Behavior of Strip Footing/s above Void in Sandy Soil

Assma Benbouza
LGC-ROI Civil Engineering Laboratory, Department of Civil Engineering, Faculty of Technology, University of Batna 2, Algeria
as.benbouza@univ-batna2.dz

Tarek Mansouri
LGC-ROI Civil Engineering Laboratory, Department of Civil Engineering, Faculty of Technology, University of Batna 2, Algeria
t.mansouri@univ-batna2.dz

Khelifa Abbeche
LGC-ROI Civil Engineering Laboratory, Department of Civil Engineering, Faculty of Technology, University of Batna 2, Algeria
k.abbeche@univ-batna2.dz

Received: 17 November 2022 | Revised: 4 December 2022 | Accepted: 12 December 2022

ABSTRACT

This paper presents a numerical study that utilizes finite element analysis under the plain strain condition performed on sand with isolated strip footing and two closely spaced strip footings above a continuous void. The Bearing Capacity Ratio (BCR) and the efficiency factor $\zeta\gamma$ were introduced to determine the effect of the void on the ultimate bearing capacity of footing/s. The influence of various parameters including spacing (S/B) (i.e. edge to edge) between the two interfering footings along with the location and the shape of the void were studied. In general, the results indicate that the presence of a void reduces the bearing capacity and affects the performance of footing/s and there is a critical value of S/B beyond which the effect of the void on the bearing capacity of the interfering footings becomes negligible.

Keywords-bearing capacity; strip footing; finite element method; underground voids; granular soil; interfering footings

I. INTRODUCTION

During the recent years, urban development has been one of the Algerian government's top priorities, which has greatly reduced the amount of land available for the construction of projects, thus, it has become impossible to avoid constructing buildings close to each other in order to make the most of the land and reduce cost, but this type of construction can have an influence on the bearing capacity of the foundations. The calculation of the bearing capacity of shallow footings is usually done using a method similar to that developed in [1] for isolated footings. In reality, footings are rarely isolated and interfere with each other to some extent. Shallow footings interference has been studied theoretically in [2] using the limit equilibrium method and the stress characteristic method. Authors in [3] presented laboratory test results for two closely spaced footings on sand. They found that the interference factors are generally lower than those predicted by theory. Authors in [4] studied the interaction between two closely spaced rough and smooth strip footings on dense and loose sand. Authors in [5] used the method of characteristics for two different failure mechanisms. Authors in [6] used the upper bound limit analysis for finding the interference effect of two nearby strip footings on sand. Authors in [7] examined the closely spaced footings on geogrid-reinforced sand, and authors in [8] studied the interference effect of strip and square footings on sand reinforced with geosynthetics. Authors in [9] studied numerically the bearing capacity of two interfering strip footings on sands. Authors in [10] studied the failure surface in granular soil under two closely spaced strip footings and concluded that the failure patterns observed for granular soil conform to those proposed by the theory in [2]. Authors in [11] experimentally studied the interference effect of two closely strip footings constructed on bi-layer soil. Authors in [12] studied experimentally and numerically the interference of closely spaced square footings on sandy soil, and concluded that the maximum interference effect is observed when the spacing between the two footings is 0.5B, and was approximately negligible when the spacing between the footings was equal to 2B, where B is the footing's width.

