

# Damage Identification of Suspension Footbridge Structures using New Hunting-based Algorithms

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## ABSTRACT

Metaheuristic algorithms have been applied to tackle challenging optimization problems in various domains, such as health, education, manufacturing, and biology. In particular, the field of Structural Health Monitoring (SHM) has received significant interest, particularly in the area of damage identification in structures. Popular optimization algorithms such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Cuckoo Search (CS), Teaching Learning Based Optimization (TLBO), Artificial Hummingbird Algorithm (AHA), Moth Flame Optimizer (MFO), among others, have been employed to address this problem. However, notwithstanding the wide recognition of the current algorithms, their constraints are commonly acknowledged. Hence, this article advocates for the adoption of innovative hunting-inspired algorithms, namely the Ant Lion Optimizer (ALO), African Vulture Optimization Algorithm (AVOA), Grey Wolf Optimizer (GWO), Marine Predator Algorithm (MPA), and Whale Optimization Algorithm (WOA), which emulate the behaviors of wildlife species, to discern the areas and magnitudes of deterioration in a suspension footbridge. Moreover, in order to reduce computational time, only natural frequencies are applied as objective functions. The obtained results indicate that all the utilized algorithms can accurately detect the damages in the considered structure.

*Keywords-structural health monitoring; metaheuristic algorithms; hunting-based; damage identification; suspension footbridge*

## I. INTRODUCTION

Structural Health Monitoring (SHM) is a crucial monitoring and evaluation system for current bridge structures. Basically, this system includes various types of sensors for data regarding acceleration, stress, displacement, deformation, temperature,

wind speed, rainfall, etc. These sensors are installed and attached to the structure during the construction to monitor and ensure the quality during construction and operation. Moreover, during the inspection and maintenance of old works and the construction of new bridges, the SHM system makes a

particularly important contribution in the management, operation, and repair.

Optimization algorithms have been studied and applied effectively in many different fields such as medical, biological, manufacturing design and many other fields [1-8]. The metaheuristic algorithms based on nature are the most researched, developed, and widely applied. They are inspired by traits, habits, or behaviors of animals, plants, humans, or laws of physics and chemistry. In the problem of damage detection, there have been many successful studies in applying these algorithms. GA was the first and laid the foundation for the development of applying optimization algorithms to the problem of diagnosing damage [9]. With a main content that adheres to the laws of genetics, the results from GA have solved the problem of diagnosing damage in some studies with highly accurate and reliable results [10]. The PSO algorithm was introduced in 1995 [11]. It is inspired by the behavior of animals that live in swarms, such as fish schools and bird flocks. Numerous research endeavors have also investigated the utilization of PSO in discerning the areas and magnitudes of deterioration in various structures [12, 13]. Other algorithms have also been studied, proposed, and applied to the problem of fault diagnosis [14-25].

During the recent years, many new algorithms have been introduced and proven effective in solving benchmark objective functions. These include Ant Lion Optimizer (ALO), African Vulture Optimization Algorithm (AVOA), Grey Wolf Optimizer (GWO), Marine Predator Algorithm (MPA), and Whale Optimization Algorithm (WOA). The common point of the above algorithms is that they are inspired by the hunting behavior of animals. Therefore, in this paper, the application of the above algorithms to the problem of damage detection in a suspension footbridge is proposed. To the best of our knowledge, these newly proposed algorithms are applied for the first time to a pedestrian suspension bridges.

## II. METHODOLOGY

### A. Ant Lion Optimizer

Ant lions is an insect species in the neuropteran family Myrmeleontidae, belonging to the order Neuroptera. ALO [26] replicates the feeding behaviors of antlions in the desert. The predatory processes of this species consist of 5 main and iterative steps: random movement, trap digging, waiting for prey, capturing prey, and digging new traps.

$$AntPosition_i^j = \frac{W_{rl}^j + W_{el}^j}{2} \quad (1)$$

where  $AntPosition_i^j$  presents position of the  $i_{th}$  ant at the  $j_{th}$  iteration,  $W_{rl}^j$  and  $W_{el}^j$  are the positions of a ant walk randomly by a roulette wheel and the elite position, respectively. Ant larvae will dig a funnel-shaped burrow about 2 inches deep and 3 inches wide to trap prey. After digging the 'grave' of the prey, the ant lion will lie down at the bottom of the funnel and bury itself in the sand, revealing only its sharp and strong jaw. When prey (usually small ants, arthropods, or spiders) steps in and with the trap's initial stable slope, a landslide occurs causing the animal to fall to the anthill. If there is any attempt to get out, it will be in vain because the ant lion at the bottom of the

hole will throw up sand, causing the sand pit to collapse and drag the prey down.

