A Simplified Energy Model Approach for the Determination of Long-Term Crack Width in Reinforced Concrete Elements

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ABSTRACT

The deformation of reinforced concrete elements is a main consideration during the design process due to its importance for the service life of structures. Deformation (deflections and cracking) control has an effect at both the design and construction stages. The determination of crack width based on simple code provisions, leads to conservative results, due to many affecting parameters being unknown. The exact determination of the crack width is a very complicated process with significant computational load. In the current paper, a simplified procedure is proposed for the prediction of crack width at different loading stages. The procedure is based on a previously published energy model that uses the integration of the moment-curvature relationship to take into consideration the load stage and most affecting design variables. The obtained results using the simplified model are then tested against previously published experimental data and a good agreement is shown.

Keywords-reinforced concrete elements; deformation; state diagrams; energy model; general and residual resources

I. INTRODUCTION

The serviceability limit state of concrete structures is one of the most important steps within the design. The determination of deflection and crack width using code provisions commonly results in conservative values due to many important variables affecting design being unknown [1-3]. The idealized stress-strain curves for both concrete and reinforcing steel as well as the equivalent stress block in concrete, represent some major causes of conservative estimations. Another design approach for crack width calculation in reinforced concrete structures under short-term loading was developed in [4], based on the actual strain distribution along the spacing between two adjacent cracks. Determination of the real stress-strain state of reinforced concrete elements at different stages of loading has always been a key task in reinforced concrete theory. Its implementation is largely associated with the development of general models of deformation. The fulfillment of most of validity requirements in the so-called “strength” models [5] has been impractical due to the use of an idealized equivalent rectangular stress block. In “deformation” models [6-8], the methodological unity of all calculations of reinforced concrete elements according to limit states can generally be ensured with the help of complete concrete stress-strain curves \( \sigma_c - \varepsilon_c \). However, when integrating these curves or even using simplified forms of these diagrams [7, 8], the internal static uncertainty of the cross-section of reinforced concrete elements must be calculated through numerous iterations, involving several empirical parameters and coefficients which result in a significant computational load.

The concrete deformation model [9] can be considered the closest one to the generalized model of reinforced concrete elements deformation today. It shows the reason for being impossible to build a generalized model of the deformation of reinforced concrete elements with just the help of a stress-strain diagram of concrete. The calculation of concrete deformation at any stage is not controlled by the concrete deformation diagrams, but by the moment-curvature relationship of the reinforced concrete element \( (M - \phi) \), which characterizes its stiffness. However, the concrete deformation model needs further development, especially in matters of assessing the actual technical condition of reinforced concrete elements and structures and calculating their capacity and deformations.

The current paper aims to predict long-term crack width based on energy modeling. The proposed procedure is largely based on the concrete deformation model [9], modified in order to simplify the calculations and lighten the computational load. The simplified model is then applied on a previously published case study.
II. MOMENT CURVATURE RELATIONSHIP

For the case of short-term loading, the relation between the moment and curvature will have the form presented in [9]:

\[
M = \frac{E_c l_c r \phi - M_{uf} (\phi / \phi_{uf})^2}{1 + [E_c l_c r / M_{uf} - 2 / \phi_{uf}] \phi}
\]  

(1)

where \(E_c\) is the concrete's modulus of elasticity, \(I_{cr}\) is the moment of inertia of the cracked section, \(M_{uf}\) is the moment capacity at failure, and \(\phi_{uf}\) is the limit value of the curvature at failure. For the case of long-term loading, the relation will follow the same curve of the short-term loading up to the value of service load (\(M_a, \phi_a\)), then it will have a constant value of moment with the increase of the curvature. The increase in curvature after applying the service load will be due to the creep of concrete.

Solving (1) for the curvature, the following expression is provided in [10]:

\[
\phi = \frac{\phi_{uf}}{2 M_{uf}} \left( 1 - \frac{M}{M_{uf}} \right) \cdot E_c l_c r \phi_{uf} + 2 M - \sqrt{\left( 1 - \frac{M}{M_{uf}} \right) \cdot E_c l_c r \phi_{uf} + 2 M - 4 M \cdot M_{uf}} \right)
\]  

(2)

For the case of short-term loading (Figure 1), three points must be defined. For a given concrete element with defined section properties, the value of moment at the three defined points in the shown moment-curvature curve can be calculated (\(M_a, \phi_a, M_{uf}\)). The curvature at failure (\(\phi_{uf}\)) can be determined using the hypothesis of concrete plane section under bending. Then the corresponding curvature at service and ultimate load (\(\phi_a, \phi_{uf}\)) can be calculated using (2). These parameters are essential for use in the energy modeling approach.