The existence of underground voids (e.g. natural or artificial caves) may be attributed to two reasons: dissolution of soluble materials (e.g. salt, dolomite, and limestone) and artificial underground activities such as tunneling, mining, subway excavations, etc. These voids could cause serious engineering problems leading to poor performance of shallow foundations, structure collapses, road settlements, etc. which need special attention in engineering practice. In many cases, footings are positioned on soil containing voids that are either not visible before construction or are formed after the construction. Several studies on the subject are available and some of them deal with cavity-footing interaction, e.g. [13], while others studied experimentally, theoretically, and numerically the effect of a void on the stability of shallow footings [14-18, 19]. Other studies investigated the yielding pressure of strip footings above multiple-shaped cavities using FEM [20, 21]. Authors in [22] adopted FE analysis to estimate the undrained bearing capacity of surface strip footings above one and two voids. Authors in [23] studied with FEM the behavior of shallow strip footing on twin voids and clarified the failure mechanism. Authors in [24] investigated the ultimate bearing capacity and failure mechanism of strip footings subjected to vertical load placed on c-φ soil with square voids. The critical and adverse locations of voids were analyzed. Authors in [25] investigated numerically the undrained stability of strip footings above voids in two-layered clays. Using finite element limit analysis, the undrained bearing capacity factor Ns of strip footings has been calculated and the effect of the thickness of the top layer and the effect of undrained shear stress ration on Ns were studied. In case of slope, a numerical study was conducted to analyze the bearing capacity behavior of strip footings on a reinforced sand slope with a single circular void [26]. Further investigations have been carried out on the effect of artificial cavities on deep foundations [27, 28].

The influence of voids on the performance of shallow interfering footings has not been well covered or is not available in the literature. For this reason, this study aims to investigate numerically the effect of interference of two closely spaced strip footings above single continuous void in sandy soil. The main purpose of this study, in order to evaluate the bearing capacity of strip footing/s above void, was to reveal the effect of various parameters, such as the spacing (S/B) (i.e. edge to edge) between two footings, the shape and the location of the void, on the ultimate bearing capacity.

II. PROBLEM DEFINITION

The geometry and the key parameters of the problem analyzed in this paper are illustrated in Figure 1. The geometry consists of an isolated rigid strip footing and two closely spaced rigid strip footings with B and S representing the footing width and the spacing between the two footings respectively. They are placed on the horizontal surface of an isotropic homogenous soil of friction angle φ and unit weight γ. A static vertical load is imposed. By considering the case of zero surcharge and zero soil cohesion, the bearing capacity formula reduces to:

$$q_u = \frac{1}{2}\gamma B N_\gamma \zeta_\gamma \qquad (1)$$

where ζ_γ is the efficiency factor which is function of S/B and φ.

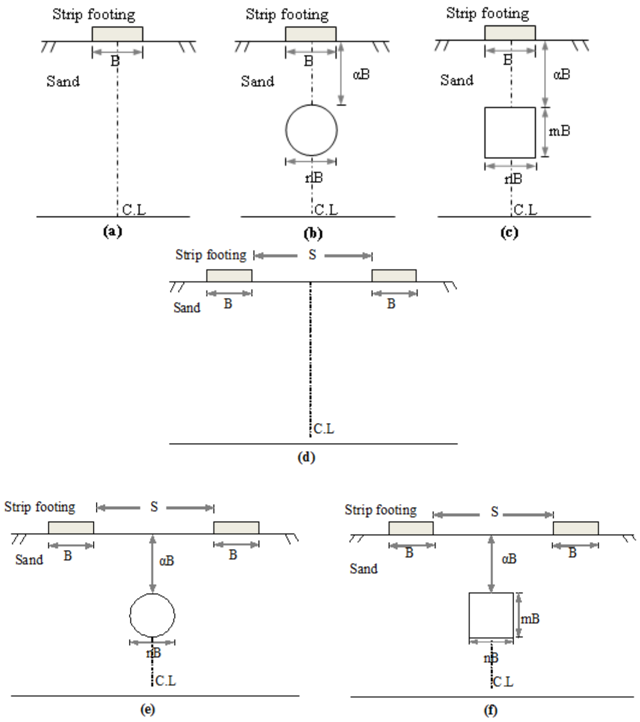

Fig. 1. Schematic view of footing/s and void.

In case of voids below the footings, one square and one circular voids were adopted in this study. Shape and location of the void and the spacing between voids are quantified in terms of dimensionless parameters, i.e. m, n, α. Parameters m and n represent the void height and width or diameter normalized by the footing width B and they are equal to 1 in this study. Parameter α designates the relative vertical distance from the centerline of the footing to the void normalized by B. The configuration of a single square or circular void is shown in Figure 1. According to [20], the void continuously extends horizontally. Furthermore, the ratio of void width to the width of the strip footing is equal to 1, which corresponds to the presupposition of [22].

