

A Survey on H_∞ Control-Based Output Feedback Techniques

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ABSTRACT

The study of 2-D discrete systems has always been a preferred choice amongst researchers and academics, due to its diversified applications in most practical applications. For more than two decades, research based on H_∞ control techniques has been the focus of attention, as it plays a key role in the design and development of various applications based on signal processing and control theory. In many practical applications, the accessibility of the state vectors is not possible, and, in such cases, output feedback techniques are most appropriate. This paper presents a detailed survey based on H_∞ control-based output feedback techniques for 2-D discrete systems.

Keywords-robust stability; asymptotic stability control; 2-D discrete system; static output feedback controller; dynamic output feedback controller; observer based output feedback controller; bounded real lemma

I. INTRODUCTION

2-D state space models got attention with the introduction of the Roesser model [1], and, since then, several studies proposed other 2-D state space models. The most popular and investigated linear state space models are Roesser's [1], Fornasini and Marchesini's (FM) first model [2], the second FM model [3], and Kurek's model [4]. The output feedback technique for 2-D discrete models can also be extended to nonlinear systems for designing robust and efficient control systems that can achieve desired performance objectives. However, the design process can be complex and requires careful consideration of the nonlinear dynamics of the system, as well as the stability and performance requirements of the applications [5-7].

The state space equation of the 2-D discrete FM first model [3] is:

$$x(i+1, j+1) = A_1 x(i, j+1) + A_2 x(i+1, j) + A_3 x(i, j) + Bu(i, j) \tag{1}$$

$$z(i, j) = Cx(i, j) \tag{2}$$

$$i \geq 0, j \geq 0 \tag{3}$$

where $x(i, j)$ is a state vector of $n \times 1$ dimension, $A_1 \in \mathbb{R}^{n \times n}$, $A_2 \in \mathbb{R}^{n \times n}$, $A_3 \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $u(i, j)$ is an input vector of $m \times 1$ dimension, $z(i, j)$ is a scalar out $1 \times n$ dimension, and $C \in \mathbb{R}^{1 \times n}$. Figure 1 shows a matrix block diagram representation for the system of (1) and (2).

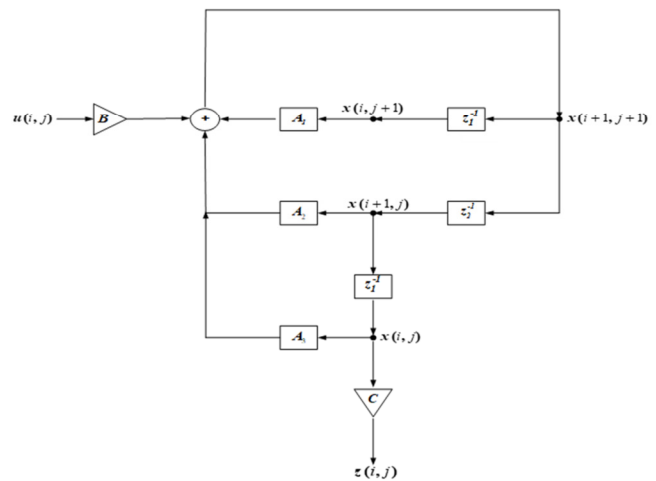


Fig. 1. Matrix block diagram representation for the system of (1) and (2).

The system has a finite set of initial conditions as defined based on the existence of two positive integers r_1 and r_2 such that:

$$x(i, 0) = 0, i \geq r_1 \tag{4}$$

$$x(0, j) = 0, j \geq r_2 \tag{5}$$

The second FM model is also a state space model representation of 2-D discrete systems [4]. The state space equation of this model is given by:

$$x(i+1, j+1) = A_1x(i, j+1) + A_2x(i+1, j) + B_1u(i, j+1) + B_2u(i+1, j) \tag{6}$$

