

# A Force-based Method for the Numerical Simulation of a Reinforced Concrete Shear Wall Building

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## ABSTRACT

Reinforced Concrete (RC) shear walls are structural elements that resist lateral loads. This research aims to present the numerical modeling of RC shear walls in order to evaluate the seismic performance of structures. Various types of numerical models of RC frame elements are implemented in nonlinear analysis packages. These numerical models are based on different theories and assumptions, something that poses a significant difficulty to practicing engineering and limits confidence in the analysis of the numerical results. In this study, inelastic force-based elements and distributed plasticity methods are used for the modeling of the inelastic behavior of these elements (infrmFB). The efficiency of the inelastic force-based element and distributed plasticity method is evaluated through the comparison with the experimental results of a shear wall structure subjected to seismic loadings. The accuracy of the numerical model is assessed in terms of top displacement, inter-story drift, base shear force, and the absolute maximum values of the overturning moment.

*Keywords-shear wall; infrmFB element; force-based; nonlinear numerical model; RC frame elements; distributed plasticity*

## I. INTRODUCTION

Shear walls are used as the primary lateral load-resistant system of Reinforced Concrete (RC) structures. Earthquake engineering is focused on simulating the nonlinear response of shear wall structures to seismic excitations, through experimental studies [1, 2] and numerical modeling [3-5]. New performance-based seismic design guidelines [6] require that structures should be analyzed using nonlinear static pushover analysis or nonlinear dynamic analysis to control global and local demands. The use of nonlinear frame analysis necessitates the availability of robust and computationally efficient models for performing computations in a reasonable amount of time [7, 22]. In the numerical modeling approach, ground motion loading analysis of RC shear wall buildings is generally conducted with beam-column elements, which should be capable of fully taking into consideration the inelastic behavior of the actual member [8, 23]. Distributed plastic hinges in

beam-column elements used to model the nonlinear behavior of RC structural elements are thus required in order to simulate the seismic response and to evaluate the performance of structural systems [9].

Several studies have investigated the performance of different nonlinear modeling strategies for the numerical simulation of the response of a RC shear wall subjected to seismic loads. Authors in [10] studied the numerical strategy 3S-R adopted in the laboratory for the simulation of the nonlinear behavior of the specimen. Multi-fiber Timoshenko beam elements were used for the finite element mesh of the RC walls. Constitutive mechanical damage models were used for RC. The proposed modeling strategy accurately describes the overall behavior of the structure. The comparison with the experimental results helped identifying the shortcomings of the original numerical model and suggested remedial measures to improve its performance. Authors in [11] studied the nonlinear dynamic behavior and response of RC walls. They used a

continuous damage model, and simplified numerical strategies were proposed. They chose two dimensional (2D) Euler beam theory for the modeling of walls whose behavior was controlled effectively by bending. For 3D problems, a multi-fiber Timoshenko beam element with higher-order interpolation functions was developed. As for walls with a small slenderness ratio, they used the Equivalent RC model. Comparisons with the experimental results of RC walls tested on a shaking table or reaction wall showed the advantages and limitations of the approach. Authors in [12] developed a constitutive model for RC under cyclic or dynamic loading based on existing experimental results. The proposed model featured hysteretic characteristics of concrete in tension, compression, shear along the crack direction, and the bond between concrete and the reinforcing bars. They also developed a non-orthogonal multi-directional smeared crack model for the realistic representation of concrete cracking under stress reversals. The applicability and efficiency of the proposed models were demonstrated through simulation with various types of RC specimens subjected to cyclic loads or seismic excitation on a shaking table. Authors in [13] studied the nonlinear response of RC moment-resisting frames considering the bond-slip effect between concrete and bars along the lengths of the beam, column, and common elements. They used the fiber model theory to simulate the behavior of RC in the nonlinear domain. The ideal bond hypothesis between concrete and bars has been removed. They showed that the proposed method can accurately model the nonlinear behavior of RC frames by comparing the experimental and numerical models of two specimens under cyclic loading.