III. FINITE ELEMENT MODEL

The commercially available finite element program PLAXIS was used to model the footings-voids system. The behavior of soil was numerically simulated as an elastic-perfectly plastic material considering Mohr–Coulomb failure criterion in conjunction with a non-associated flow rule (i. e. $\varphi \neq \Psi$).The values of the geotechnical properties are: Young's modulus E = 30MPa, Poisson's ratio ν = 0.2, bulk unit weight γ = 20KN/m^3, and friction angle φ =35°. The soil was modeled with 15-node triangular elements. Furthermore, Figure 1 shows two shapes of void: the continuous circular void which is considered as a tunnel without lining and the continuous square void which is introduced by excavation of the soil at the desired depth, to model a natural cavity. Thus the vertical and horizontal limits of this model are chosen appropriately far to avoid any influence on the results [17]. The dimensions of the area for this analysis are 15B in vertical and 30B in horizontal taking that B=1m. Figure 2 shows a schematic of the boundary conditions and the finite element mesh used in the current study. The vertical borders have been fixed in the horizontal direction and full fixities have taken place at the bottom of the model. The mesh is refined in the area adjacent to the footings and around voids to enhance the accuracy of the numerical results. The footings are considered very stiff and rough. In this study, instead of modeling the footing itself, the settlement of the footing is imposed by means of a uniform indentation at the top of the sandy layer until the ground reaches the failure state. The uniformly displacement is automatically decided with trial calculations in the program. Because of the symmetrical nature of the problem and in order to reduce the time required for each run, only half of the model was taken in the numerical simulation.

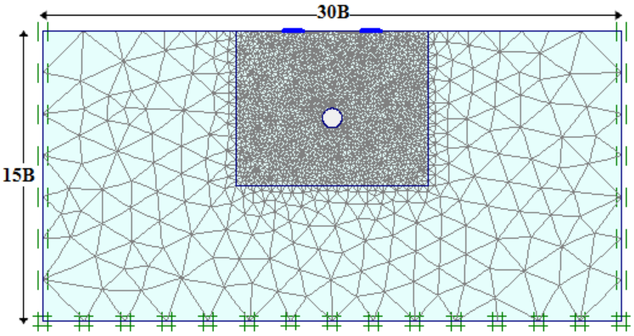

Fig. 2. Finite element mesh with boundary conditions.

IV. TEST PROGRAM

A total of 98 parametric tests were performed on the load-bearing capacity of rigorous strip foundations resting on one cavity of circular or square shape, as shown in Table I.

TABLE I. DETAILS OF NUMERICALLY MODEL TESTS

Test series	**Footings number**	**Shape of voids**	**Variable parameters**		**Fixed parameters**
			α	**S/B**	
A	Isolated footing	Without void	/	/	*n*=*m*=B=1m *φ*=35° *L*/B=1
		With a circular void	1 - 2 - 3 - 4 - 5 - 6 - 7 and 8	/	
		With a square void	1 - 2 - 3 - 4 - 5 - 6 - 7 and 8	/	
B	Two interfering footings	Without void	/	0 - 0,5 - 1 - 1,5 - 2 - 3 - 4 - 5 and 6	
		With a circular void	1 - 3 - 5 and 7	0 - 0,5 - 1 - 1,5 - 2 - 3 - 4 - 5 and 6	
		With a square void	1 - 3 - 5 and 7	0 - 0,5 - 1 - 1,5 - 2 - 3 - 4 - 5 and 6	

The efficiency factor ζ_γ was introduced to determine the influence of the void on the ultimate bearing capacity. ζ_γ is expressed as:

$$\zeta_\gamma = \frac{q_{u\,int\,v}}{q_{u\,isol}} \qquad (2)$$

where $q_{u\,int\,v}$ is the ultimate bearing capacity of two strip footings with or without void, and $q_{u\,isol}$ is the ultimate bearing capacity of an isolated strip footing without void.