III. A SIMPLIFIED APPROACH ON ENERGY MODELING

In general, it is known that in the deformation-force model, the main deformation and force parameters of the state of the element are interconnected by utilizing stiffness \(EI = M / \phi\) [11-13]. The mentioned parameters can be combined using another characteristic: the energy consumed on the deformation of a reinforced concrete element per unit volume \(W' = M \cdot \phi\) [14]. Under such circumstances, the methodology for the calculation of the total energy of reinforced concrete elements can be used to determine the deformations directly. The energy modeling approach has been developed in [9, 10] and is summarized as follows: the total energy consumed at any stage of short-term loading can be determined by integrating (1) from null to the curvature value at the stage under consideration. As explained in [10], three integrations must be performed for the three defined stages on the moment curvature curve, at the service load, at the ultimate load and at the failure load. These three integration results are called \(W_1(W_a), W_u\) and \(W_{uf}\) respectively (Figures 2-4).

The energy consumed in both cases of the short-term and the long-term loading must be equal [9]. The following relations can be used to determine both \(W_u\) and \(W_f\):

\(\)
\[ W_u = W_1 + W_2 \]  
\[ W_{uf} = W_1 + W_2 + W_3 \]  
\( W_u \) and \( W_{uf} \) are the moment-curvature relationships for the short-term and long-term loading cases, respectively.

The value of long-term loading curvature can be determined for both values of long-term (\( \phi_1 \)) and ultimate long-term curvature (\( \phi_{uf} \)) using:

\[ \phi_1 = \frac{W_1}{M_0} + \phi_a \]  
\[ \phi_{uf} = \frac{W_{uf} - W_0}{M_0} + \phi_a \]

The application of the integrations presented in [10] for the determination of the consumed energy \( W_1, W_u, \) and \( W_{uf} \), has a significant computational load. Using a rough approach for the determination of the consumed energy, the following equations can be derived for the calculation of \( W_2 \) and \( W_3 \):

\[ W_2 = \frac{M_0 + Mu}{2} \cdot (\phi_u - \phi_a) \]  
\[ W_3 = \frac{M_0 + Mu}{2} \cdot (\phi_{uf} - \phi_a) \]

Solving (5) with (7) and (6) with (8), the corresponding curvatures for the long-term loading case will be:

\[ \phi_1 = \frac{1}{2} \left(1 + \frac{M_0}{M_a}\right) \left(\phi_u - \phi_a\right) + \phi_a \]  
\[ \phi_{uf} = \frac{1}{2} \left(1 + \frac{M_0}{M_a}\right) \left(\phi_{uf} - \phi_u\right) + \phi_1 \]

Following this scheme, once the moment and the corresponding curvature at the three defined stages of short-term loading have been defined, the values of long-term curvature \( \phi_1 \) and \( \phi_{uf} \) can be determined directly using (9) and (10). Knowing the curvature, the following formula can be used to determine the crack width [15]:

\[ \phi = \frac{\varepsilon_{cm} + \varepsilon_{cr} + \varepsilon_{ctu,m}}{d} \]

where \( \varepsilon_{cr} \) and \( \varepsilon_{ctu} \) are the normal compressive and ultimate strain in concrete at the tension side [9], respectively, and \( d \) is the effective depth of the concrete section.

The corresponding curvature \( \phi_1 \) is related to the crack width \( w_1 \) and the average strain of compressed concrete \( \varepsilon_{cl,m} \), as follows:

\[ w_1 = \left(\phi_1 \cdot d - \varepsilon_{cl,m} - \varepsilon_{ctu,m}\right) \cdot S_{cr} \]  
\[ \varepsilon_{cl,m} = \frac{1}{2} \left(1 + \phi(t, t_0)\right) \]

where \( \phi(t, t_0) \) is the value of the creep of compressed concrete under the action of an external load with a duration of \( (t - t_0) \) [7]. The value of crack width \( w_1 \) can be determined from (10).

The crack width at failure under long-term loading will have the form:

\[ w_{ul,cr} = \left(d \cdot \phi_{ul} - \varepsilon_{ctu,m} - \varepsilon_{ctu,m}\right) \cdot S_{cr} \]

IV. COMPARISON WITH EXPERIMENTAL RESULTS

The theoretical prediction of long-term crack width has been performed for the 4 beams tested in [16]. The selected beams are simply supported with spans of 3500 mm and are reinforced by two longitudinal bars of 16 mm diameter. The concrete compressive strength at 28 days was 24.8 MPa. The cross-section of the beams is shown in Figure 5 and the details are given in Table I. The maximum crack width measured from the tests under long-term loading at 14 and 400 days are also presented in Table I along with the predicted values of crack width. The detailed results of the theoretically predicted long-term crack widths were compared with the experimental ones and the result is presented in Figures 6 and 7. The theoretically predicted crack width at different ages correspond well with experimental results. From both the theoretical and experimental results, it has been observed that the crack width increases rapidly at early ages up to the first 100 days, and then slowly increases. The maximum long-term crack width has been performed at an average age of 200 days.
and achieved accuracy should be investigated. Further, the exact tradeoff between computational load should focus on testing the simplified model in other case agreement with the experimental results. Future research published in [16]. As shown, the simplified model shows good agreement with the experimental results. Future research should focus on testing the simplified model in other case studies. Further, the exact tradeoff between computational load and achieved accuracy should be investigated.

REFERENCES


