Validated Finite Element Modeling of Lightweight Concrete Floors Stiffened and Strengthened with FRP

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ABSTRACT
The main challenge in designing Light-Weight Concrete (LWC) is to adapt most of the design, production, and execution rules from normal-weight concrete. Carbon Fiber-Reinforced Polymer (CFRP) composites provide strength and stiffness to the composite system. This study investigated the stiffness of an LWC flat slab with CFRP when subjected to human-induced vibration. This was determined by finding the natural frequency of the slab and comparing it with the acceleration limit ratio (human perception of vibration) of 0.5% g. In most cases, vibration characteristics are examined using commercial software based on Finite Element Analysis (FEA) methods that are powerful tools, but the user needs to understand the underlying assumptions and methods implemented, especially for reinforced concrete floor systems where inherent attributes, such as cracking, play an important role in the determination of vibration characteristics. This study used Abaqus CAE. The main idea of this study was that such software cannot detect the behavior of cracks in structures over the years and the effect on frequencies, as stiffness depends on the modulus of elasticity and not on the moment of inertia. Therefore, the natural frequency equation has a component that constantly accounts for the level of cracking on concrete slabs. This component was theoretically determined with detailed calculations that are not provided in the Design Guide for Vibrations of Reinforced Concrete Floor Systems. Then, the constant that accounts for the level of cracking $k_1$ was multiplied by the modulus of elasticity $E$ and substituted in the latter's place in Abaqus to ensure the right behavior of the slab with and without CFRP. This study also investigated the properties of CFRP and how to represent it in the Abaqus. The numerical results showed good agreement with FEA and the acceptance criteria for walking excitations increased when using CFRP on a floor system.

Keywords-light weight concrete; validation; FRP; floor stiffness

I. INTRODUCTION
Since the 1990s, engineers and researchers have been aware of the advantages of combining concrete with FRP materials, as concrete aids in compressive resistance and stability and Fiber Reinforced Polymers (FRPs) provide tensile resistance [1]. This study considered a slab with and without FRP stiffening. A preliminary Finite Element Model (FEM) was prepared for the slab considered but with normal-weight concrete in the Design Guide for Vibrations of Reinforced Concrete Floor Systems [2]. This model was validated by comparing its results with the semi-empirical results of [2]. Subsequently, the validated model was adopted to show how Light-Weight Concrete (LWC) and CFRP affect the vibrational aspects of the slab. There are many advantages to having low density, as it helps reduce dead load, increases building progress, and reduces transportation and handling costs [3]. Also, by reducing the cross sections of the components, there is an increase in the demand for LWCs in many modern architectural constructions [4]. Figure 1 shows the flat plate reinforced concrete system considered in this study. For validation purposes, the indicated dimensions were the metric equivalent of the corresponding customary United States dimensions in [2].

The Design Guide for Vibrations of Reinforced Concrete Floor Systems was written with two objectives. It aimed to support structural engineers in selecting a suitable reinforced concrete floor system subjected to human-induced vibration and to present simplified approaches on estimating the vibration characteristics of reinforced concrete floor systems that can be used to evaluate whether the anticipated vibration is acceptable or not, by computing the natural frequency and comparing it with the ratio aspects.
The peak acceleration as a fraction of the acceleration of gravity $a_p/g$ is less than or equal to the acceleration limit $a_0/g$ for the appropriate occupancy, as shown in (1) and (2), respectively. The natural frequency is the quantity of the floor system's response to the sources that can cause vibration and is related to how occupants will perceive such vibrations. Many methods and resources are available to determine this property.

$$f_1 = \frac{k_2 h^3}{2m_c \left\{12\eta(1-v^2)\right\}^{1/2}}$$

$$a_p = \frac{65e^{-0.35m_0}}{\beta w} \frac{a_p}{g}$$

II. CARBON FIBER REINFORCEMENT POLYMER

A. Overview

Carbon fibers, also called graphite fibers, are lightweight and strong fibers with excellent chemical resistance. They dominate the aerospace market and are considered an orthotropic material [9]. CFRP composites are extremely stiff and brittle and susceptible to galvanic corrosion. Their use in field applications has grown since the late 1980s due to their low cost and importance in strengthening improperly or insufficiently designed structural elements, especially in seismic areas [10]. The use of CFRP strengthening techniques improves the stiffness and flexural strength of RC members and reduces crack spacing and crack width [11].