V. VALIDATION

To validate the numerical model, the bearing capacity factor N_γ of a single strip footing on sand is calculated. The obtained N_γ value is compared with the results from the literature while the friction angle is equal to 35°. As shown in Table II, the result of this study is remarkably close to that given in the literature. This good accordance can be taken as a validation of the present numerical model.

TABLE II. VALUES OF BEARING CAPACITY FACTOR N_γ

Reference	[1]	[29]	[30]	[31]	Current study
N_γ	45.41	37.15	48.03	33.92	45.72

VI. RESULTS AND DESCUTION

A. *Single Strip Footing in Soil with and without Void*

The bearing pressure-displacement curves for the strip footing in soil without void and with a circular or square void are shown in Figure 3. For all cases, the footing reaches a clear limit load, which was taken as the ultimate bearing capacity. The presence of the void has a big influence on the bearing pressure. The magnitude of bearing pressure is higher for the soil without a void, therefore the presence of voids in the soil reduces bearing capacity. In addition, the magnitude of bearing pressure for a soil with circular void is slightly higher than for a square void.

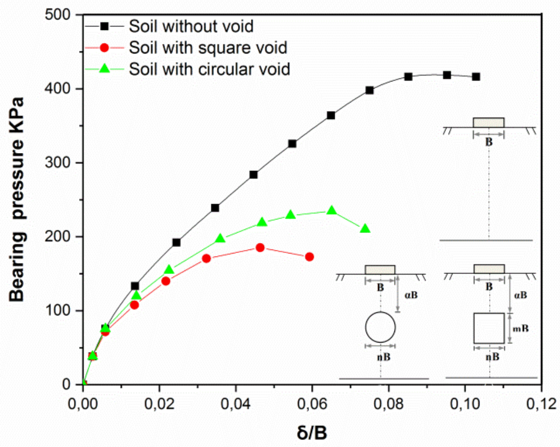

Fig. 3. Bearing pressure-displacement (α=3).

1) *Effect of Void Depth from the Single Footing Base*

The influence of α on the bearing capacity is presented in Figure 4 which shows the relationship between the Bearing

Capacity Ratio (BCR) as a function of the void depth α. BCR is defined as:

$$BCR=\frac{q_{u\,isol\,v}}{q_{u\,isol}} \quad (3)$$

where $q_{u\,isol\,v}$ is the ultimate bearing capacity of an isolated strip footing with void and $q_{u\,isol}$ is the ultimate bearing capacity of an isolated strip footing without void.

As can be noticed in Figure 4 the bearing capacity ratio increases linearly when the value of α increases from1 to 4 for circular void and from 1 to 5 for square void and remains almost constant thereafter for either circular or square void. The BCR recorded is 0.14 to 1 for α ranging from 1 to 8. This indicates that the existence of circular or square void under the foundation at a depth equal to or greater than 6 times the foundation width does not affect the bearing capacity of the footing and the void impact is eliminated. These results are in accordance with the findings of [24]. Also, it is observed from Figure 4 that the values of the BCR for the circular void case are larger than those of the BCR for the square void case.

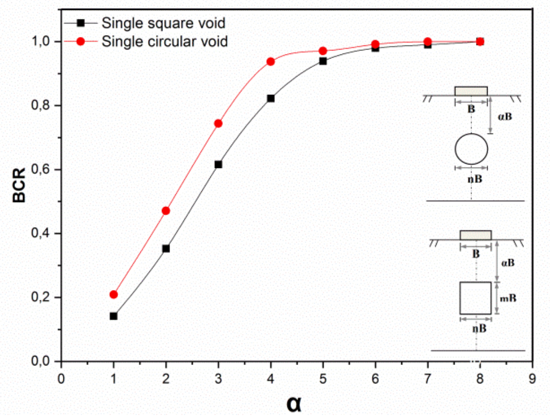

Fig. 4. Variation of BCR as a function of α.

