A Study regarding the Technical-Economical Optimization of Structural Components for enhancing the Buckling Resistance in Stiffened Cylindrical Shells

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Received: 23 June 2023 | Revised: 12 July 2023 | Accepted: 16 July 2023

DOI: https://doi.org/10.48084/etasr.6135

ABSTRACT

This paper presents a technical-economical optimization by maximizing the ratio between the critical buckling pressure (technical characteristic) and the production cost (economic characteristic) of stiffened cylindrical shells, a basic concept of value analysis. Critical buckling load values were determined using both the Finite Element Method (FEM) and analytical calculations to validate the accuracy of the results obtained. The maximum difference between the analytical and numerical results was 10%. Technical-economic optimization was carried out using the design of experiments method with MINITAB 19 and allowed to select the optimal input parameters, stiffener dimensional ratio 0.10, shell wall thickness 2.50 mm, and distance between circumferential stiffeners 400 mm, and identify the main factors that impact the output response. For the optimal constructive configuration, the ratio between the critical buckling load and the production cost of the stiffened cylindrical shells was maximized by 199%.

Keywords-buckling; thin walled cylindrical shell; stiffeners; finite element analysis; design of experiments; optimization; value analysis

I. INTRODUCTION

Thin-walled structures, widely used in aerospace, automotive, and construction industries, are very susceptible to buckling, a common phenomenon in such structures [1-5]. To increase the buckling strength of thin-walled structures, additional structural components, such as stiffeners, can be used. Although increasing the thickness of the main structure is a commonly used method to improve its strength, stiffeners offer an alternative solution that allows achieving the same level of stiffness and strength with significantly less material consumption. Several studies [4, 6-12] have investigated the buckling characteristics of stiffened cylindrical shells.

In [13], the Ritz method was introduced as a means of analyzing the elastic buckling behavior of shells with ring stiffeners under various pressure loadings, considering ring stiffeners with flexible cross-sections that allow for different shapes. The rings were positioned in arbitrary distributions along the length of the shell. In [14], the implementation of optimal reinforcements was investigated using the PANDA2 computer program, which facilitates the minimum weight design of cylindrical shells reinforced with rings and stringers. In [12], an optimization study was presented that focused on improving the buckling performance of a simply supported cylindrical shell stiffened by inner rings when subjected to external pressure. The objective was to maximize the critical buckling load by optimizing the design parameters, such as the dimensions and distribution of the inner rings. In [15], topology optimization was carried out to determine the optimal pattern of stiffeners across the shell surface, aiming to maximize buckling strength. The buckling load capacities of cylindrical shells with cutouts were numerically examined with and without stiffeners. In [16], Finite Element (FE) analysis was used to predict the collapse behavior of ring-stiffened cylinders under hydrostatic loading, considering non-linear elastoplastic deformations. Various experimental test models were analyzed using FE models, which incorporated measured as-built shape data, including out-of-circularity, frame alignment, tilt, and other scantlings. Additionally, the FE models considered residual stresses induced by cold bending. In [7], FEM was used to predict shell and general instability failure modes for ring-stiffened thin cylindrical shells under external pressure.

This study focused on carrying out a technical-economical optimization regarding the buckling load of stiffened thin-walled cylindrical shells under uniform external pressure, evaluated using numerical simulations and validated with analytical calculation methods. The Design of Experiment (DOE) approach was implemented to ensure the optimization of the parameters that influence buckling load (stiffeners dimensions, stiffeners distance, and shell wall thickness), based on a comprehensive and innovative research method using the value analysis concept. By considering both technical and economic factors, this study provides a comprehensive perspective on the optimization process. This approach enables the balance between performance and cost-effectiveness to be evaluated, ultimately leading to informed design choices and efficient resource allocation when designing thin-walled stiffened cylindrical shells under uniform external pressure. The findings can be applied practically in industries, and the novel approaches used, such as validated simulations, DOE methodology, and value analysis concept, contribute to its significance and potential impact in the field of stiffened thin-walled cylindrical shells under uniform external pressure.

