined:

$$f(y_2, y_1, V_1, V_2, m, b, \alpha_{01}, \alpha_{02}, \rho, \mu, g) = 0 \quad (3)$$

By reducing similar characteristic parameters $\alpha_{0i} \approx 1$ (turbulent flow) and using the Pi theory of Buckingham, the following equations are obtained:

$$\beta\left(\frac{y_2}{y_1}, \frac{b}{y_1}, Re, m, Fr_1\right) = 0 \quad (4)$$

$$\frac{y_2}{y_1} = \Psi\left(\frac{b}{y_1}, Re, m, Fr_1\right) \quad (5)$$

where V is the velocity (m/s), m is a side wall slope, ρ is the mass density of the water (kg/m^3), μ is fluid viscosity (kg/m.s), g is the gravitational acceleration (m/s^2), Re is a Reynolds number, M_1 is the side wall constant in the trapezoidal channel ($M_1 = \frac{m \cdot y_1}{b}$), Fr_1 is the inflow Froude number ($Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ (or

$$Fr_1 = \frac{V_1}{\sqrt{g \cdot D}}$$

), and D is the hydraulic depth (m). Considering the characteristics of the cross-sectional coefficient, it is observed that for very large Reynolds numbers ($Re > 20000$), its influence can be ignored [32]. Therefore, (5) can be simplified to (1), while (5) effectively represents the factors influencing the y_2/y_1 of the hydraulic jump. In the case of studying the jump in a rectangular channel ($m = 0$), (5) exhibits similarities to Bélanger's equation [6].

IV. EXPERIMENTAL SETUP

A. Experimental Model Structure

The physical model was constructed at the Key Laboratory of River and Coastal Engineering (KLORCE), in Vietnam Academy for Water Resources (VAWR). Figure 1 shows the model, that consists of a reservoir, an ogee spillway, a trapezoidal channel, and a control gate at its end. The energy dissipation structure was the trapezoidal channel, having a length of 4.0 m, bed width of 0.335 m and 0.55 m, height of 0.65 m, and side slope of 1:1 (angle 45°).

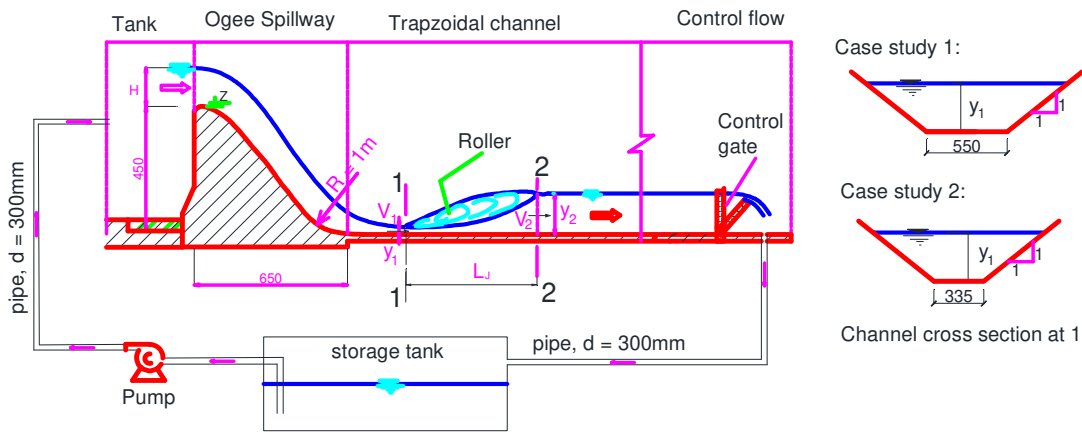


Fig. 1. Plan of the experimental equipment.



Fig. 2. The model from downstream to upstream (bed width 55 cm).

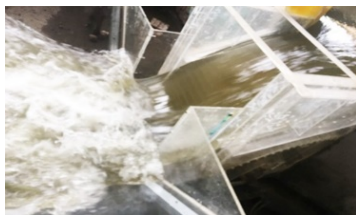


Fig. 3. Spillway and the hydraulic jump in the trapezoidal channel (bed width 55 cm, $Q = 165 \text{ l/s}$, and $Fr_1 = 5.72$).

B. Measuring Data

Level surveying and leveling staff are used to measure the depth of the flow. The technique involves having one person hold the leveling staff while the other reads the information from the surveying apparatus, as shown in Figure 4. The observed data are shown in Table I. The collected data are shown in Table II, with a total of 86 datasets (with $Fr_1 \geq 4.0$) from the experimental implementation on two physical models (bed width is 55 and 33.5 cm). These data were regulated to establish the relationship between y_2/y_1 and Fr_1 , as shown in Figure 5.