$$z(i, j) = Cx(i, j) + Du(i, j) \tag{7}$$

$$i \geq 0, j \geq 0 \tag{8}$$

where $x(i, j)$ is a state vector of $n \times 1$ dimension, $A_1 \in \mathbb{R}^{n \times n}$, $A_2 \in \mathbb{R}^{n \times n}$, $B_1 \in \mathbb{R}^{n \times m}$, $B_2 \in \mathbb{R}^{n \times m}$, $u(i, j)$ is an input vector of $m \times 1$ dimension, $z(i, j)$ is a scalar out $1 \times n$ dimension, and $C \in \mathbb{R}^{1 \times n}$. Figure 2 shows a matrix block diagram representation for the system in (6),(7). The system has a finite set of initial conditions as defined based on the existence of two positive integers r_1 and r_2 such that:

$$x(i, 0) = 0, i \geq r_1 \tag{9}$$

$$x(0, j) = 0, j \geq r_2 \tag{10}$$

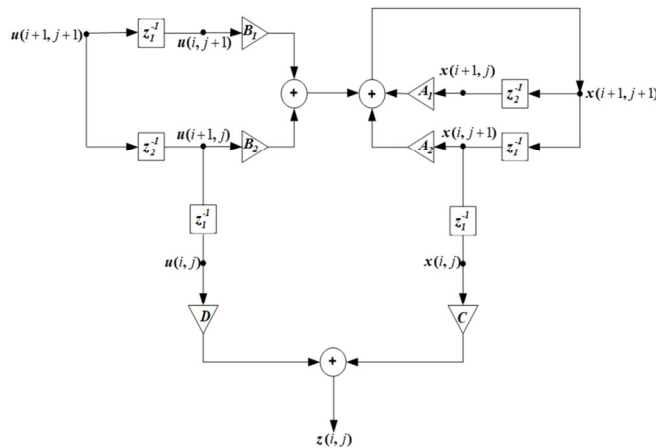


Fig. 2. Matrix block diagram representation for the system of (6)-(8).

The state space form of a 2-D discrete system can also be described by the Roesser model [1] as:

$$\begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(i, j) \tag{11}$$

$$z(i, j) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + Du(i, j) \tag{12}$$

$$i \geq 0, j \geq 0 \tag{13}$$

In a condensed way, it can be represented as:

$$x_{11}(i, j) = Ax(i, j) + Bu(i, j) \tag{14}$$

$$z(i, j) = Cx(i, j) + Du(i, j) \tag{15}$$

$$i \geq 0, j \geq 0 \tag{16}$$

where $x^h(i, j) \in \mathbb{R}^m$ is the horizontal state vector, $x^v(i, j) \in \mathbb{R}^n$ is the vertical state vector, $A_{11} \in \mathbb{R}^{m \times m}$, $A_{12} \in \mathbb{R}^{m \times n}$, $A_{21} \in \mathbb{R}^{n \times m}$, $A_{22} \in \mathbb{R}^{n \times n}$, $B_1 \in \mathbb{R}^{m \times q}$, $B_2 \in \mathbb{R}^{n \times q}$, $u(i, j) \in \mathbb{R}^q$, $z(i, j)$ is a scalar output, $C_1 \in \mathbb{R}^{1 \times m}$, $C_2 \in \mathbb{R}^{1 \times n}$, and $D \in \mathbb{R}^{1 \times q}$. Figure 3 shows a matrix block diagram representation for the system (11). The system has a finite set of initial conditions, as defined based on the existence of two positive integers r_1 and r_2 such that:

$$x(i, 0) = 0, i \geq r_1 \tag{17}$$

$$x(0, j) = 0, j \geq r_2 \tag{18}$$

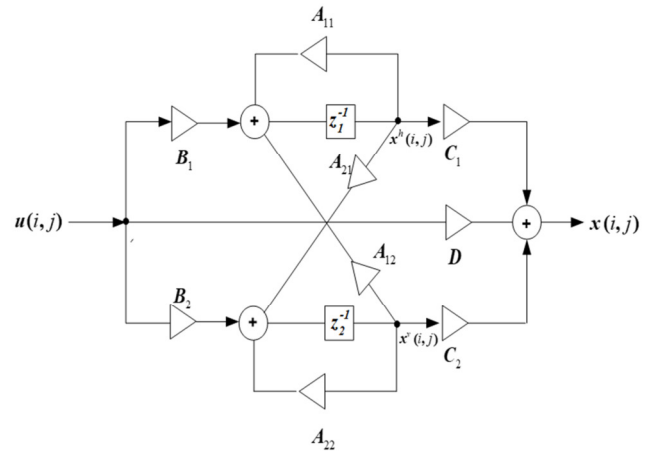


Fig. 3. Matrix block diagram representation for the system (11) and (12).