B. Interfering Effect of Two Strip Footings with and without Void

1) Two Adjacent Strip Footings without Void

The results of numerical analysis obtained by the Plaxis code shown in Figure 5 prove that for $0 \leq S/B \leq 1$, the efficiency factor ($\zeta\gamma$) magnitude increased to its maximum value, which means that the ultimate bearing capacity of each strip footing increases almost by 60% and for $1 \leq S/B \leq 4$, $\zeta\gamma$ decreases with an increase in spacing ratio. Finally, for $S/B \geq 4$, $\zeta\gamma$ remains constant. This means that for a spacing ratio greater than 4B, no interference effect was observed and each footing acted as an isolated footing.

To verify the accuracy of the results obtained by the present study, they were compared with the obtained numerical analysis results from [8, 9], theoretical analysis [2], and experimental test [3], which are presented in Figure 5.

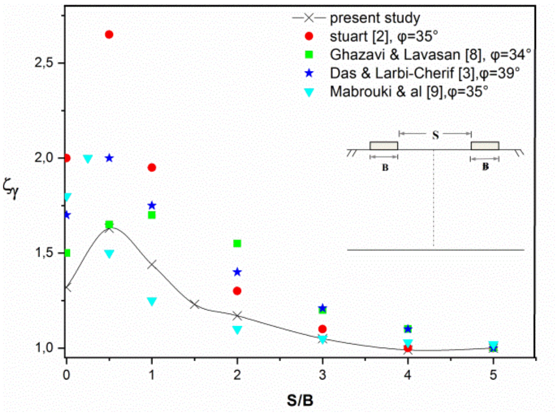

Fig. 5. Comparison between the literature values and present study.

It appears that the general trend of interference factor variations found in this study is similar to those predicted by other studies, but there is a large variation in amplitudes between theory, the experimental, and the numerical results. From this Figure, the numerical results agree very well with the experimental test results [10].

2) Two Adjacent Strip Footings with a Single, Square or Circular Void

Figure 6 indicates the variation of ζ_γ with different spacing ratios S/B for two interfering strip footings above a single circular or square void for a variation of void depths α=1, 3, 5, and 7 at 2m intervals, for a sandy soil with a void located in the centerline of the model. It can be noted that for S/B varying from 0 to 1.5 for circular void and from 0 to 2 for square void, the value of $\zeta\gamma$ increases with increasing α until the void is present within a certain critical depth α which is about 7 for a circular void and a little higher for a square void. The influence of the void gradually becomes insignificant so only the interference effect of the two footings remains. When the footings are located away from the centerline of the model (i.e. when the S/B value increases), the efficiency factor magnitude increases with an increase in α and reaches the maximum value when S/B = 1 for α = 1, 5, or 7 and S/B = 1.5 for α = 3. In most cases $\zeta\gamma$ decreases until the ultimate bearing capacity remains at 100% for S/B almost equal to 4, indicating no effect of the void on footing stability while there is not much interference effect. The only exception to this observation occurs when α = 1 where $\zeta\gamma$ values are significantly lower and keep invariant approximately for S/B varying between 1.5 and 2.5 for a circular void and between 1 and 1.5 for a square void, particularity due to the effect of the stability of the soil above the void which is more dominant than that of the effect of footings' interference. $\zeta\gamma$ continues its increase to reach 1 at S/B=6 for a circular void and almost 1 at S/B=6 for a square void, so the void effect is neglected and there is not much interference effect. On the other hand, for the case of a circular void, the $\zeta\gamma$ for all values of S/B and α is more than that for the case of the square void shown in Figure 6. It clearly can be observed that same remark in Figures 3, 5, and 6, in which the values of bearing capacity for the circular void are larger than those of the bearing capacity for the square void, due to the section of the square void which is greater than that of the circular void and not to the shape of the void, therefore the effect of the void's shape can be neglected [32].