### B. African Vulture Optimization Algorithm

Vulture is the common name for a group of carnivorous and carrion birds that live on every continent except Antarctica and Oceania. One of the characteristics of vultures is that the head is often bald, without feathers due to the habit of eating carrion by sticking its head into the carcass of the animal to eat, so the head is stained with blood and body fluids. Vultures are divided into 2 groups. The Old World vultures in Africa, Asia, and Asia belong to the family Accipitridae. This family includes Eagles, Hawks, Eagles, and Magpies. AVOA [27] was inspired by the hunting habits of African vultures. Normally, in the wild, vultures are divided into 2 groups and the first stage of the algorithm is to identify the best individual in each group. Next is to evaluate the survival rate through the energy in the body to decide the next stage is exploitation or exploration. The AVOA metric is:

$$V(j) = B(i) - \alpha + r_1 \times [(UB - LB) \times r_2 + LB] \quad (2)$$

where  $V(j)$ ,  $B(i)$  are respectively the position of a vulture at  $j = i + 1$  and  $i$  iteration,  $\alpha$  presents the satisfaction ratio,  $UB$  and  $LB$  are the upper and lower boundary,  $r_1$  is a random number in  $[0,1]$ , and  $r_2$  has a value approximately equal to 1.

### C. Grey Wolf Optimizer

The grey wolf is a carnivorous mammal native to Eurasia and North America. The grey wolf is the largest member of the Canidae family and the most famous wolf species. Grey wolves are social animals, they usually live in packs of about 5-11 children with 1-2 leaders including 1-2 adult wolves, 3-6 juvenile wolves, and 1-3 cubs or sometimes 2 or 3 such families, with particularly large herds consisting of up to the maximum known of 42 wolves. The grey wolf algorithm was introduced in 2014. The social hierarchy of GWO was modeled into 4 types: the alpha ( $\alpha$ ), beta ( $\beta$ ), delta ( $\delta$ ), and omega ( $\omega$ ). Accordingly, the hunting process includes stages such as: locating prey, approaching, encountering, or facing, ambush, chase.

$$\vec{G}(t+1) = \frac{\vec{\alpha} + \vec{\beta} + \vec{\delta}}{3} \quad (3)$$

In (3),  $\vec{G}(t+1)$  is the best value of  $(t+1)$  iteration,  $\vec{\alpha}$ ,  $\vec{\beta}$ , and  $\vec{\delta}$  are the position of a grey wolf at current iteration.

### D. Marine Predator Algorithm

MPA was first introduced in 2020 [28]. The fundamental concept behind MPA is the adoption of optimal foraging strategies observed in ocean predators, particularly through Lévy and Brownian movements, as well as the encounter rate policy in predator-prey biological interactions. MPA adheres to the principles that govern these strategies and policies in marine ecosystems. When considering the speed ratio of the hunter to prey, MPA basically consists of 3 stages: (1) the high-speed ratio or when the hunter moves slower than prey, (2) the speed ratio unit velocity or when both the predator and the prey are moving at roughly the same speed, and (3) at the low-velocity ratio when the prey is moving slower than the predator.

$$H_0 = \text{Min}V + \rho \tag{4}$$

Basically, MPA starts with (4) and uses other equations on each stage. In the initial stage, the first best value  $H_0$  is calculated with the minimum value of variables  $\text{Min}V$ , and  $\rho$  is a random number from 0 to 1.