In this study, the structural response is assessed by nonlinear dynamic (time history) analysis to estimate the seismic response using the force-based inelastic frame elements (infrmFB) method. Emphasis is made on the applicability of the representative numerical model of the RC shear wall. Its efficiency is evaluated by comparison with the experimental results for a 7-story RC structural wall [14] that can be readily performed with existing software packages.

## II. NUMERICAL TOOL AND MODELING STRATEGIES

The numerical analysis developed and described in this paper and the nonlinear modeling strategies were conducted using the SeismoStruct v7 (SeismoSoft and 2021) [15]. The program includes models for the representation of the behavior of spatial frames under static and dynamic loading, considering both material and geometric nonlinearities. Seven types of analysis can be performed: dynamic and static time history analysis, conventional and adaptive pushover, incremental dynamic analysis, modal analysis, and static analysis (possibly nonlinear) under quasi-permanent loading. All the features are based on force or displacement formulations. While the evaluated numerical models are based on different assumptions, the input parameters for these elements are primarily physical properties, such as section geometry and the uniaxial behavior of materials. Hence, as long as the limitation of the element can be clearly defined, engineers can use the

elements with confidence and without much effort to calibrate model parameters. Therefore, in this study, the numerical simulation of NEES-UCSD RC shear wall structure was investigated using the nonlinear modeling strategy of force-based inelastic frame element (infrmFB). The experimental results obtained from previous shaking table tests of these shear wall structures are compared with the results of the numerical nonlinear seismic analysis. The experimental study uses the NEES-UCSD design [16, 17, 24, 25]. The response of an RC structure consisting of shear walls is simulated under the seismic loading recorded during the 1994 Northridge earthquake.

## III. ELEMENT FORMULATION – THE FLEXIBILITY (FORCE) METHOD

In this study, the force fields are described by the flexibility method and the nonlinear analysis of the RC shear wall structures is conducted using the infrmFB method. RC structures that consist of shear walls are simulated by a 7-story system available from the European Laboratory for Structural Assessment (ELSA) [14]. The numerical analysis results are compared with the experimental results in terms of absolute values of base overturning moment, base shear force, and tip displacement. A time history plot of the top displacement response of the shear wall structure is also used for comparison. This can easily be done with existing software packages. The structural response is evaluated by nonlinear dynamic (time history) analysis to estimate the seismic response:

$$D(x) = b(x) Q \quad (1)$$

where  $b(x)$  contains the force interpolation functions, which relates the generalized nodal forces  $Q$  to the internal forces  $D(x)$ . Replacing  $\Delta D$  from the incremental form of (1) in the inverse from the constitutive relation  $\Delta D = k \Delta d$ , namely  $\Delta d(x) = K^{-1} \Delta D(x)$  yields the incremental deformation field [18]:

$$\Delta d(x) = f(x) \Delta D(x) = f(x) b(x) \Delta Q \quad (2)$$

where  $f(x) = K^{-1}(x)$  is the section flexibility matrix. The principle of virtual forces leads of the compatibility condition are described by (3):

$$q = \int_0^L b^T(x) d(x) dx \quad (3)$$

and its linearization:

$$F \Delta Q = r \quad (4)$$

in the form of a displacement-force relation, where  $q$  = element end displacement and

$$F = \frac{\partial q}{\partial Q} = \int_0^L b^T(x) f(x) b(x) dx \quad (5)$$

is the element flexibility matrix, while  $\Delta Q$  and  $r$  are the vectors of force increments and residual displacement, respectively. Note that a meaningful expression for the flexibility matrix  $F$  can only be derived for the beam element without rigid-body modes [19].

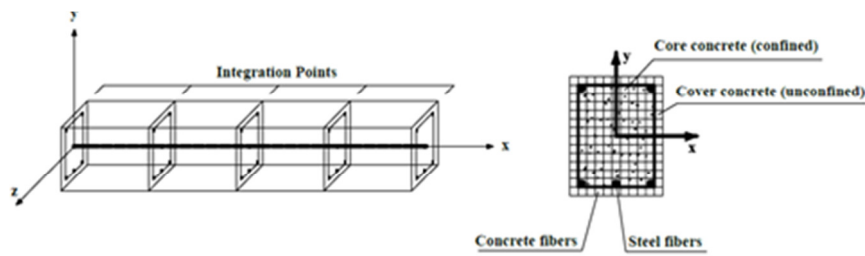


Fig. 1. Discretisation of the cross-section and section fiber.