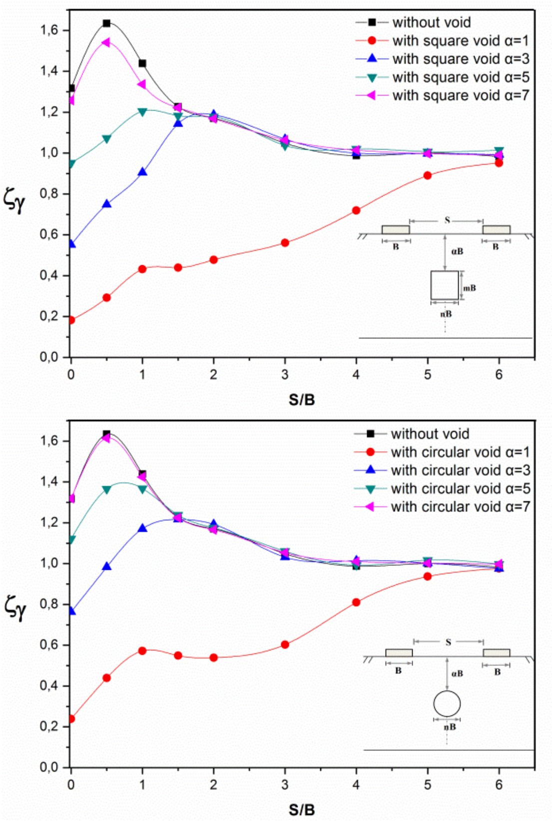

Fig. 6. Variation of the efficiency factor ζγ versus spacing ratio S/B for two interfering strip footings above a single circular or square void.

VII. CONCLUSION

In this paper, the behavior of strip footings above voids in sandy soil has been investigated numerically, and the BCR and ζγ factors of footings have been calculated. In addition, the critical location of the void and the spacing between footings have also been discussed. The main conclusions of the current study are:

- The presence of void has a big influence on bearing pressure. The magnitude of the bearing pressure is higher for a soil without voids, therefore the presence of voids in the soil reduces bearing capacity.
- With single strip footing, the existence of a circular or square void under the foundation at a depth equal to or greater than 6 times the foundation width does not affect the bearing capacity of the footing and the void impact is eliminated.
- With two interfering strip footings without void the results of numerical analysis proved that for a spacing ratio greater than 4B, no interference effect was observed and each footing acted as an isolated footing.
- With two interfering strip footings with a void located in the centerline of the model, it can be noted that for values of S/B varying from 0 to 1.5 for circular void and from 0 to 2 for square void, the value of ζγ increases with increasing α until the void is present within a certain critical depth α which is about 7 for a circular void and a little higher for a square void. The influence of the void becomes insignificant so only the interference effect of the two footings remains.
- There is a critical value of S/B beyond which the effect of the void on the bearing capacity of the interfering footings becomes negligible.
- The values of bearing capacity for the circular void case are larger than those for the square void case, due to the section of the square void which is greater than that of the circular void and not to the shape of void, therefore the effect of the void shape can be neglected.

REFERENCES

[1] K. Terzaghi, *Theoretical Soil Mechanics*. New York, NY, USA: Wiley, 1943.

[2] J. G. Stuart, "Interference Between Foundations, with Special Reference to Surface Footings in Sand," *Géotechnique*, vol. 12, no. 1, pp. 15–22, Mar. 1962, https://doi.org/10.1680/geot.1962.12.1.15.

[3] B. M. Das and S. Larbi-Cherif, "Bearing Capacity of Two Closely-Spaced Shallow Foundations on Sand," *Soils and Foundations*, vol. 23, no. 1, pp. 1–7, Mar. 1983, https://doi.org/10.3208/sandf1972.23.1.

[4] E. C. J. Hazell, "Interaction of closely spaced strip footings," Department of Engineering Science, University of Oxford, Final year project report, 2004.

[5] J. Kumar and P. Ghosh, "Ultimate Bearing Capacity of Two Interfering Rough Strip Footings," *International Journal of Geomechanics*, vol. 7, no. 1, pp. 53–62, Jan. 2007, https://doi.org/10.1061/(asce)1532-3641(2007)7:1(53).