TABLE I. RANGE OF EXPERIMENTAL DATA

Parameter	Symbol	Unit	Max	Min
Discharge	Q	m^3/s	0.201	0.04
Initial depth of hydraulic jump	y_1	m	0.041	0.092
Secondary depth of hydraulic jump	y_2	m	0.488	0.182
Bed width of the channel	b	cm	55	33.5

TABLE II. RANGE OF DIMENSIONLESS PARAMETERS

Values	y_2/y_1	Fr_{D1}	Fr_1	M_1
Max	9.396	8.40	7.98	0.275
Min	4.397	4.19	4.01	0.06



Fig. 4. Measuring water level data by the surveying equipment and the leveling staff (bed width $b = 33.5$ cm, $Q = 136$ l/s, and $Fr_1 = 4.83$).

V. ESTABLISHING AN EMPIRICAL EQUATION OF THE CONJUGATE DEPTH

A. Relationship between y_2/y_1 and Fr_1

Considering the relationship between the y_2/y_1 with the Fr_1 based on the observed values of this and other studies, gives the following:

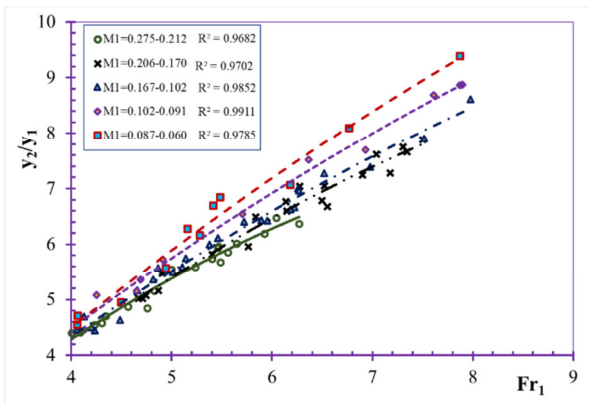


Fig. 5. Relationship between the y_2/y_1 and the Fr_1 according to M_1 .

Figure 5 shows that y_2/y_1 and Fr_1 tend to converge in each group of M_1 values (M_1 groups are referenced from [10]), so y_2/y_1 is a function of Fr_1 . This tendency, however, depends on M_1 and does not merely demonstrate the role of Fr_1 in the y_2/y_1 variation rule. Therefore, it is essential to completely take into account the upstream Fr_1 and coefficient M_1 of the hydraulic leap in the smooth horizontal trapezoidal channel while developing the connection to determine the conjugate depth. Equation (1), the equation structure of other studies, and experimental data of the leap on two physical models (physical model with bed widths of 55 and 33.5 cm) were adopted to develop the empirical equation for estimating y_2/y_1 .

B. Establishing an Empirical Equation

Using the measurement data in Table II, the study combined the influencing factors according to (1) and the structure of the existing equations. Table III shows the study combinations.

TABLE III. THE EMPIRICAL EQUATIONS FOR THE SEQUENT DEPTH OF THE HYDRAULIC JUMP

Symbol	Equations	R^2	Ref.
(Y_1)	$\frac{y_2}{y_1} = 0.721Fr_1^{1.151} + 0.875$	0.953	[12-15,17],
(Y_2)	$\frac{y_2}{y_1} = 0.888M_1^{-0.088}Fr_1^{1.003} + 0.204$	0.981	[10-11,16]
(Y_3)	$\frac{y_2}{y_1} = 1.051M_1^{-0.154}(Fr_{D1} - 1)^{0.871} + 0.464$	0.980	[25-27]
(Y_4)	$\frac{y_2}{y_1} = \frac{1}{2} \sqrt{1 + 9.286Fr_{D1}^{1.729} - 1}$	0.738	[7-9]

Figure 6 shows that the measured values are evenly distributed on both sides of the best-fit line ($y = x$), but (Y_4) , shown in Figure 6 (d) has a one-sided bias and a large error (the maximum error is 17.5%). Equations (Y_2) and (Y_3) , shown in Figure 6 (b)-(c), have a small error (7.8% and 7.4%, respectively). The values calculated by (Y_3) have $\pm 8\%$ difference from the measured data. Thus, in terms of statistical indicators, (Y_3) has better computational efficiency and is proposed.