The state space representation of a 2-D discrete linear shift-invariant can also be represented by the general model [5]:

$$x(i+1, j+1) = A_1x(i, j+1) + A_2x(i+1, j) + A_3x(i, j) + B_1u(i, j+1) + B_2u(i+1, j) + B_3u(i, j) \tag{19}$$

$$z(i, j) = Cx(i, j) + Du(i, j) \tag{20}$$

$$i \geq 0, j \geq 0 \tag{21}$$

where $x(i, j)$ is an $n \times 1$ state vector, $A_1 \in \mathbb{R}^{n \times n}$, $A_2 \in \mathbb{R}^{n \times n}$, $A_3 \in \mathbb{R}^{n \times n}$, $B_1 \in \mathbb{R}^{n \times m}$, $B_2 \in \mathbb{R}^{n \times m}$, $B_3 \in \mathbb{R}^{n \times m}$, $u(i, j)$ is an input vector of dimension $m \times 1$, $z(i, j)$ is a scalar output of dimension $1 \times n$, $C \in \mathbb{R}^{1 \times n}$, and $D \in \mathbb{R}^{1 \times m}$. Figure 4 shows a matrix block diagram representation for the system (19)-(20).

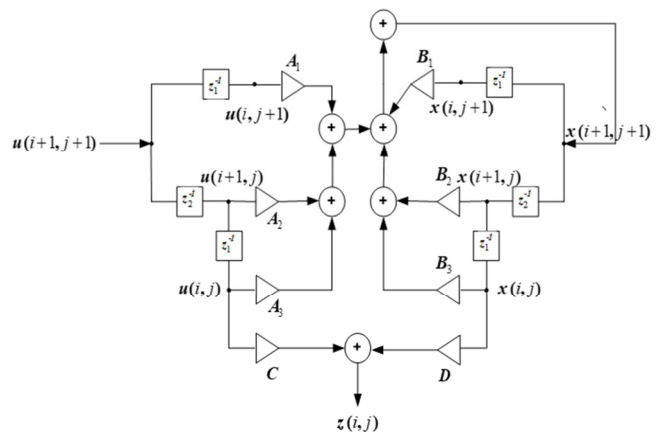


Fig. 4. Matrix block diagram representation for the system (19) and (20).

In addition, this system has a finite set of initial conditions, as defined based on the existence of two positive integers r_1 and r_2 such that:

$$x(i, 0) = 0, i \geq r_1 \tag{22}$$

$$x(0, j) = 0, j \geq r_2 \tag{23}$$

II. BACKGROUND

A. H_∞ Performance

The initiation of the H_∞ control theory began when in [8] the problem of sensitivity reduction was observed by the feedback mechanism in the basic input-output setting of the system, as shown in Figure 5.

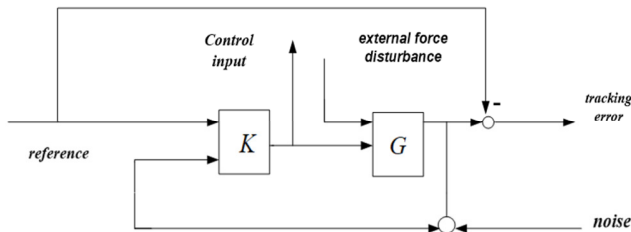


Fig. 5. Block diagram of a typical closed-loop performance objective.