E. Whale Optimization Algorithm

WOA was introduced in 2019 [29]. The whale is considered to be the world's largest mammal, the largest adult species discovered is the blue whale in Antarctica. They can weigh up to more than 180 tons and possess a length of up to 30 meters. Because of their large mass, the daily food requirement is very large, so they have a very special hunting habit. This routine is known as the "bubble-net feeding method". Basically, it consists of 3 main steps: enveloping prey (mostly feeding on a school of fish and cephalopods), bubble-net attacking method (exploitation), and finally searching for prey (exploration).

$$\vec{W}(i + 1) = \vec{W}_{rd} - \vec{X} \times |\vec{Y} \times \vec{W}_{rd} - \vec{W}(i)| \tag{5}$$

where  $\vec{W}(i + 1)$ ,  $\vec{W}(i)$  are the vector presenting the current position at  $(i + 1)$  and  $i$  iteration, respectively,  $\vec{W}_{rd}$  is a random vector solution at the  $i$  iteration, and  $\vec{X}$  and  $\vec{Y}$  are the factor vectors.

III. CASE STUDY

A. General Information and Finite Element Model

Na Xa bridge (Chau Hoan commune, Quy Chau district, Nghe An province) is a suspension bridge built to serve residents in mountainous and remote areas. The bridge was built in August 2008 and put into operation in May 2009 with a main length of 78 m, excluding the mooring lines at the 2 ends of the bridge, and 2 planes of suspended cables with compartments of different dimensions. The longitudinal beam is I150-shaped steel and the crossbeam is I200, combined with D20 steel wind bracing. The main cable has an outer diameter of 56 mm, the hangers are D20 steel bars. In addition, the bridge also has an anti-shake system with a D21.5 cable that is cross-connected at the two tower legs. The bridge tower is assembled from many different types of steel with a height of 8m and embedded in the ground for about 1m as shown in Figure 1. This is the typical design of the residential suspension bridges in Vietnam. Vibration characteristics such as frequencies and mode shapes of the bridge are extracted by using the StaBIL toolbox [30] in MATLAB. The model is updated based on the actual measurement results.



Fig. 1. General view of the Na Xa footbridge.

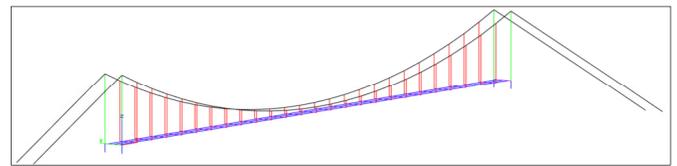


Fig. 2. Finite element model of the Na Xa bridge.

The model includes 198 nodes representing the intersections of the elements, 300 beam elements, namely longitudinal beams and transverse beams, main cables, wind bracing, suspension bars, and bridge towers. The finite element model of Na Xa bridge is shown in Figure 2. The boundary conditions of the structure are arranged according to the actual bridge structure. Steel is the main material with elastic modulus  $E = 21 \times 10^4$  MPa, Poisson ratio  $\nu = 0.3$  and density  $\rho = 7850$  kg/m<sup>3</sup>. The first 6 mode shapes of the bridge are depicted in Figure 3.

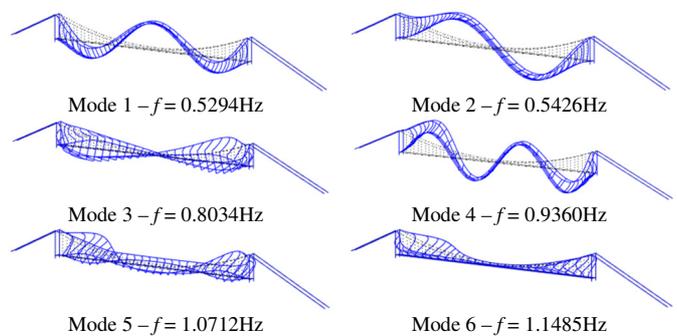


Fig. 3. The 6 first mode shapes.

B. Damage Identification

In this section, we assume two cases of damage as shown in Figure 4:

- Case 1 (single damage): there is only one damaged element in the structure with a loss of elastic modulus of 23%.
- Case 2 (multiple damage): there are two or more damaged elements in the structure with loss of elastic modulus of 26% and 27%, respectively.