[6] J. Kumar and P. Ghosh, "Upper bound limit analysis for finding interference effect of two nearby strip footings on sand," *Geotechnical and Geological Engineering*, vol. 25, no. 5, pp. 499–507, May 2007, https://doi.org/10.1007/s10706-007-9124-9.

[7] A. Kumar and S. Saran, "Closely Spaced Footings on Geogrid-Reinforced Sand," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 129, no. 7, pp. 660–664, Jul. 2003, https://doi.org/10.1061/(ASCE)1090-0241(2003)129:7(660).

[8] M. Ghazavi and A. A. Lavasan, "Interference effect of shallow foundations constructed on sand reinforced with geosynthetics," *Geotextiles and Geomembranes*, vol. 26, no. 5, pp. 404–415, Oct. 2008, https://doi.org/10.1016/j.geotexmem.2008.02.003.

[9] A. Mabrouki, D. Benmeddour, R. Frank, and M. Mellas, "Numerical study of the bearing capacity for two interfering strip footings on sands," *Computers and Geotechnics*, vol. 37, no. 4, pp. 431–439, Jun. 2010, https://doi.org/10.1016/j.compgeo.2009.12.007.

[10] A. Benbouza, L. Arabet, and K. Abbeche, "Numerical Study of the Failure Surface in Granular Soil Under Two Closely Spaced Strip Footings," in *Sustainable Civil Infrastructures*, Egypt, Jul. 2017, pp. 165–172, https://doi.org/10.1007/978-3-319-61905-7_14.

[11] R. Boufarh, K. Abbeche, and A. Abdi, "Experimental Investigation of Interference Between Adjacent Footings on Layered Cohesionless Soil," *Soil Mechanics and Foundation Engineering*, vol. 56, no. 2, pp. 128–135, May 2019, https://doi.org/10.1007/s11204-019-09580-z.

[12] A. Gupta and T. G. Sitharam, "Experimental and numerical investigations on interference of closely spaced square footings on sand," *International Journal of Geotechnical Engineering*, vol. 14, no. 2, pp. 142–150, Apr. 2018, https://doi.org/10.1080/19386362.2018.1454386.

[13] A. Badie and M. C. Wang, "Interaction Between Strip Footing and Soft Ground Tunnel," in *13th International Conference on Soil Mechanics and Foundation Engineering*, New Delhi, India, 1994, pp. 571–574.

[14] R. L. Baus and M. C. Wang, "Bearing Capacity of Strip Footing above Void," *Journal of Geotechnical Engineering*, vol. 109, no. 1, pp. 1–14, Jan. 1983, https://doi.org/10.1061/(ASCE)0733-9410(1983)109:1(1).

[15] L. A. Wood and W. J. Carnach, "The Behaviour of Footings Located Above Voids," in *Proceedings of the Eleventh International Conference on Soil Mechanics and Foundation Engineering*, San Francisco, CA, USA, Aug. 1985.

[16] A. Badie and M. C. Wang, "Stability of Spread Footing Above Void in Clay," *Journal of Geotechnical Engineering*, vol. 110, no. 11, pp. 1591–1605, Nov. 1984, https://doi.org/10.1061/(ASCE)0733-9410(1984)110:11(1591).

[17] M. C. Wang and A. Badie, "Effect of Underground Void on Foundation Stability," *Journal of Geotechnical Engineering*, vol. 111, no. 8, pp. 1008–1019, Aug. 1985, https://doi.org/10.1061/(ASCE)0733-9410(1985)111:8(1008).

[18] M. C. Wang and C. W. Hsieh, "Collapse load of strip footing above circular void," *Journal of Geotechnical Engineering*, vol. 113, no. 5, May 1987, https://doi.org/org/10.1061/(ASCE)0733-9410(1987)113:5(511).