II. METHODOLOGY

The analyzed thin-walled cylindrical shell had external radius $R_e = 1734$ mm, length $L = 6000$ mm, and was made of steel with Young modulus $E = 2.1 \cdot 10^5$ MPa and Poisson coefficient $\upsilon = 0.3$. Circular stiffeners were considered to be steel equal angle profiles with the dimensional characteristics presented in Table I. Stiffener's dimensional ratio was calculated by dividing the stiffener's width by its length. The optimization was made using a full factorial design method with Minitab 19, which provides a comprehensive set of tools and features that allow users to analyze data, visualize results, and make data-driven decisions. This study considered three input parameters, the stiffener's dimensional ratio, shell wall thickness, and distance between two consecutive circumferential stiffeners, with three levels, as presented in Table II.

TABLE I. DIMENSIONAL CHARACTERISTICS OF STIFFENERS

Stiffener type	Stiffener dimensions (mm)	Stiffener dimensional ratio	Stiffener weight (kg/m)
S1	30×30×3	0.10	1.2996
S2	25×25×3	0.12	1.1000
S3	20×20x3	0.15	0.9576

TABLE II. PARAMETERS AND LEVELS USED IN DOE ANALYSIS.

Parameter	Level		
	1	2	3
Stiffener dimensional ratio	0.10	0.12	0.15
Shell wall thickness, mm	2.50	3.00	4.00
Stiffeners distance, mm	400	500	600

The number of simulations that should be performed was determined by n^k, where n is the number of input factors and k is the number of levels. In this case, an orthogonal array of $3^3=27$ tests was used. The optimization was based on the concept of implementation of the value analysis, considering the fundamental relation [17-25]:

$$\frac{V_i}{C_p} \rightarrow max. \tag{1}$$

where V_i is the use value and C_p is the production cost of the product. In this application, the use value can be considered the technical characteristic (critical buckling pressure) and the production cost an economical characteristic. According to (1), the ratio between the technical and economic characteristics should be maximized.

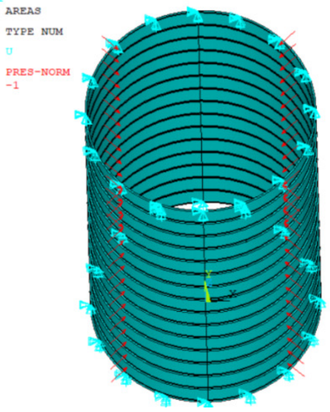

Fig. 1. The loads and boundary conditions used for FE calculation.

FEM was used to evaluate the critical buckling pressure for the 27 sets of data. The parametric model of the stiffened shell was created and written in the ANSYS Parametric Design Language (APDL) using the FE SHELL181 for cylindrical shells and circumferential stiffeners. Linear buckling analysis was implemented and an external pressure unit load was applied, as shown in Figure 1, so the load factor calculated during the analysis is equal to the critical buckling pressure, as in [6]. Simply supported boundary conditions were considered. To validate the numerical results, the critical buckling pressure was calculated using the formula [9]:

$$\begin{aligned}&\lambda\left(0{,}5m^2\left(\frac{\pi R}{L}\right)^2+n^2\right)=\xi_2(-2n^2-n^3b_n)+m^4\left(\frac{\pi R}{L}\right)^4\\&+(2+\eta_{t2})m^2n^2\left(\frac{\pi R}{L}\right)^2+(1+\eta_{02})n^4\\&+12\left(\frac{R}{t}\right)^2\left[(1+\mu_2)(1+nb_n)+\nu m\frac{\pi R}{L}a_n\right]\end{aligned} \tag{2}$$

where the external pressure q appears in the form of the dimensionless parameter λ:

$$\lambda = \frac{R^3}{D} \cdot q \qquad (3)$$

where R is the cylinder's mean radius, t is the shell's wall thickness, and L is the shell length. Parameter D is defined as:

$$D = \frac{Et^3}{12(1-\nu^2)} \qquad (4)$$

The other terms contain expressions calculated as functions of the stiffener's geometrical characteristics and the distance between successive circumferential stiffeners, described in [9]. Critical buckling pressure was obtained by minimizing the expression of λ for the integer values of m and n, which are the number of circumferential and axial waves, respectively.