VI. TEST AND EVALUATION OF THE PROPOSED EQUATIONS

A. Metrics for Evaluating Calculated Results

The statistical indicators used to evaluate the effectiveness of the proposed equations were [33-35]:

Mean Absolute Error (MAE):

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - x_i| \tag{6}$$

Mean Squared Error (MSE):

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - x_i)^2 \tag{7}$$

Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - x_i)^2} \tag{8}$$

R squared (R^2):

$$R^2 = 1 - \frac{\sum(y_i - x_i)^2}{\sum(y_i - \bar{x})^2} \tag{9}$$

Mean Absolute Percentage Error (MAPE, %):

$$MAPE = \frac{100}{n} \sum \frac{|x_i - y_i|}{x_i} \tag{10}$$

Error percentage (ε , %):

$$\varepsilon = \frac{|y_i - x_i|}{x_i} \times 100 \tag{11}$$

where y and x are the calculated values and the observed values, respectively, \bar{x} is the average observed value, and n is

the number of observations. To evaluate the calculation efficiency of the equations, the statistical indicators *MAE*, *MSE*, *RMSE*, *MAPE*, and ϵ should be as close to zero as possible

(ideal point), and the larger the R^2 value, the higher the calculation efficiency of the equation.

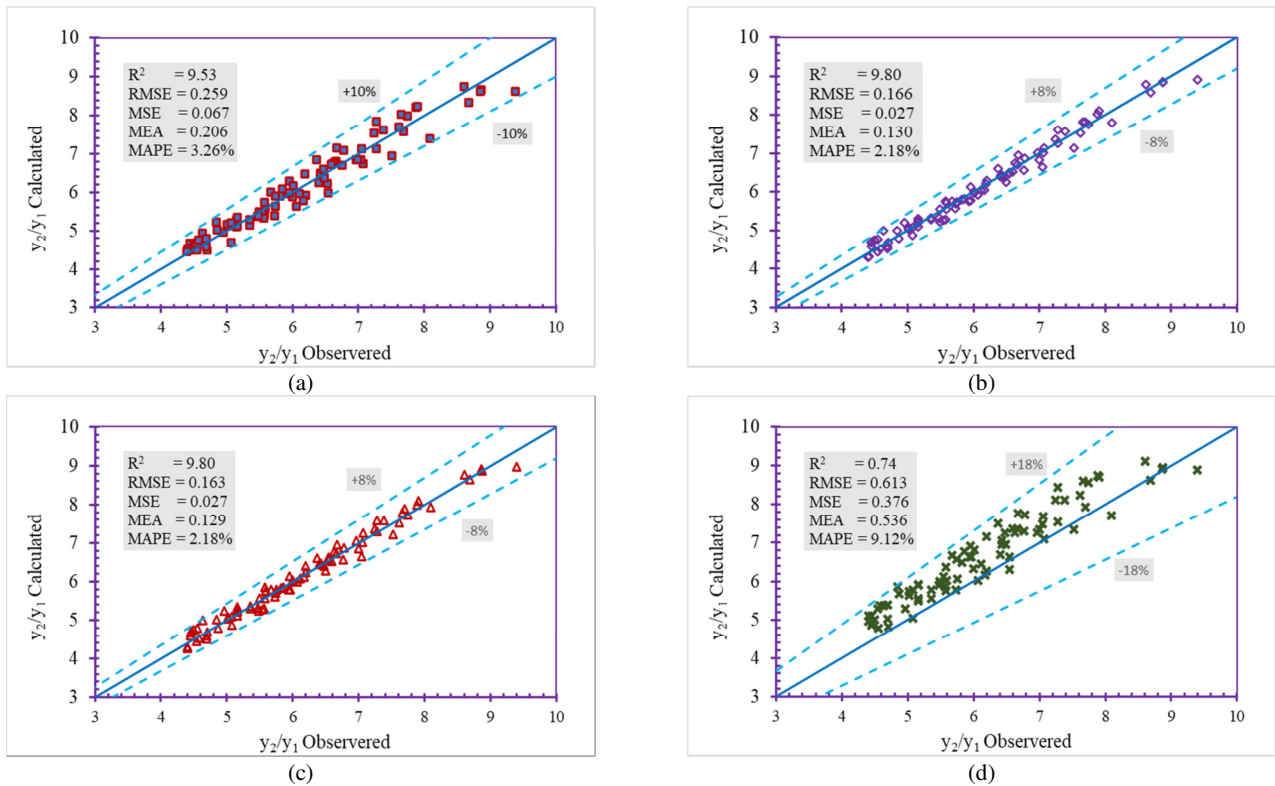


Fig. 6. Comparison of the observed values with the predicted values by the proposed equations. (a) Using (Y_1), (b) using (Y_2), (c) using (Y_3), (d) Using (Y_4).