The main objective is to keep the value of K to maintain the tracking errors and control input value as low as possible for all sensor noises, reference commands, and external force disturbances. From the outside influences to the regulated variables, let the closed-loop mapping denoted by T , then,

$$\begin{bmatrix} \text{tracking error} \\ \text{control input} \end{bmatrix} = T \begin{bmatrix} \text{reference} \\ \text{external force} \\ \text{noise} \end{bmatrix}$$

regulated variables *outside influences*

The performance can be assessed in terms of measurement of gain from outside influences to regulated variables. If the value of T is small, it means that the performance is good. As the closed-loop system is a Multi-Input, Multi-Output (MIMO) dynamic system, there are two different aspects on which the gain of T depends:

- Spatial (vector disturbances/vector errors)
- Temporal (dynamic relationship between input/output signals). Therefore, the performance criterion should take into consideration:
 - The relative magnitude of outside influences
 - The frequency dependence of signals
 - The relative importance of the magnitudes of regulated variables

The performance objective in the form of a matrix norm is expressed as a weighted matrix norm given by:

$$\|W_L T W_R\| \tag{24}$$

where W_L and W_R are frequency-dependent weighting functions responsible for the bandwidth constraints and the spectral content of exogenous signals. The H_∞ norm is a natural (mathematical) way to characterize acceptable performance in terms of MIMO $\|\cdot\|_\infty$. In optimal control theory, there are two popular performance measures: H_2 and H_∞ norms. The H_2 norm is useful when exogenous signals are fixed or have a fixed power spectrum. There are two assumptions on which the H_2 filtering approach (also called Kalman) is based [9-10]. The

first is that the system under consideration is exactly known and, secondly, there is a priori information on the external noises. The H_∞ norm is useful when the disturbance is not a fixed signal but can be represented as weighted balls of exogenous signals, meaning that when all the outside influences such as reference commands, sensor noise, and external force disturbances are mapped into regulated variables, then a weighting function W is used that acts as a design parameter. This weighting function matrix W is frequency dependent and accounts for bandwidth constraints and spectral content of exogenous signals. The value of W should be small for good performance [11]. The signal l_2 norm is given by:

$$\|\mathbf{x}\|_2 = \sqrt{\sum_{k=0}^{\infty} \|\mathbf{x}(k)\|^2} \tag{25}$$

Bounding the l_2 -induced norm from input to output leads to H_∞ optimization.

B. Output Feedback H_∞ Control

The motivation behind using output feedback techniques is that the state vector is frequently not fully accessible in practice. However, in many practical applications, accessibility of the state vectors is not always possible and sometimes the state vector measurement is too expensive. Controlling through the output feedback technique is most appropriate in such situations. This has led many studies [12-27] to investigate and analyze output feedback techniques for 2-D discrete systems.

In an optimal H_∞ controller, all admissible K controllers should be found to minimize $\|T_{zw}\|_\infty$ [28]. However, it is difficult to find such an optimal H_∞ controller [29]. This is not the case with the H_2 theory, where the optimal controller can be found by solving two Riccati equations without iterations. The optimal H_∞ controller can be found through the chain of successive solutions of H_∞ suboptimal problems [30] with γ approaching the γ_{min} value. Theoretically, it is not necessary to design an optimal H_∞ controller [31], despite the importance of the optimal H_∞ norm. Furthermore, it is always cheaper to design suboptimal controllers in the norm sense, as they are quite close to the optimal. The bandwidth requirement is also less in designing a suboptimal controller. Given a scalar number $\gamma > 0$, the effort is to discover all the admissible controllers K such that $\|T_{zw}\|_\infty < \gamma$. In this way, a suboptimal controller can be designed. In an H_∞ control, the following conditions are needed to design an asymptotically stable output feedback controller:

For the continuous-time case:

- The closed-loop system is asymptotically stable when $w(t)=0$.
- The closed-loop system has a prescribed level γ of H_∞ noise attenuation, i.e., under the zero initial condition:

$$\int_0^\infty z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty w^T(t)w(t) dt \tag{26}$$

is satisfied for any nonzero $w(t) \in L_2[0, \infty)$.