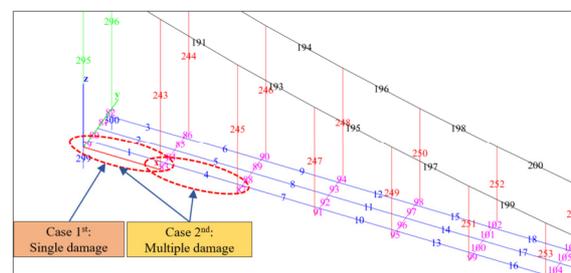


Fig. 4. Two cases of the damage identification issue.

In this study, the natural frequencies are used as the objective function because they are sensitive to damage:

$$ObjectiveFunction = \sum_{i=1}^{n=6} \frac{(f_{FEM} - \tilde{f}_{FEM\_damaged})^2}{\tilde{f}_{FEM\_damaged}^2}$$

where  $n = 6$  is the corresponding frequency,  $f_{FEM}$  is the initial simulation frequency, and  $\tilde{f}_{FEM\_damaged}$  is the damaged structure frequency. The parameters used in the algorithms include: the number of spaces is  $nPop = 200$ , the number of iterations is  $nIter = 300$ , and boundary  $[LB:UB] = [0.00:0.99]$ . The computer configuration used in the calculation is 12th Intel® Core™ i7-12700F 32GB RAM, NVIDIA GeForce 3060 RTX. Figure 5 illustrates the convergence of the 5 considered algorithms. The best convergence is basically achieved at the 35<sup>th</sup> iteration and is stable until the end of the loop.

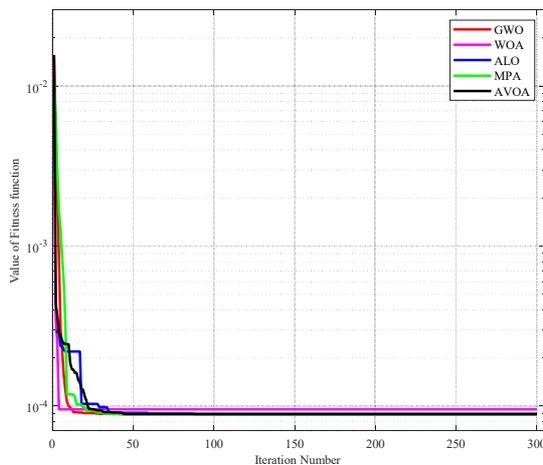


Fig. 5. Convergence curve of case 1: single damage.

The results of damage location and extent determination in the structure are depicted in Figure 6. They show high accuracy for GWO, ALO, MPA, and AVOA. However, WOA misidentifies the damage level with an error of approximately 8.962%. In terms of computation time, as shown in Figure 7, the algorithms take around 2300-2900 s to find the optimal result, except for MPA that spend 4623 s for this process. In terms of the multi-damage case, as shown in Figure 8, it can be seen that, MPA has the highest convergence, followed by ALO, GWO, WOA, and AVOA.

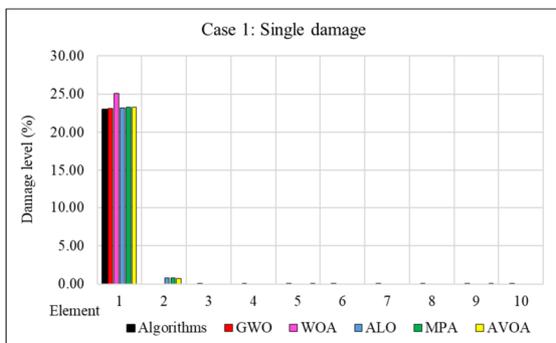


Fig. 6. Results of case 1: single damage.