IV. INELASTIC FORCE-BASED FRAME ELEMENT

This force-based 3D beam-column element type can model spatial frame members with geometric and material nonlinearities. Cross-section-related stress-strain states of beam-column elements are obtained by integrating the nonlinear uniaxial material responses of individual fibers whose cross-sections are divided by the depth of the cross-section. The infrmFB element is the most accurate of the four SeismoStruct frame element types, due to the fact that the inelastic behavior can be captured over the entire structural element length, even when using one element per structure. Therefore, it provides very accurate analysis results and allows users to output element chord rotations for seismic code validation (e.g. Eurocode 8, NTC-08, KANEPE, FEMA-356, ATC-40, etc.). One must define the number of section fibers used in the equilibrium calculations performed on each integrated section of the element. The ideal number of profile fibers sufficient to adequately reproduce the stress-strain distribution across the cross-section of the element depends on the element's geometry and material properties and on the element's degree of inelasticity. As a rough rule of thumb, the user can assume that sections made of a single material are typically well represented by 100 fibers. In more complex sections, subjected to high inelasticity, typically 200 or more fibers should be used. A sensitivity study on a case-by-case basis can undoubtedly determine the optimum number of cut fibers [20].

V. STRUCTURE, MATERIALS, AND LOADS

In the present study, a full-scale 7-story structure with two main perpendicular walls: the web wall and the flange wall linked with slabs (Figure 2), was modeled. A pre-cast column is needed to limit torsional behavior and gravity columns to support the slabs are also present. The RC frame was tested on the NEES (Earthquake Engineering Simulation Network) Large High-Performance Outdoor Shake Table at the University of California at San Diego (UCSD)'s Englekirk Structural Engineering under dynamic conditions [12] by applying 4 subsequent uniaxial ground motions. The structure has been designed with the displacement-based capacity approach for a site in Los Angeles. Hence, the lateral design forces are smaller than those currently specified in U.S. building codes for regions of high seismic risk.

A. Structural Geometry

A 7-story RC earthquake-resistant wall designed by the NEES was considered in this study. It was seismically loaded on a shaking table at the UCSD [16]. This structure comprises

two types of shear walls located on the central axis and the pier columns on the corners of the structure. One of the shear walls is called the web wall, while the other is the flange wall. The walls are built perpendicular to each other. The slab is connected to the flange wall, and slotted connections join the two shear walls. The basement and floor heights are 0.76m and 2.74m, respectively. The total height of the structure is 19.96m. The structure is fastened to the ground, and its mass is 226 tons. The cross-sections of the flange wall and the web wall are different. The width of the flange wall is 4.88m and the thicknesses of the flange wall are 203mm and 152mm for the last floor and the other floors, respectively. The width of the web wall is 3.65m and its thickness is 203mm for the first and the last floor, while it is 152mm is for the other floors. The columns and shear walls supported 3.65x8.13m slabs on each floor. Post-tensioned precast piers are joined to the web wall and slabs via bracing. The full-scale RC shear wall structure details are shown in Figure 3. The geometry of the structure is summarized in Table I. The mass values of the structure are summarized in Table II. Two different views of the Finite Element (FE) model are presented in Figure 3.