III. RESULTS AND DISCUSSION

A. *Evaluation of Critical Buckling Loads*

Figure 2 presents the comparison between analytical and numerical values of critical buckling pressure and also the values of the ratio between the critical buckling pressure and the production cost, as defined by (1).

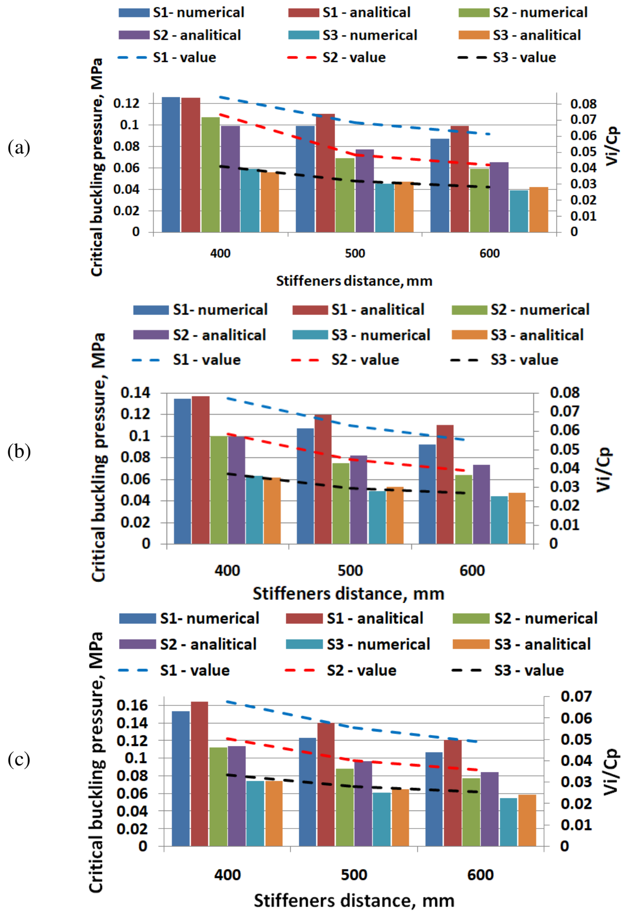

Fig. 2. Comparison between analytical and numerical results: (a) t = 2.5 mm, (b) t = 3 mm, and (c) t = 4mm.

Figure 3 shows one example of the deformed shape of the stiffened shell under external pressure. The graphs shown in Figure 2 validate the accuracy of the numerical results obtained. The maximum difference between the analytical and numerical values of the critical buckling pressure was 10%. Similar differences (8-22%) were obtained in [7]. In [6], the highest deviation between analytical and numerical results regarding the critical buckling pressure was 9%.

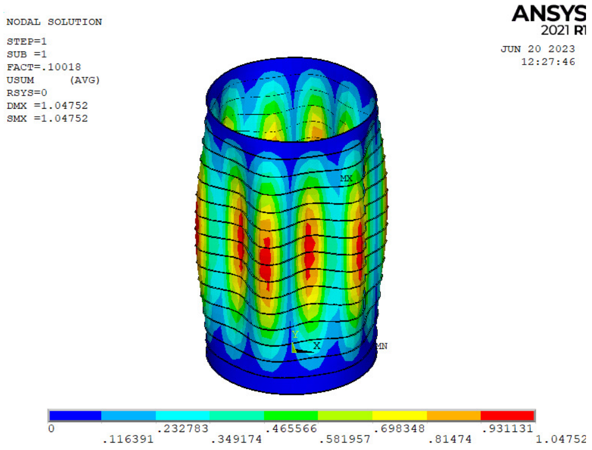

Fig. 3. Deformed shape of stiffened cylindrical shell under uniform external pressure.