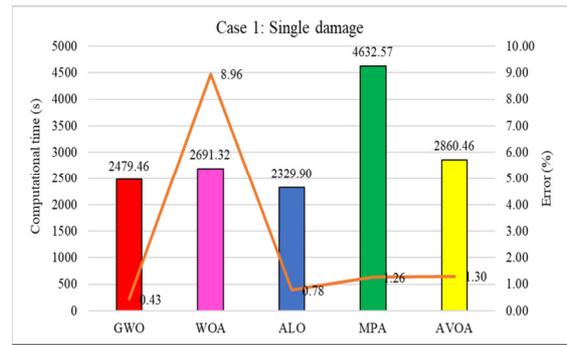


Fig. 7. The computational time of case 1: single damage.

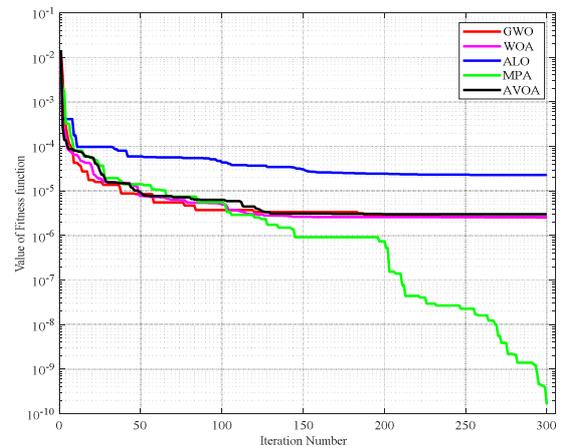


Fig. 8. Convergence curve of case 2: multiple damage.

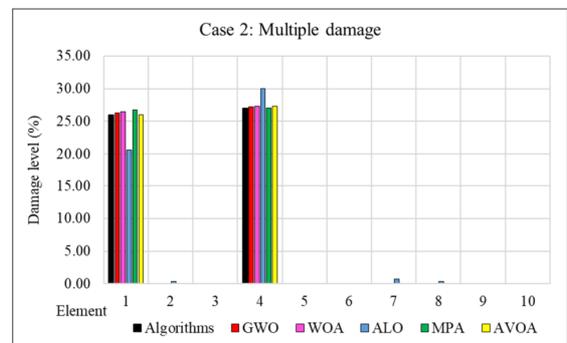


Fig. 9. Results of case 2: multiple damage.

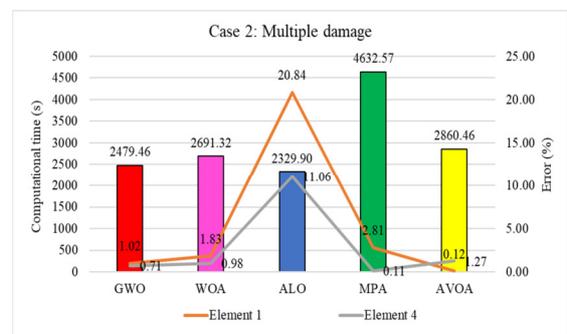


Fig. 10. The computational time of case 2: multiple damage.

MPA demonstrates the best convergence but requires approximately 5000 s for damage identification in the structure. GWO and AVOA exhibit accurate results in terms of location and extent of damages. ALO produces inaccurate results for damage identification in the case of two damaged elements.

#### IV. CONCLUSION

This paper proposes the utilization of algorithms based on predatory animal behaviors for damage detection in a suspension bridge structure. From the obtained results, several conclusions can be drawn:

- The considered algorithms, namely GWO, WOA, ALO, MPA, and AVOA are capable of accurately detecting the location of faults. However, fine-tuning input parameters may be necessary to achieve the highest accuracy rate. Hence, each algorithm may be suitable for specific cases with an appropriate level of confidence.
- Further research is recommended to explore the application of these algorithms to other types of structures. Additionally, incorporating additional shape modes in the objective function could potentially enhance the accuracy of the results.
- Overall, the findings suggest the potential of using these hunting-based algorithms for damage diagnosis in suspension bridge structures, with GWO being the most promising algorithm among the ones tested. Further investigations and refinements can contribute to the advancement of this field of research.
- In this paper, we only applied algorithms to the numerical model. Therefore, only natural frequencies are used as the objective function. In future studies, to increase the accuracy of SHM problems, mode shapes are vital to taken into account.

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