B. Orthotropic Material

An orthotropic material has three planes of symmetry, as shown in Figure 2, which coincide with the coordinate planes.
A unidirectional fiber-reinforced composite can be considered orthotropic. One plane of symmetry is perpendicular to the fiber direction, and the other two can be any pair of planes parallel to the fiber direction and orthogonal among themselves. Only nine constants are required to describe an orthotropic material: $E_1$, $E_2$, $E_3$, $G_{12}$, $G_{13}$, $G_{23}$, $v_{12}$, $v_{13}$, and $v_{23}$ [9].

![Orthotropic material](image)

**Fig. 2.** Orthotropic material.

### C. Mechanical Properties

The modulus of elasticity value of carbon fibers is approximately 240 GPa [8]. Based on modulus and strength, carbon fibers can be divided into [8]:

- Ultra-High-Modulus (UHM) for $> 450$ GPa
- High-Modulus (HM) between 350-450 GPa
- Intermediate-Modulus (IM) between 200-350 GPa
- Super High-Tensile (SHT) for tensile strength $> 4.5$ GPa

In the mechanics of materials, both fibers and matrix are assumed to be isotropic. Isotropic material stiffness is completely represented by two properties: modulus of elasticity and Poisson's ratio. Using micromechanics, the combination of two isotropic materials, fiber and matrix, is represented as an equivalent, homogeneous, and anisotropic material [9]. The two isotropic materials, fiber and matrix, is represented as an orthotropic material:

$$E_1$$ is the modulus of elasticity in the fiber direction, $E_2$ is the modulus of elasticity in the direction transverse to the fibers, $G_{12}$ is the in-plane shear modulus, $G_{13}$ is the out-of-plane shear modulus, and $v_{12}$ is the in-plane Poisson's ratio.

### III. VALUES IN ABAQUUS

#### A. Part Module

Three main parts were defined for the concrete, longitudinal rebars, and CFRP: solid extrusion, wire planar, and shell planar, respectively.

#### B. Property Module

Three sections were defined, where solid and homogeneous were defined and assigned to the concrete mass and two truss sections were defined and assigned to the longitudinal rebars and stirrups. For concrete, the values of Young's modulus and Poisson's ratio of 6713.8 and 0.2 were inserted, respectively. The $E$ with the effect of crack was:

$$E_c = k_1 \times p c^{1.5} \times 0.043 \sqrt{f_c} = 0.54 \times 1440^{1.5} \times 0.043 \times \sqrt{28} = 6713.8 \text{ MPa}$$

Table II shows the values inserted for CFRP.

### TABLE II. VALUES INSERTED FOR CFRP

<table>
<thead>
<tr>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$G_{12}$</th>
<th>$G_{13}$</th>
<th>$G_{23}$</th>
<th>$v_{12}$</th>
<th>$v_{13}$</th>
<th>$v_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>170000</td>
<td>90000</td>
<td>0.34</td>
<td>4800</td>
<td>4800</td>
<td>4500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### IV. STEEL REINFORCEMENT OF SLAB

The maximum spacing of the main bar of the slab must be at least 300 mm or three times the effective depth, and the maximum spacing of the distribution bar of the slab must be at least 450 mm or five times the effective depth. Meanwhile:

$$\text{Effective Depth} = 150 - 20(\text{cover}) - \frac{10}{2} (\text{rein.}) = 125\text{mm}$$

Therefore, the spacing used was 300 mm in two ways.

### V. DYNAMIC LOADS GENERATED BY HUMAN ACTIVITIES

The dynamic loads generated by human activities can be grouped into two types according to the person-structure interaction. The first is when there is a loss of contact with the structure, such as running and jumping, and the second is when there is no loss of contact with the structure, such as walking. A human being produces a dynamic load during walking or running that can be represented by a force that varies in time. For walking, its frequency range has not been the same comprehensive investigation for running has been stated to be between 1.6 and 2.4 Hz. Although there has not been the same comprehensive investigation for running or jumping activities, the typical frequency range for running is considered 2.0 to 3.5 Hz while for jumping it is 1.8 to 3.4 Hz [1]. People are exposed daily to vibrations on floor slabs or on footbridges caused by different sources of excitation. The evaluation of human sensitivity to these vibrations involves psychological and physical aspects. Human-induced vibrations can cause serviceability problems and discomfort to users. The main factors that influence human sensitivity are position, such as standing, sitting, or lying, and the type of activity that
depends on age, gender, mood, vibration frequency, displacement amplitudes, damping, and finally the acceleration of dynamic excitation [1]. The natural frequency of two-way reinforced concrete floor systems can be obtained using some fundamental concepts in plate theory. The simplifying assumptions made in the following discussion enable the natural frequency to be determined straightforwardly and consistently for all two-way systems [2].