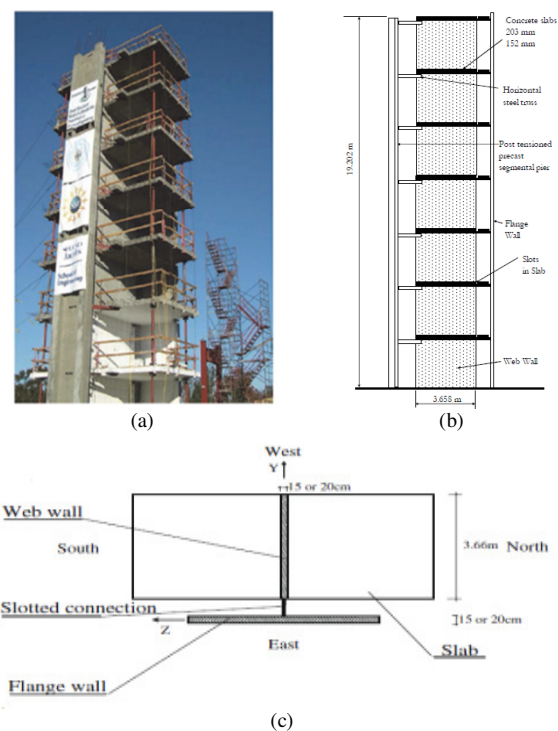


Fig. 2. The 7-story RC shear wall building tested at the NEES.

TABLE I. GEOMETRY OF WALLS AND GRAVITY COLUMNS

| Story | Web wall (m <sup>2</sup> ) | Flange wall (m <sup>2</sup> ) | Gravity volumns         |
|-------|----------------------------|-------------------------------|-------------------------|
| 7     | 3.658×0.203                | 4.877×0.152                   | d= 0.102 m, t = 0.033 m |
| 6     | 3.658×0.152                | 4.877×0.152                   | d= 0.102 m, t = 0.028 m |
| 5     | 3.658×0.152                | 4.877×0.152                   | d= 0.102 m, t = 0.028 m |
| 4     | 3.658×0.152                | 4.877×0.152                   | d= 0.102 m, t = 0.028 m |
| 3     | 3.658×0.152                | 4.877×0.152                   | d= 0.102 m, t = 0.028 m |
| 2     | 3.658×0.152                | 4.877×0.152                   | d= 0.102 m, t = 0.028 m |
| 1     | 3.658×0.203                | 4.877×0.152                   | d= 0.102 m, t = 0.028 m |

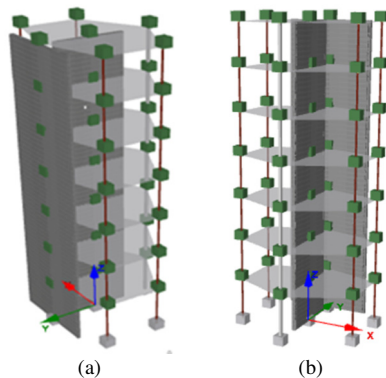


Fig. 3. FE model view.

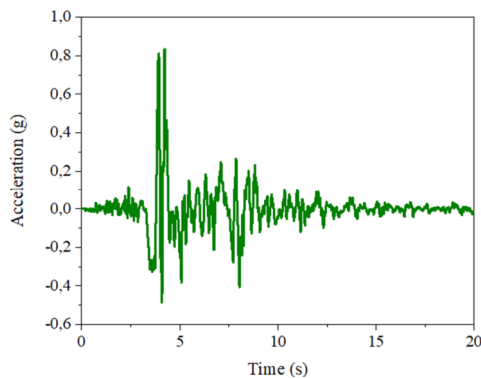


Fig. 4. Acceleration time history.

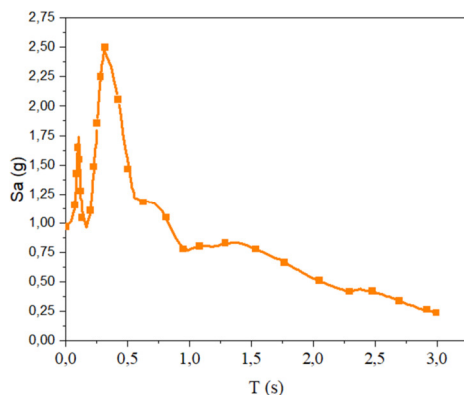


Fig. 5. Response spectrum of the accelerograms—5% damping.