For the discrete-time case:

- The closed-loop system is asymptotically stable when $w(k)=0$.

- The closed-loop system has a prescribed level γ of H_∞ noise attenuation, i.e. under the zero initial condition:

$$\sum_{k=0}^{\infty} z^T(k)z(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k) \quad (27)$$

is satisfied for any nonzero $w(k) \in L_2[0, \infty)$.

where $z(t)z(k)$ and $w(t)w(k)$ denote the system-controlled output variable and noise signal, respectively.

1) Finite Horizon H_∞ Control Problem

The H_∞ control problem aims to minimize the H_∞ norm. The H_∞ norm value is the maximum of all disturbances w of the portion of the amount of energy entering and leaving the system. This energy is measured over an infinite time interval $[0, \infty)$ in the standard H_∞ control problem. However, in practical applications, this might not always be realistic. In the case of a finite horizon H_∞ control problem, the same norm is to be minimized except that the energy is measured over a finite time interval $[0, T]$ for some given $T > 0$.

2) A Brief Survey Report

In [27], using a Dynamic Output Feedback (DOF) controller, asymptotic stability achieved a desired value of the H_∞ norm from disturbance input to controlled output for an uncertain 2-D discrete second FM model. A different version of the 2-D bounded lemma was derived, which was an extension of the bounded real lemma used in [32], to satisfy the solvability criteria for H_∞ control of discrete 2-D systems. When building a DOF controller, the non-convexity form was mitigated using the modification of variables as in [33]. In [28], a dynamic output feedback controller was designed for an uncertain 2-D discrete Roesser model, so that the closed-loop system not only achieves asymptotic stability but also the specified H_∞ performance. This solution was reorganized into a convex optimization problem that was efficiently solved using the Matlab LMI control toolbox. In [27], in order to address the H_∞ control problem for an uncertain 2-D discrete general model, a DOF was constructed so that the closed-loop system was asymptotically stable to achieve the specified H_∞ performance level. The H_∞ control issue was modified to be LMI solvable, and so the desired controller parameters can be found by solving certain LMIs. In [25], the delay-independent and delay-dependent H_∞ control problems were considered for the second FM model. A delay-dependent bounded real lemma was proposed for 2-D state delayed systems, developed based on the definition of the H_∞ disturbance attenuation level. A delay-dependent DOF controller was developed, as the induced results were not in a strict LMI form, therefore, some limitation was put on certain matrices to make it LMI solvable. The experimental results showed that a smaller H_∞ performance level can be obtained using a delay-dependent approach compared to the delay-independent.

In [28], design methods for the control of H_2 and mixed H_2/H_∞ 2-D discrete systems described by the Roesser model were developed, extending the definition of the H_2 performance specification earlier developed for 1-D systems to 2-D. All design methods were LMI-solvable. The full-order DOF controller was devised and used to transform nonlinear matrix inequalities into linear ones using the change of variable method [34]. The Static Output Feedback (SOF) H_∞ control

problem was explored in [12] for the 2-D Roesser and the second FM model. Using the 2-D bounded real lemma, as described in [17] and [12], and combined with a slack variable approach, sufficient conditions based on linear matrix inequalities were established for SOF controllers. In [19], the issue of sufficient SOF control conditions was considered for the 2-D discrete Roesser model. While the design criteria for a 2-D H_∞ SOF controller were explicitly defined in the form of linear matrix inequalities, an iterative algorithm based on Cone Complementary Linearization (CCL) [35] was proposed to solve linear matrix inequalities. In [36], the robust H_∞ control issue was investigated for an uncertain 2-D singular Roesser model. The uncertainties considered were a parametric time-invariant norm bounded in the state matrix. A 2-D bounded real lemma was established for the 2-D singular Roesser model, which was an extent of the result of the bounded real lemma for 1-D [37]. A necessary condition for the creation of SOF controllers was established in terms of matrix inequalities. In [37], an observer-based DOF controller was presented for an uncertain 2-D discrete Roesser model with time-varying state delay, external disturbances, and actuator saturation. To address this issue, using H_∞ control synthesis, a state feedback method was used to design an observer-based controller to stabilize the system by mitigating the saturation nonlinearity using a convex hull approach. In [13], the issue of filtering and dissipative control was addressed for a 2-D discrete Roesser model. Using an LMI insertion approach, sufficient necessary computational complexity criteria were satisfied while rendering it in a two-dimensional $(Q, S, R) - \alpha$ dissipative. Furthermore, a less conservative two-dimensional $(Q, S, R) - \alpha$ dissipative output feedback approach was suggested by employing a slack variable approach.