Assume that a reinforced concrete flat plate or a voided slab system can be modeled as a rectangular, isotropic plate where the primary vertical deflection is due to flexure. Since the slab is supported only by columns, it is free to deflect at any point except at the locations of the columns, where it is generally assumed that vertical displacements are negligible and that rotations are not restricted. Equation (1) can be used to determine the natural frequency \( f_c \), which applies to rectangular plates with corner supports.

\[
f_c = \frac{k_2 \lambda_1^2}{2\pi \gamma} \left[ \frac{k_1 e h^3}{12\gamma(1-\nu^2)} \right]^{1/2}
\]

where \( h \) is the total thickness of the flat plate or voided slab, \( v \) is the mass per unit area of the plate, \( \nu \) is the Poisson’s ratio, \( l_1 \) is the longest of the two center-to-center span lengths of the plate panel, and \( \lambda_1 \) is a function of the panel aspect ratio \( l_1/l_2 \). Table III shows the values of \( \lambda_1 \) for the fundamental mode of vibration. In [13-15] there are values of this parameter for other modes and panel aspect ratios.

### Table III. Values of \( \lambda_1^2 \) for a Corner-Supported, Rectangular Plate [2].

<table>
<thead>
<tr>
<th>( l_1/l_2 )</th>
<th>( \lambda_1^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>7.12</td>
</tr>
<tr>
<td>1.5</td>
<td>8.92</td>
</tr>
<tr>
<td>2.0</td>
<td>9.29</td>
</tr>
</tbody>
</table>

The constant \( k_2 \) accounts for the effect of rigidity at the joint between the slab and the columns, which are cast monolithically in typical reinforced concrete structures. Column size has a direct impact on joint rigidity: the larger the column size, the greater the slab stiffness and fundamental frequency. Instead of a more exact analysis, the following are approximate values of \( k_2 \) that can be used in (1) and are based on column size \( c_1 \) [16]:

\[
k_2 = \begin{cases} 
1.9 & \text{for } c_1 \leq 24 \text{ in.} \\
2.1 & \text{for } c_1 > 24 \text{ in.}
\end{cases}
\]

(3)

where the constant \( k_1 \) accounts for the level of cracking in the concrete slab and can be determined by dividing the effective moment of inertia \( I_c \) by the gross moment of inertia \( I_e \). When \( k_1 \) is less than 1.0, the stiffness of the slab is less than the gross stiffness, and the natural frequency of the system is reduced [2]. For a flat plate, \( I_c \) for a panel is approximated by adding the average \( I_{c1} \) of the column strip in one direction to the average \( I_{c2} \) of the middle strip in the orthogonal direction. The average \( I_{c1} \) for the column and middle strips is used to take into account positive and negative regions in the strips. The following equation is applicable for square panels that have both ends continuous [17]:

\[
\text{Average } I_{c1} = 0.7I_m + 0.15(I_{c1} + I_{c2})
\]

(4)

where \( I_m \) refers to \( I_c \) at the midspan section and \( I_{c1} \) and \( I_{c2} \) refer to \( I_c \) at the end sections of the panel [2]. For a rectangular panels system, \( I_c \) for a panel is approximated by taking the average of the average \( I_{c1} \) values in the column strips (cs) and middle strips (ms) in both the \( x \) and \( y \) directions:

\[
I_{c1} = \frac{(I_{c1,x} + I_{c1,y}) + (I_{c1,x} + I_{c1,y})}{2}
\]

(5)

In (4), the effective moment of inertia \( I_e \) can be determined for sections with relatively low reinforcement ratios. The contribution of shrinkage restraint to cracking should also be considered in such systems. To account for this effect, the use of a two-thirds factor applied to \( M_{cr} \) was recommended in [18], which is equivalent to using a reduced modulus of rupture \( f_r = 5.0\sqrt{f_c} \), while in [19] it was proposed using \( f_r = 4.0\sqrt{f_c} \) to determine \( I_e \).

\[
I_{c1} = \frac{I_{c1,x} + I_{c1,y}}{1-(f_{cr}^2/f_{cr}^2)} \leq I_g
\]

(6)

A reduced modulus of rupture \( f_r = 4.5\sqrt{f_c} \) was adopted for vibration analysis in [2] for flat plate and voided slab systems.