[19] G. Azam, C. W. Hsieh, and M. C. Wang, "Performance of Strip Footing on Stratified Soil Deposit with Void," *Journal of Geotechnical Engineering*, vol. 117, no. 5, pp. 753–772, May 1991, https://doi.org/10.1061/(ASCE)0733-9410(1991)117:5(753).

[20] M. Kiyosumi, O. Kusakabe, M. Ohuchi, and F. Le Peng, "Yielding Pressure of Spread Footing above Multiple Voids," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 133, no. 12, pp. 1522–1531, Dec. 2007, https://doi.org/10.1061/(ASCE)1090-0241(2007)133:12(1522).

[21] M. Kiyosumi, O. Kusakabe, and M. Ohuchi, "Model Tests and Analyses of Bearing Capacity of Strip Footing on Stiff Ground with Voids," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 137, no. 4, pp. 363–375, Apr. 2011, https://doi.org/10.1061/(asce)gt.1943-5606.0000440.

[22] J. K. Lee, S. Jeong, and J. Ko, "Undrained stability of surface strip footings above voids," *Computers and Geotechnics*, vol. 62, pp. 128–135, Oct. 2014, https://doi.org/10.1016/j.compgeo.2014.07.009.

[23] A. A. Lavasan, A. Talsaz, M. Ghazavi, and T. Schanz, "Behavior of Shallow Strip Footing on Twin Voids," *Geotechnical and Geological Engineering*, vol. 34, no. 6, pp. 1791–1805, Dec. 2016, https://doi.org/10.1007/s10706-016-9989-6.

[24] H. Zhou, G. Zheng, X. He, X. Xu, T. Zhang, and X. Yang, "Bearing capacity of strip footings on c–φ soils with square voids," *Acta Geotechnica*, vol. 13, no. 3, pp. 747–755, Feb. 2018, https://doi.org/10.1007/s11440-018-0630-0.

[25] Y. Xiao, M. Zhao, and H. Zhao, "Undrained stability of strip footing above voids in two-layered clays by finite element limit analysis," *Computers and Geotechnics*, vol. 97, pp. 124–133, May 2018, https://doi.org/10.1016/j.compgeo.2018.01.005.

[26] B. Mazouz, T. Mansouri, M. Baazouzi, and K. Abbeche, "Assessing the Effect of Underground Void on Strip Footing Sitting on a Reinforced Sand Slope with Numerical Modeling," *Engineering, Technology & Applied Science Research*, vol. 12, no. 4, pp. 9005–9011, Aug. 2022, https://doi.org/10.48084/etasr.5131.

[27] M. A. Soomro, M. A. Keerio, M. A. Soomro, and D. K. Bangwar, "3D Centrifuge Modeling of the Effect of Twin Tunneling to an Existing Pile Group," *Engineering, Technology & Applied Science Research*, vol. 7, no. 5, pp. 2030–2040, Oct. 2017, https://doi.org/10.48084/etasr.1393.

[28] M. A. Soomro, D. K. Bangwar, M. A. Soomro, and M. A. Keerio, "3D Numerical Analysis of the Effects of an Advancing Tunnel on an Existing Loaded Pile Group," *Engineering, Technology & Applied Science Research*, vol. 8, no. 1, pp. 2520–2525, Feb. 2018, https://doi.org/10.48084/etasr.1693.

[29] G. G. Meyerhof, "Some Recent Research on the Bearing Capacity of Foundations," *Canadian Geotechnical Journal*, vol. 1, no. 1, pp. 16–26, Sep. 1963, https://doi.org/10.1139/t63-003.

[30] A. S. Vesic, "Analysis of Ultimate Loads of Shallow Foundations," *Journal of the Soil Mechanics and Foundations Division*, vol. 99, no. 1, pp. 45–73, Jan. 1973, https://doi.org/10.1061/JSFEAQ.0001846.

[31] J. B. Hansen, "A General Formula for Bearing Capacity," *Ingeniøren - International Edition*, 1961.

[32] A. Al-Tabbaa, "Model tests of footings above shallow cavities," *Ground Engineering*, vol. 22, no. 7, pp. 39–42, Oct. 1989.