**B. Input Accelerations and Spectrum**

The 1994 Northridge earthquake acceleration record, obtained from the Sylmar Olive View Medical Centre, was

used as the seismic input parallel to the web wall and was successfully applied to the shaking table with amplifying acceleration magnitude. The acceleration record of the earthquake is shown in Figure 4, with a peak ground acceleration of 0.85g. During the test, global and local measurements were made (displacements, accelerations, moments, etc.). The apparent mode was measured after the examination to follow the stiffness evolution. Detailed information on the experimental program and results are described in [20]. The response spectrum of these motions corresponding to 5% viscous damping is shown in Figure 5.

TABLE II. MASS OF DIFFERENT COMPONENTS

| Components      | Mass (tons) |
|-----------------|-------------|
| Foundation      | 21.432      |
| Slabs           | 101.604     |
| Web wall        | 24.947      |
| Flange wall     | 33.263      |
| PT column       | 30.158      |
| Gravity columns | 4.717       |
| Braces          | 0.181       |

**C. Material Properties**

Authors in [21] proposed a unified stress–strain approach for confined concrete applicable to both circular and rectangular-shaped transverse reinforcement. Its characteristic parameters are:  $f_c = 16300\text{kPa}$ ,  $f_t = 1900\text{kPa}$ ,  $\epsilon_c = 0.002\text{m/m}$ , and  $E_c = 18975\text{MPa}$ . The Menegotto-Pinto steel model is employed for defining the steel with the following properties:  $E_s = 2.00\text{E}+ 008 \text{ kPa}$ ,  $f_y = 343000 \text{ kPa}$ ,  $\mu = 0.0024$ . The materials considered at the design phase were: low-strength concrete of class C16/20 (CEN, 1991) and smooth reinforcement steel class Fe B22 k (Italian standards). The latter refers to smooth bars with a yield stress of 235MPa and ultimate strength of 365MPa.

**VI. RESULTS AND DISCUSSION**

**A. Displacement**

To evaluate the level of accuracy obtained with the modeling strategy, the first step is to assess the global response of the structure under seismic action. The roof time-displacement trends for the studied frame are presented in Figure 6.

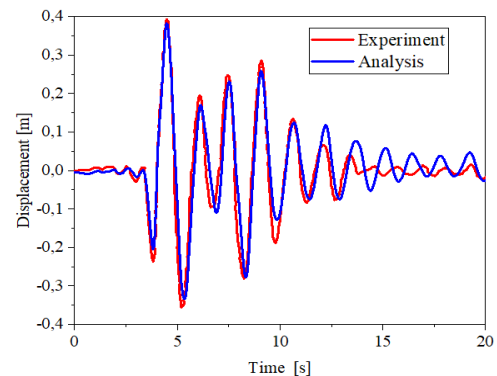


Fig. 6. Lateral displacements at the top versus time.

They are obtained by using inelastic force-based elements with distributed plasticity. The results indicate a good agreement with the experimental results when the time is less than 12s. For a time greater than 12s, the results diverge from the experimental ones. The lateral displacements from numerical analysis for each story are close to those of the experimental ones, see Figure 7.

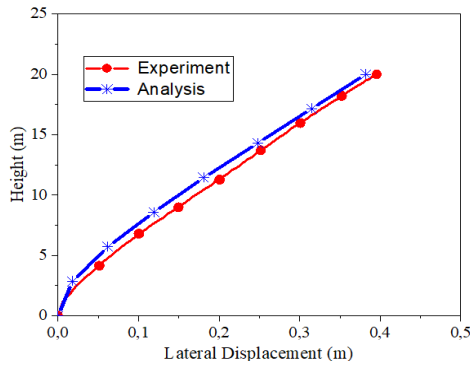


Fig. 7. Maximum lateral displacement.