### A. Flat Plate System Walking Excitation

An existing flat floor system residential building will be converted into an office building. The vibration of the system was evaluated assuming normal occupancy of the office. Table IV shows the design data. It was also assumed that the office space will have some non-structural components [2]. Notice that \( w_c = 1440 \text{ kg/m}^3 \) for LWC.

### Table IV. Design Data [2].

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Compressive strength ( f' = 4,000 \text{ psi} )</th>
<th>( f' = 28 \text{ MPa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ( w ) = 150 psf</td>
<td>( f' = 412 \text{ MPa} )</td>
<td></td>
</tr>
<tr>
<td>Reinforcing steel</td>
<td>Yield strength ( f_y = 60,000 \text{ psi} )</td>
<td>( f_y = 412 \text{ MPa} )</td>
</tr>
<tr>
<td>Loads</td>
<td>Superimposed dead load = 20 psf</td>
<td></td>
</tr>
<tr>
<td>Live load (design) = 65 psf (includes partition)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Live load vibration = 11 psf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spans</td>
<td>Typical bay: 25’-0”x20’-0”</td>
<td>7620x6096mm</td>
</tr>
<tr>
<td>Members sizes</td>
<td>Slab thickness ( h = 9.5” )</td>
<td>76.20mm</td>
</tr>
<tr>
<td>Slabs ( 22” \times 22” )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>560x560mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### B. Determination of Concrete Properties

The modulus of elasticity is determined as:

\[
E_c = \frac{w}{1.5} \times 33\sqrt{f_c}
\]

(7)

Dynamic \( E_c = 1.2 \sqrt{f_c} \)

### C. Determination of the Bending Moments at the Critical Sections for a Typical Interior Bay

Since the limitations of ACI 13.6.1 are satisfied, the direct design method can be used in the bending moment determination at the critical negative and positive sections in the column and middle strips along the span. The bending moments are determined by calculating the total static moment in each direction and then applying the appropriate coefficients given in ACI 13.6.3. Table V gives a summary of the service
bending moments. It was assumed that the 25-foot span is in
the north-south direction [2].

Determination of the Effective Moments of Inertia \(I_e\) and the Crack Coefficient \(k_f\).

The effective moment of inertia of the column and middle
strips is determined using (7). For this floor system with
rectangular panels, \(I_e\) for the panel is determined by (4),
where the average of the average \(I_e\) values in the column strips (cs) and middle strips (ms) in both directions are used. Detailed
calculations are provided below for the section containing the
negative flexural reinforcement in a column strip in the north-
south design strip. Calculations for the other sections are
similar [2].

For a column strip width of 10 ft, the section properties \(y_t\)
and \(I_e\) are 4.75 in and 8574 in\(^4\), respectively. \(I_e\) can be
determined by:

\[
a_1 = \frac{column \ strip \ width}{nA_s} \quad (8)
\]

\[
k_d = \frac{2da_s + 1 - 1}{a_1} \quad (9)
\]

\[
I_{cr} = \frac{(Column \ strip \ width)kd^3}{3} + nA_s(d - kd)^2 \quad (10)
\]

\[
M_{cr} = \frac{f_r\gamma_f}{\gamma_t} \quad (11)
\]

\[
I_e = \frac{l_{cr}I_{cr}}{1 - \left(\frac{M_{cr}}{M_o}\right)} \leq I_{o} = \frac{3043}{1 - \left(\frac{32.1}{87.3 + 42.7}\right)} \leq 3160 \text{ in}^4 \quad (12)
\]

Similarly, \(I_e = 2071 \text{ in}^4\) for the positive section in this
column strip. Using (5), the average \(I_e\) for this column strip in
the north-south direction is \(I_{e|cs,N-S} = (0.7 \times 2071) + 0.15 (2 \times 3160) = 2398 \text{ in}^4\). The average \(I_e\) for the middle strip in the north-
south direction and the column and middle strips in the east-west
direction are determined by (7) similarly: \(I_{e|ms,E-W} = 3126 \text{ in}^4\),
\(I_{e|cs,E-W} = 2546 \text{ in}^4\), and \(I_{e|ms,N-S} = 12861 \text{ in}^4\). The effective
moment of the panel is determined by (6) as \(I_{e|panel} = 10466 \text{ in}^4\).