In [14], DOF controller stabilization, based on H_∞ , was carried out for a 2-D discrete switched system, described by the FM LSS model. Exponential stability was ensured with a specified weighted H_∞ disturbance attenuation level by employing the average dwell time approach. In [18], an H_2 control problem was investigated for a 2-D discrete switched system of the Roesser model form. Based on the multiple Lyapunov function method, an LMI framework-based sufficient condition was established to guarantee asymptotic stability and H_2 performance analysis. An H_2 DOF controller was developed for the system under consideration. In [20], using the bounded real lemma, the H_∞ control SOF problem of 2-D discrete systems was investigated as represented by the second FM model. Using the 2-D bounded real lemma, the 2-D control-based SOF problem was formulated as a BMI problem. Combining the slack variable technique with two existing LMI methods, based on the choices of coordinate transformation matrices, fewer conservative sufficient LMI conditions were proposed for the BMI formulation. In [38], the slack variables were of block diagonal structure, however, the matrix structure of the used slack variable had a block-triangular structure, providing more freedom in the choice of decision variables when solving the LMI condition, reducing the conservatism.

The study of optimal H_2 control theory has gained importance for both continuous and discrete-time systems in system theory. The aim was to reduce the energy of the system when it experiences a unit impulse input, or equivalently when

the input has white noise of a unit variance. In [18], the stabilization of a 2-D discrete switched system represented by the Roesser model, was studied. Using a multiple Lyapunov function, sufficient criteria were established for the existence of the DOF controller to examine the H_2 effectiveness of 2-D discrete switched systems. In [16], based on the 2-D bounded real lemma for Finsler's lemma, a new necessary condition for the H_∞ evolved, relying on a 2-D discrete Roesser model. A SOF controller design was presented to ensure asymptotic stability and H_∞ disturbance attenuation level in terms of linear matrix inequalities. In [22], the H_∞ control problem was considered on a 2-D Takagi Sugeno (TS) system, described by the second FM model with time delays in both horizontal and vertical directions, bounded noise, and probabilistic missing measurements. Using the CCL algorithm, some equality constraints were successfully transformed into LMIs, and the parameters of the desired fuzzy controller were obtained. Finally, sufficient conditions were established to design the closed loop 2-D fuzzy system with SOF controller to the specified prescribed H_∞ performance criterion.

In [22], novel slack matrices for DOF controller construct criteria were presented for a 2-D TS fuzzy model as described by the second FM model, where there were no restrictions on the system matrices, i.e., they were not required to be row or column full rank. Sufficient criteria were established in terms of an LMI, which guarantees an H_∞ noise attenuation level of the resulting closed-loop system. In [21], an observer-based H_∞ controller was designed for a 2-D discrete fuzzy TS model described by the FM second model. In this study, a Luenberger observer was used to estimate the state. An LMI condition for