B. Inter Story Drift

The lateral deformability of structural systems is measured through the horizontal drift. In buildings, story drift  $\Delta$  is the absolute displacement of any floor relative to the base. In contrast, inter-story drift  $\delta$  defines the relative lateral displacement between two consecutive floors. The inter-story drift is generally expressed as the ratio  $\delta/h$  of the displacement to the story height  $h$ . The drift of the roof  $\Delta$  normalized by the total height  $H$  of the building (roof drift,  $\Delta/H$ ) is also used to quantify the lateral stiffness of structural systems. Figure 8 shows the vertical distribution of the maximum story drift of the main frame. The analytical model can predict the results of the 7 floors. However, at the base, the results are not close to the experimental values. For example, the difference in inter-story drift is about 0.67% in the first story, in the second story the difference is about 0.28%, and above this floor, the difference is negligible.

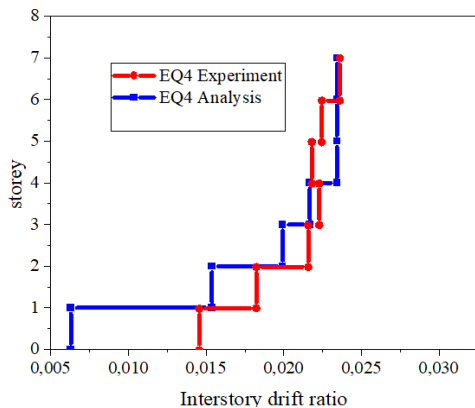


Fig. 8. Maximum inter-story drift ratio in the structure.

C. Base Overturning Moment

The base moment was determined by summing the moments at the base of the web and flange walls and the couple generated by the gravity columns that had about 10–24% contribution to the base overturning moment. Figure 9 compares the experimental and analytical values for a base moment using inelastic force-based elements with distributed plasticity. The results indicate that the peak values of the numerical and experimental results varied by about 25%.

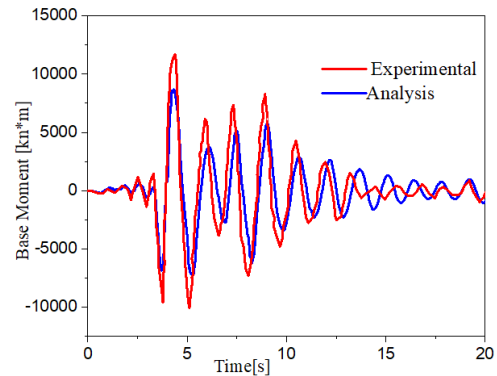


Fig. 9. Comparison of experimental and analytical base moment values.

D. Story Shear Forces

Figure 10 compares the experimental and analytical values for base shear using inelastic force-based elements and distributed plasticity. Again, the numerical model underestimates the maximum measured base shear force value during the input of the motion by about 12%.

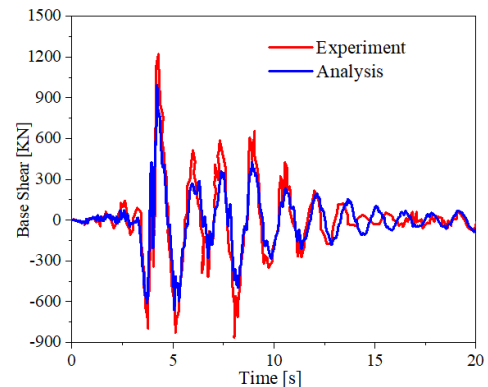


Fig. 10. Comparison of the experimental and analytical values of the shear force at the wall base.

VII. CONCLUSIONS

In this study, the numerical simulation of reinforced concrete shear wall structure is undertaken in order to evaluate the efficiency of the numerical model inelastic force-based element with distributed plasticity. The numerical analysis results for the strongest input motion are compared with the experimental shaking table test results and the accuracy of the numerical model is assessed, considering the time-history

graphs of top displacement, inter-story drift, base shear force, and the absolute maximum values of the overturning moment. The main outcomes for the considered method are:

- The numerical simulation predicts a satisfactory maximum displacement, especially in the y direction, where the loading is more severe. There is a slight shifting in the time history curves, the same remarks apply to the inter-story drift curve, the base shear force, and the absolute maximum values of the overturning moment.
- The model was able to reproduce the response of the structure with good approximation. This confirms that the level of discretization and the type of numerical elements adopted in the model are sufficient to describe the nonlinear behavior of the reinforced concrete shear wall structures subjected to severe ground motion.