B. Analysis According to the Statistical Indicators

The sequent depths ratio y_2/y_1 was calculated using the equations in Table III, with the study data shown in Table I, and Table IV shows the statistical indicators.

TABLE IV. STATISTICAL INDICATORS

Eq.	MEA	MSE	RMSE	R^2	MAPE (%)
Y_1	0.206	0.067	0.259	0.95	3.27
Y_2	0.130	0.027	0.166	0.98	2.18
Y_3	0.129	0.027	0.163	0.98	2.18
Y_4	0.536	0.376	0.613	0.74	9.12

Most of the proposed equations had good coefficients ($R^2 > 0.9$), indicating a strong correlation [33]. The other statistical indicators were also very small (close to zero). Equations (Y_2) and (Y_3) have strong indicators, such as the coefficient $R^2 = 0.98$ which is the largest, and the other statistical indicators were also lower compared to those of the other equations.

C. Test data

The proposed equations were tested on the data from [10], in which an experimental study of the hydraulic jump was executed in a smooth horizontal trapezoidal channel with a bed width of 0.2 m and a 1:1 side slope, which is similar to this study's side slope. This study investigated experimental cases with $4.0 \leq Fr_1 < 10$ and $y_1 \geq 3$ cm. The total test data [10] were

39 datasets. After removing the data that did not match the research criteria, there were 19 datasets left, as shown in Table V. Figure 7 shows the relationship between y_2/y_1 and the Fr_1 based on the M_1 values (the group of M_1 is classified according to the author's experimental data).

TABLE V. RANGE OF EXPERIMENTAL DATA OF [10]

Values	Q (l/s)	Fr_{D1}	Fr_1	M_1	y_2/y_1
Max	98.0	10.10	9.34	0.41	9.00
Min	7.5	4.70	4.14	0.10	4.33

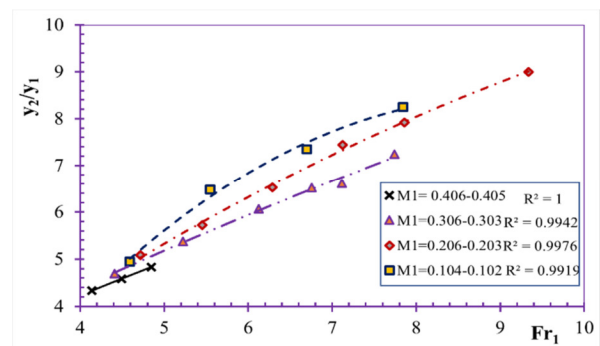


Fig. 7. The sequent depth depends on Fr_1 and M_1 .

Figure 7 shows that the relationship between y_2/y_1 and Fr_1 based on the quadratic function has a very strong correlation coefficient ($R^2 > 0.97$), but the division into many different groups to establish the relationship system and then building a general theoretical equation to reckon the y_2/y_1 is very difficult to ensure high accuracy.

D. Testing with the Observed Values of [10]

The proposed equations were used with the test data (Table V) from [10] to determine the calculated values, and the statistical indicators were used to evaluate their efficiency in calculation, as shown in Table VI.

TABLE VI. ANALYZING THE STATISTICAL INDICATORS FOR THE TEST DATA

Eq.	MEA	MSE	RMSE	R ²	MAPE (%)
(Y ₁)	0.491	0.386	0.621	0.78	7.50
(Y ₂)	0.259	0.121	0.348	0.93	3.80
(Y ₃)	0.250	0.107	0.327	0.94	3.70
(Y ₂)	1.088	1.468	1.212	0.61	17.78

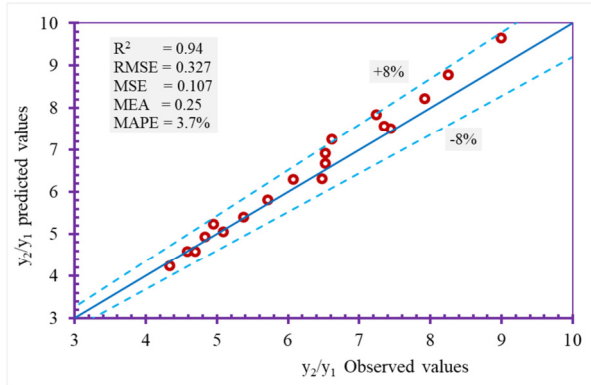


Fig. 8. Comparison of the observed with the predicted values by (Y₃).