B. *Optimization Results*

1) *Main Effects Plots*

The main effects plot is used to establish how the input parameters influence the ratio between the critical buckling pressure and production cost.

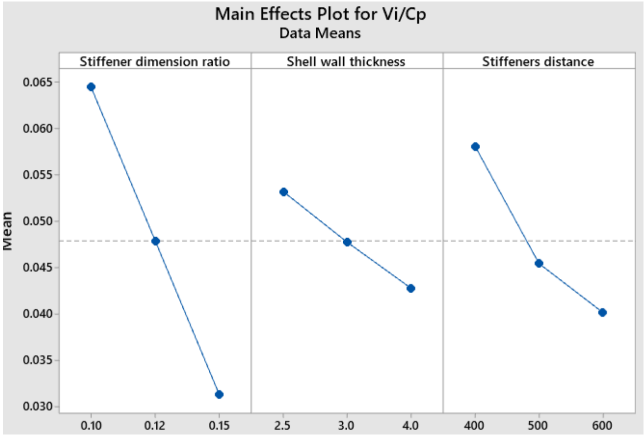

Fig. 4. Main effects plots.

As shown in Figure 4, the stiffener dimension ratio had the most significant effect on the response, followed by stiffener distance and shell wall thickness. The same conclusion can be taken from the Pareto chart shown in Figure 5. These graphs show how the ratio between the critical buckling pressure and production cost increases with increasing stiffeners' dimensions and their number while decreasing shell wall thickness.

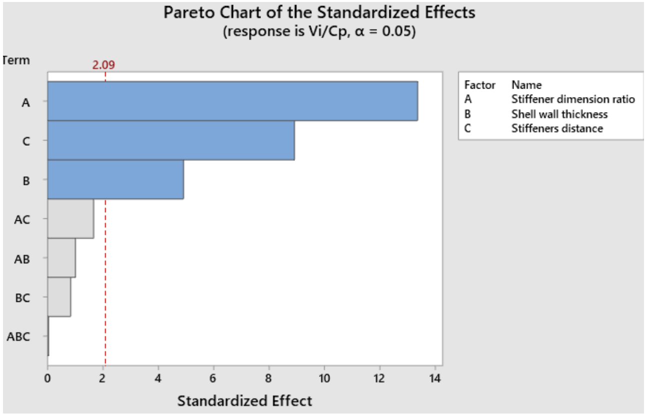

Fig. 5. Pareto Chart of the standardized effects.

Interaction plots serve as a valuable tool for assessing the presence of interactions, illustrating the means of each factor level while keeping the level of another factor constant. Interactions occur when the response at a particular factor level depends on the levels of other factors. Parallel lines in an interaction plot indicate the absence of interaction. On the contrary, the greater the deviation of the lines from parallel, the stronger the interaction. In this analysis of the interaction plot, shown in Figure 6, it can be seen that the lines representing shell wall thickness and thickness dimensional ratio are almost parallel, particularly for the first two stiffener dimensional ratio values, suggesting a lack of significant interactions between them. However, there is a slight change when considering the relationship between the stiffener distance and the stiffener dimensions. In this case, there is a notable departure of the lines from the parallel state, indicating the presence of significant interactions between these factors.