#### REFERENCES

- [1] S.-T. Huang *et al.*, "Experimental study on seismic behaviour of an innovative composite shear wall," *Journal of Constructional Steel Research*, vol. 148, pp. 165–179, Sep. 2018, <https://doi.org/10.1016/j.jcsr.2018.05.003>.
- [2] S.-T. Huang *et al.*, "Experimental study on seismic behaviour of an innovative composite shear wall," *Journal of Constructional Steel Research*, vol. 148, pp. 165–179, Sep. 2018, <https://doi.org/10.1016/j.jcsr.2018.05.003>.
- [3] F. Dashti, R. P. Dhakal, and S. Pampanin, "Numerical Modeling of Rectangular Reinforced Concrete Structural Walls," *Journal of Structural Engineering*, vol. 143, no. 6, Jun. 2017, Art. no. 04017031, [https://doi.org/10.1061/\(ASCE\)ST.1943-541X.0001729](https://doi.org/10.1061/(ASCE)ST.1943-541X.0001729).
- [4] S. Zhang, M. Cheng, J. Wang, and J.-Y. Wu, "Modeling the hysteretic responses of RC shear walls under cyclic loading by an energy-based plastic-damage model for concrete," *International Journal of Damage Mechanics*, vol. 29, no. 1, pp. 184–200, Jan. 2020, <https://doi.org/10.1177/1056789519889103>.
- [5] F. C. Filippou and G. L. Fenves, "Methods of Analysis for Earthquake-Resistant Structures," Y. Bozorgnia and V. V. Bertero, Eds. Boca Raton, FL, USA: CRC Press, 2004, pp. 316–394.
- [6] *NEHRP Guidelines for the Seismic Rehabilitation of Buildings*. Washington, DC, USA: FEMA, 1997.
- [7] J. Coleman and E. Spacone, "Localization Issues in Force-Based Frame Elements," *Journal of Structural Engineering*, vol. 127, no. 11, pp. 1257–1265, Nov. 2001, [https://doi.org/10.1061/\(ASCE\)0733-9445\(2001\)127:11\(1257\)](https://doi.org/10.1061/(ASCE)0733-9445(2001)127:11(1257)).
- [8] J. P. Almeida, S. Das, and R. Pinho, "RC Frame Analysis with a New Damage-following Model," presented at the 15 WCEE, Lisboa, Portugal, 2012.
- [9] M. H. Scott and G. L. Fenves, "Plastic Hinge Integration Methods for Force-Based Beam–Column Elements," *Journal of Structural Engineering*, vol. 132, no. 2, pp. 244–252, Feb. 2006, [https://doi.org/10.1061/\(ASCE\)0733-9445\(2006\)132:2\(244\)](https://doi.org/10.1061/(ASCE)0733-9445(2006)132:2(244)).
- [10] S. Grange, P. Kotronis, and J. Mazars, "Numerical modelling of the seismic behaviour of a 7-story building: NEES benchmark," *Materials and Structures*, vol. 42, no. 10, pp. 1433–1442, Dec. 2009, <https://doi.org/10.1617/s11527-008-9462-y>.
- [11] P. Kotronis and J. Mazars, "Simplified Modelling Strategies to Simulate the Dynamic Behaviour of R/C Walls," *Journal of Earthquake Engineering*, vol. 9, no. 2, pp. 285–306, Mar. 2005, <https://doi.org/10.1080/13632460509350543>.
- [12] K. Naganuma, K. Yonezawa, O. Kurimoto, and H. Eto, "Simulation of Nonlinear Dynamic Response of Reinforced Concrete Scaled Model Using Three-Dimensional Finite Element Method," in *13th World Conference on Earthquake Engineering*, Vancouver, BC, Canada, Aug. 2004, Art. no. 586.
- [13] S. S. Hashemi and M. Vaghefi, "Cyclic analysis of RC frames with respect to employing different methods in the fiber model for consideration of bond-slip effect," *Turkish Journal of Engineering and Environmental Sciences*, vol. 36, pp. 1–18, 2012.