The gross moment of inertia of the panel is averaged in the
same way as the effective moment of inertia:

\[
I_{o|panel} = \frac{I_{e|cs,N-S} + I_{e|ms,N-S} + I_{e|cs,E-W} + I_{e|ms,E-W}}{4} = 19292 \text{ in}^4.
\]

The constant \(k_f\) accounts for the level of cracking in the
slab and is determined by dividing the effective moment of inertia
of the panel by the gross moment of inertia of the panel
\(k_f = \frac{I_{e|panel}}{I_{o|panel}} = 0.54\).

F. Determination of the Natural Frequency.

The fundamental frequency for a flat plate floor system is
estimated by (1). The quantities in this equation that have not
been previously determined are as follows [2]:

\[k_1 = 1.9\text{, since the column is less than 24 in.}\]

\[\lambda_f^2 = 8.02\text{ by linear interpolation from Table III for } l/f_1 = 1.25.\]

\[\gamma = 3.175\text{ slugs/ft}^2\]

Therefore,

\[
f_1 = \frac{1.9 \times 8.02}{2\pi \times 25^2} \left[\frac{0.54 \times 307929600 \times (9.5/12)^3}{12 \times 3.175(1-0.2^2)}\right]^{1/2} = 5.83 \text{ Hz.}\]

For comparison purposes, an FEA was performed to
determine \(f_1\) for this flat plate system. Using the nonlinear
option which includes the effects due to cracking, \(f_1\) was found
to be 8.7 Hz with a crack coefficient equal to 0.71. The crack
coefficient was determined by dividing the maximum
deflection obtained from the elastic analysis by the deflection
from the inelastic analysis of the system. Thus, the simplified
method yielded a conservative estimate for \(f_1\), approximately a
10% difference [2].

---

**TABLE V.** SERVICE BENDING MOMENTS (FT-KIPS) [2]

<table>
<thead>
<tr>
<th>North-South</th>
<th>East-West</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Strip</td>
<td>Middle Strip</td>
</tr>
<tr>
<td>Negative D</td>
<td>91.2</td>
</tr>
<tr>
<td>Positive L</td>
<td>39.1</td>
</tr>
<tr>
<td>L</td>
<td>18.3</td>
</tr>
</tbody>
</table>

**TABLE VI.** REQUIRED FLEXURAL REINFORCEMENT (in\(^2\)) [2]

<table>
<thead>
<tr>
<th>North-South</th>
<th>East-West</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Strip</td>
<td>Middle Strip</td>
</tr>
<tr>
<td>Negative 3.39</td>
<td>2.05</td>
</tr>
<tr>
<td>Positive 2.31</td>
<td>2.05</td>
</tr>
</tbody>
</table>

---
VI. RESULTS AND DISCUSSION

According to (2), a floor system is satisfactory if the peak acceleration as a fraction of the acceleration of gravity $a_p/g$ is less than or equal to the acceleration limit $a_0/g$ for the appropriate occupancy [2]. For an office occupancy, the damping ratio $\beta$ can be taken as 0.03 for offices that have some nonstructural components but no full-height partitions. Thus, 

$$a_p = \frac{66 \times 0.35 \times 5.83}{0.03 \times 51124} = 0.0055$$ for $f_i$ calculations without CFRP,

and 

$$\beta = \frac{66 \times 0.35 \times 6.18}{0.03 \times 51124} = 0.00487$$ for $f_i$ from Abaqus with CFRP.

Fig. 4. $f_i$ calculation with CFRP

Fig. 5. $f_i$ calculation without CFRP.

VII. CONCLUSION

After detailed calculations for the natural frequency according to [1], it was cleared that the natural frequency of the LWC flat plate without FRP is higher than the frequency applied due to human activities, while for walking excitations, $a_p$ is approximately equal to 0.55% of $g$. This acceleration is more than the acceleration limit $a_0$, which is equal to 0.50% of $g$ for office occupancies [2]. On the other hand, the natural frequency of an LWC flat plate with CFRP in FEM is 0.487%. Therefore, this flat plate system is satisfactory for walking excitations (human perception of vibration) only with CFRP, which is useful in increasing the stiffness and strength of concrete structures. It should be noticed that this LWC satisfied the strength limits even without FRP.

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