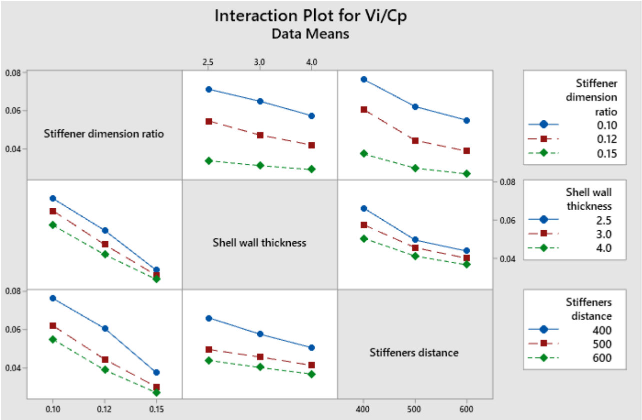

Fig. 6. Interaction plot of different input parameters.

Based on the examination of the residual plots in Figure 7, it is evident that the normal probability plot is close to a straight line, indicating that the residuals follow a normal distribution. However, the overall trends can be considered linear, except for some data points that deviate from a straight line. This deviation is reflected in the histogram, which does not exhibit a perfect bell-shaped distribution.

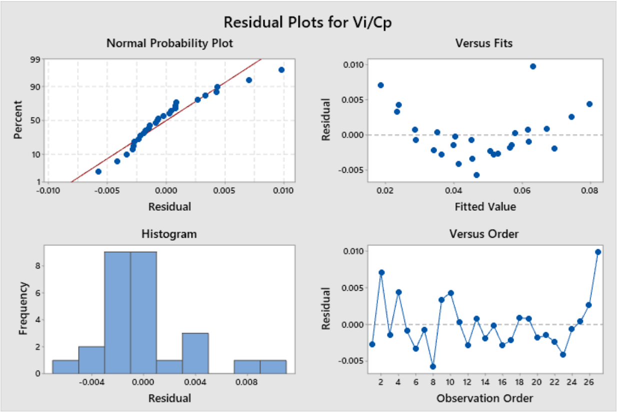

Fig. 7. Residual plots.

2) *Contour Plot*

Figure 8 shows the contour plots of the ratio variation between the critical buckling pressure and the production cost for the input parameters.

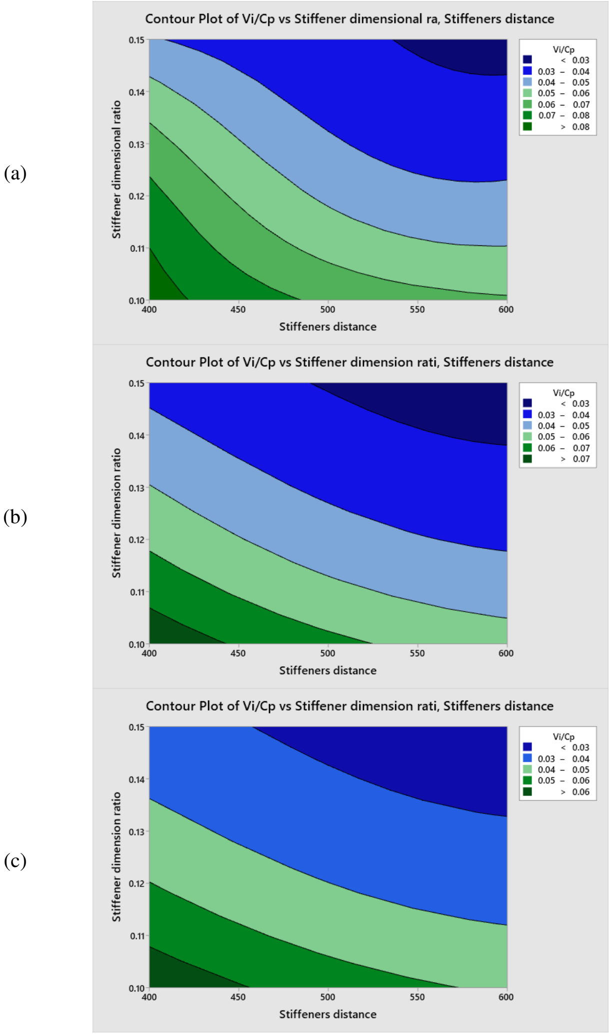

Fig. 8. Contour plots: (a) *t* = 2.5 mm, (b) *t* = 3 mm, and (c) *t* = 4mm.