the existence of the fuzzy observer and the fuzzy controller was given so that the whole system was stabilizable with a specified H_∞ performance γ . Authors in [15] focused on investigating H_∞ fuzzy output-feedback controllers to ensure that the closed-loop fuzzy TS control system is exponentially stable in the mean square. The H_∞ control was proposed for a class of discrete-time fuzzy systems. Using the H_∞ performance index, disturbance rejection attenuation was constrained to a given level. At first, the model was proposed, which according to the Bernoulli distribution describes multiple probabilistic communication delays of different sizes. Second, based on partial sensor outputs, multiple missing measurements with a missing probability were considered over the interval [0, 1]. A robust H_∞ fuzzy dynamic output feedback controller was devised, so that the closed-loop fuzzy system was exponentially stable with guaranteed H_∞ performance. When calculating the controller parameters, the equality conditions were modified into strict LMIs, using the CCL algorithm [35-39], which can be easily solved in the Matlab LMI toolbox [40-41]. The study in [39] began with a brief overview of four types of matrix inequalities, followed by a discussion of two unresolved challenges with H_∞ SOF management of continuous-time systems. The two open issues were shown to be fundamentally identical to the problem of selecting a Coordinate Transformation Matrix (CTM) by establishing the links between matrix inequalities. A two-step optimization strategy was developed to answer this open challenge. In [42], a cone complementarity was constructed but did not give a full evaluation of all the major contributions to the field, only specific LMI/NMI scenarios of interest were examined. The challenge of selecting a CTM is an obstacle.

TABLE I. COMPARATIVE ANALYSIS OF DIFFERENT FINDINGS OF THE H_∞ APPROACH

Ref.	Year	Findings	Pros	Cons
[43]	2018	Proposed a novel H_∞ output feedback controller for 2-D systems.	A distributed hybrid active control scheme for output formation-containment of interacted heterogeneous linear systems.	Requires knowledge of the system matrices, which may not be available in practice.
[44]	2021	New LMI conditions for H_∞/H_2 output feedback control of linear discrete-time systems.	Simulations showed that the new conditions were effective, achieving good performance in terms of stability and robustness.	The new conditions are more complex than some existing - have not yet been implemented in a real-world system.
[45]	2017	Controllers were effective in simulation studies, achieving good performance in terms of stability and robustness.	The new criterion can be used to design output feedback controllers for discrete time-delay systems.	Design of output feedback controllers is more complex than for systems without time delay - difficult to implement in practice.
[46]	2019	Anew approach to output feedback H_∞ control for discrete-time systems.	Is more general than traditional output feedback H_∞ control methods, as it can handle systems with uncertain parameters.	The approach is more complex than traditional output feedback H_∞ control methods.
[47]	2020	The H_∞ optimal linear matrix inequality technique was used to achieve the objectives.	Improved power-sharing accuracy and was a robust controller.	It is a computationally complex controller and not easy as others.
[48]	2019	Used a Kalman filter to estimate the states of the system and an LQR controller to control it.	Robust controller that can handle variable load conditions.	Requires more computations than traditional LQR controllers.
[49]	2016	New method for SOF stabilization of fractional-order systems in TS fuzzy models.	Based on LMIs, which makes it relatively easy to implement. Also, able to handle systems with multiple fuzzy subsystems.	Requires knowledge of the system's fractional order, which may not be always available. Also, requires the computation of LMI solutions, which can be computationally expensive for large systems.
[50]	2021	Method for co-designing an event-triggered mechanism and a dissipative-based output feedback controller for 2-D systems.	Can significantly reduce the communication overhead while ensuring the stability and performance of the closed-loop system.	Requires knowledge of the system parameters, which may not be always available - Relies on the quadratic Lyapunov function, which may not be suitable for all systems.
[51]	2023	Provides more details on the implementation of the proposed SMC scheme.	Proposed a sliding mode control scheme for uncertain 2-D fractional-order MIMO systems under stochastic scheduling.	Requires knowledge of the system parameters, which may not be available in practice.

III. CONCLUSIONS

This paper presented a detailed survey report on the application of H_∞ control-based output feedback techniques for the stability analysis of 2-D discrete systems. All the different approaches that have been presented so far for the stability analysis of 2-D discrete systems were systematically put together in a compiled form. In conclusion, output feedback control systems have several advantages, including increased robustness, disturbance rejection, better tracking performance, and simpler implementation. Such systems can be designed using various control algorithms such as LQR, H-infinity, and SMC. However, designing an output feedback control system requires careful consideration of the system's dynamics, stability, and observability. In general, output feedback control is an effective and widely used technique in modern control engineering, enabling the design of sophisticated and reliable control systems for a wide range of applications.

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