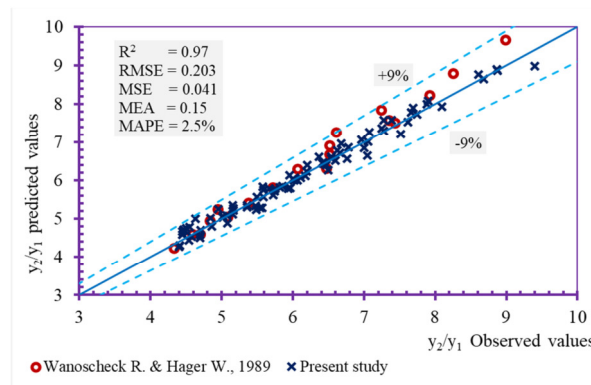


Fig. 9. Comparison of the calculated values by (Y₃) with all data (present study and [10]).

As shown in Table VI, (Y₂) and (Y₃) have good and equivalent statistical indicators, which is similar to the data of this study shown in Table IV. As shown in Tables IV and VI, (Y₃) has the best statistical indicators, which is emphasized by the coefficient $R^2 > 0.9$ and the other statistical indicators that approach zero, especially RMSE (0.163 and 0.327 for the present data and the test data, respectively) and MAPE (7.35%

for present data and 8.85% for the test data) are the best. Therefore, this study proposes the use of (Y₃) to estimate the sequent depth ratio y_2/y_1 of the jump in a horizontal trapezoidal channel with the side slope 1:1.

Figures 8 and 9 show that the calculated values exhibit a favorable agreement, falling within the agreement line, while all the data lie within the error line of $\pm 9\%$ of the measured data. Figure 8 shows that when using (Y₃) to calculate the y_2/y_1 , there is a strong agreement, with only one data point out of 19 falling outside the error band of $\pm 8\%$. This indicates that y_2/y_1 is influenced by Fr_1 , as observed in other studies. Additionally, for trapezoidal channels, the parameter M_1 , which encompasses the side slope coefficient m and the bed width of the channel, also affects y_2/y_1 . Thus, it was possible to remove the influence of the scale model (the influence of the bed width on different models) on the observed values in the physical model. When experimenting, the parameters y_1 , b , and Fr_1 always have a relationship with each other, which is adjusted through the discharge Q . Therefore, these results can be commonly controlled to calculate the conjugate depth of the hydraulic jump in the smooth horizontal trapezoidal channels with side slope 1:1.

VII. CONCLUSION

Research on the sequent depth of the jump in rectangular channels has a clear theoretical basis and its equation is popular and widely applied. For trapezoidal channels, the theoretical equation has not yet been developed, but has often been established by empirical equations. However, these equations do not yet describe all the effects of hydraulic factors. This study analyzed existing research and combined it with theoretical analysis to fully simulate the factors that directly affect the sequent depth y_2/y_1 of the jump on the trapezoidal channel. The objective function was obtained by establishing an empirical equation to calculate the sequent depth y_2/y_1 . This study also proposed different equations to determine depth. Using experimental data to establish and test the equations, their effectiveness was evaluated and the most effective equation was proposed. This study drew some key conclusions:

- Buckingham's Pi theory is effective in identifying important factors when establishing empirical equations. From that point forward, it guides some forms of the experimental equations and helps to propose quick equations that ensure scientificity.
- This study carried out experiments based on three models of different sizes that eliminated the condition of the influence of the physical model size on the research results.
- The foundation of this study lies in the structural analysis of existing formulas. Moreover, this study expanded upon these findings to develop an empirical equation specifically aimed at calculating the sequent depth y_2/y_1 of hydraulic jumps occurring in trapezoidal channels. Four equations were determined to calculate the sequent depth ratio y_2/y_1 . Basic statistical indicators (R^2 , MSE, RMSE, MAE, and MAPE) were used to evaluate the four proposed equations, showing that (Y₂) and (Y₃) had the best calculation efficiency.

- The proposed equations were tested on data from this study and [10], showing that (Y_3) had the best statistical indicators.
- Equation (Y_3) was adopted to estimate the sequent depth of the steady jump ($4.0 \leq Fr_1 \leq 9.0$) in an isosceles trapezoidal channel with a still basin and a side slope equal to 1.

This study carried out one type of physical experiment for the steady jump on an isosceles trapezoidal channel. More research is needed to determine the empirical equation for divergent side slopes of the trapezoidal channel. Despite the limitations of this study, the results clarified the relationship between the sequent depth y_2/y_1 of the jump and the hydraulic factors Fr_1 and M_1 .

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