The dark blue region corresponds to the lower ratio values, and the dark green corresponds to the greater ones. The contour had a similar pattern for all shell wall thicknesses considered. The highest values of the ratio between critical buckling pressure and production cost were obtained for a 0.1 stiffener dimensional ratio and a 400 mm stiffener distance, corresponding to 15 circumferential stiffeners. Figure 8 shows that the ratio between the use value and production cost of a stiffened cylindrical shell can be increased by 199% for shell thickness $t = 2.5$ mm, 187% for shell thickness $t = 3$ mm, and 164% for shell thickness $t = 4$ mm when adopting the optimal constructive configuration.

C. *Response Optimization*

The optimal configuration of the structure corresponding to the maximum ratio between the critical buckling pressure and production cost can be determined using the MINITAB 19 software, as can be seen in Figure 9. The optimized values of input parameters for the maximization of response, i.e., the ratio between the critical buckling pressure and production cost, were: stiffener dimensional ratio 0.1, shell wall thickness 2.5 mm, and distance between circumferential stiffeners 400 mm. Therefore it is preferable to use a thinner shell wall strengthened by additional structural components, leading to significantly less material usage, as also concluded in [12].

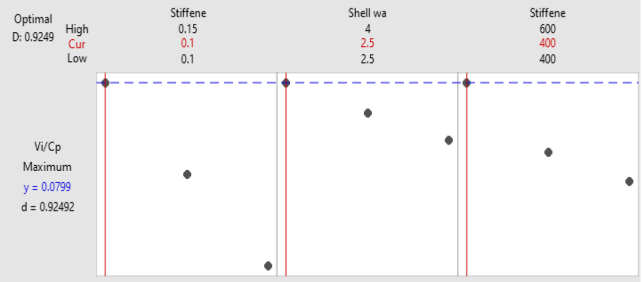

Fig. 9. Structure optimization for the maximum value of the ratio between critical buckling pressure and production cost.

IV. CONCLUSIONS

This study investigated the behavior of thin-walled cylindrical shells stiffened in a circumferential direction and subjected to uniform external pressure. Using the basic concepts of value analysis, an optimization study was carried out to establish the dimensional configuration of stiffened cylindrical shells to maximize the ratio between the critical buckling load (technical characteristic) and the production cost of a stiffened cylindrical shell (economic characteristic). The critical buckling load was calculated using FEM and eigenvalue linear buckling analysis with Ansys software. The accuracy of the results was verified by comparing them with the analytical results obtained using a complex methodology from the literature, and the maximum difference obtained was 10%. A total of 27 simulations were carried out for different stiffened shell configurations, corresponding to three input factors, the stiffener dimensional ratio, shell wall thickness, and stiffener distance, and three levels for each factor. After evaluating critical buckling pressure and the cost of structure production for each geometric configuration considered, the results were optimized using a full factorial orthogonal array. The main-effects plots highlighted that the most significant factor for maximizing the ratio between critical buckling load and production cost of a stiffened cylindrical shell was the stiffener's dimension ratio, followed by stiffeners distance and shell wall thickness, as indicated also by a Pareto chart.

This study also found the best parameter combinations for different structural configurations in terms of technical-economical optimization by maximizing the ratio between the use value and the production cost. The ratio between critical buckling load and production cost of stiffened cylindrical structures was maximized by 199% for the most efficient constructive configuration. On the basis of the performed analysis, it is recommended to adopt a strategy that involves using a thinner shell wall strengthened by additional structural components. This approach offers several advantages, including a substantial decrease in material consumption while maintaining the required strength and integrity of the structure.

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