- [14] M. Panagiotou, J. I. Restrepo, and R. E. Englekirk, "Experimental Seismic Response of a Full Scale Reinforced Concrete Wall Building," in *First European Conference on Earthquake Engineering and Seismology*, Geneva, Switzerland, Sep. 2006.
- [15] "Seismosoft Earthquake Engineering Software Solutions," *Seismosoft*. <https://seismosoft.com/>.
- [16] P. Martinelli, "Shaking Table Tests on RC Shear Walls: Significance of Numerical Modeling," Ph.D. dissertation, Politecnico di Milano, Milan, Italy, 2007.
- [17] P. Martinelli and F. C. Filippou, "Simulation of the shaking table test of a seven-story shear wall building," *Earthquake Engineering & Structural Dynamics*, vol. 38, no. 5, pp. 587–607, 2009, <https://doi.org/10.1002/eqe.897>.
- [18] S. Limkatanyu and E. Spacone, "Reinforced Concrete Frame Element with Bond Interfaces. I: Displacement-Based, Force-Based, and Mixed Formulations," *Journal of Structural Engineering*, vol. 128, no. 3, pp. 346–355, Mar. 2002, [https://doi.org/10.1061/\(ASCE\)0733-9445\(2002\)128:3\(346\)](https://doi.org/10.1061/(ASCE)0733-9445(2002)128:3(346)).
- [19] M. H. Scott, P. Franchin, G. L. Fenves, and F. C. Filippou, "Response Sensitivity for Nonlinear Beam–Column Elements," *Journal of Structural Engineering*, vol. 130, no. 9, pp. 1281–1288, Sep. 2004, [https://doi.org/10.1061/\(ASCE\)0733-9445\(2004\)130:9\(1281\)](https://doi.org/10.1061/(ASCE)0733-9445(2004)130:9(1281)).
- [20] "NHERI @ UC San Diego." <http://nees.ucsd.edu/>.
- [21] J. B. Mander, M. J. N. Priestley, and R. Park, "Theoretical Stress-Strain Model for Confined Concrete," *Journal of Structural Engineering*, vol. 114, no. 8, pp. 1804–1826, Aug. 1988, [https://doi.org/10.1061/\(ASCE\)0733-9445\(1988\)114:8\(1804\)](https://doi.org/10.1061/(ASCE)0733-9445(1988)114:8(1804)).
- [22] H. I. Polat, "Analysis of a Frame-Shear Wall Concrete Structure by Using Base Isolation and Evaluation of Structure-Soil Interaction," *Engineering, Technology & Applied Science Research*, vol. 7, no. 6, pp. 2282–2287, Dec. 2017, <https://doi.org/10.48084/etasr.1611>.
- [23] F. Abdelhamid, D. Yahiaoui, M. Saadi, and N. Lahbari, "Lateral Reliability Assessment of Eccentrically Braced Frames Including Horizontal and Vertical Links Under Seismic Loading," *Engineering, Technology & Applied Science Research*, vol. 12, no. 2, pp. 8278–8283, Apr. 2022, <https://doi.org/10.48084/etasr.4749>.
- [24] M. Saadi and D. Yahiaoui, "The Effectiveness of Retrofitting RC Frames with a Combination of Different Techniques," *Engineering, Technology & Applied Science Research*, vol. 12, no. 3, pp. 8723–8727, Jun. 2022, <https://doi.org/10.48084/etasr.4979>.
- [25] A. Zendaoui, A. Kadid, and D. Yahiaoui, "Comparison of Different Numerical Models of RC Elements for Predicting the Seismic Performance of Structures," *International Journal of Concrete Structures and Materials*, vol. 10, no. 4, pp. 461–478, Dec. 2016, <https://doi.org/10.1007/s40069-016-0170-7>.