Buckling and Vibration Estimation of Girder Steel Portal Frames using the Bayesian Updating Methods

Fahmi H. Fahmi
Department of Civil Engineering, College of Engineering, University of Baghdad, Iraq
f.fahmi1901m@coeng.uobaghdad.edu.iq
(corresponding author)

Salah Al-Zaidee
Department of Civil Engineering, College of Engineering, University of Baghdad, Iraq
salah.r.al.zaidee@coeng.uobaghdad.edu.iq

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ABSTRACT

In practice, structural systems are complicated. When dealing with such systems, the use of analytical simulations is or may be impossible to apply, since the problem includes many variables with partial differential formulations. The stability and dynamic response of structures is an important aspect that must be paid particular attention to in order to ensure safety against collapse. A physical phenomenon of a reasonably straight, slender member bending laterally (usually abruptly) from its longitudinal position due to compression is referred to as buckling. Two kinds of buckling can be distinguished: (1) bifurcation-type buckling and (2) deflection-amplification-type buckling. In vibration, frequencies and mode shapes must be known in order to use some methods of dynamic response analysis. To calculate frequencies and modes, an eigenvalue problem is solved in algebraic form. In structural dynamic problems, only the lowermost eigenpairs are of important interest. The highest eigenpairs are not needed and are not accurate due to discretization errors.

Keywords-interior supporting frame; Bayesian method; sample data; prior data; posterior data; first order; MCs

I. INTRODUCTION

The majority of engineering designs are based on deterministic variables and often do not consider the variations in the material properties and the geometry of the structure [1]. Uncertainty is vital to the analysis and design of an engineering system and the effects of uncertainties must be quantified and propagated. Increasing amounts of data in engineering systems are collected and stored. This information can and should be used to decrease the uncertainty in engineering models and optimize the management of these systems. A coherent and effective way for merging new with existing information is provided by the Bayesian method, in which prior probabilistic information is updated with new data and observations. The Bayesian framework enables the combination of uncertain and incomplete information with models from different sources and provides probabilistic information on the accuracy of the updated model [2]. The current study emphasizes the analysis of the buckling and vibration of steel portal frames, where new data have been acquired for the independent variables of the system. Subsequently, the Bayesian method presents an efficient way to update the prior knowledge regarding the stability and natural frequency of the frame. The first-order approximation method and Monte Carlo (MC) simulation have been adopted to investigate the statistical properties of the buckling and vibration.

II. THE BAYESIAN METHOD

In engineering, one often needs to use whatever information is available in formulating a sound basis for making decisions. This may include observed data (field or experimental), information derived from theoretical models, and judgments based on experience. Moreover, the available information may need to be updated as new information or data are acquired [3]. The proper tool for combining and updating the available information is embodied in the Bayesian approach. Parameter estimation in the Bayesian approach is based on the updating formula:

$$f(\theta) = cL(\theta)p(\theta) \qquad (1)$$

where $p(\theta)$ is the prior Probability Density Function (PDF) representing the initial state of knowledge about the unknown parameters θ, $L(\theta)$ is the likelihood function representing the knowledge gained from a set of observations, and the constant c is a normalizing factor. The posterior PDF $f(\theta)$ represents the updated state of knowledge regarding the parameters θ.

A. Prior Distribution

The prior distribution represents the distribution of possible parameter values from which the parameter has been drawn. The selection of the prior distribution is an important matter in Bayesian modeling. Since the choice of the prior distribution has a major effect on the resulting inference, this choice must be conducted with the utmost care [4].

B. Posterior Distribution

The current information about the parameter is contained in the posterior probability distribution. It merges the information of the prior distribution and the likelihood. This results in a strong representation of the information than separate sources of information [5]. The prior and posterior densities are presented in Table I and the mean and variance of the posterior parameters in Table II.

TABLE I. RANDOM VARIABLES AND THEIR PRIOR AND POSTERIOR DENSITIY DISTRIBUTIONS [3]

Basic random variables	Parameter	Prior and posterior distribution
Normal $f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$	μ	Normal $\mu''_\mu=\frac{\mu'_\mu(\sigma^2/n)+\bar{x}{\sigma'}^2_\mu}{\sigma^2/n+(\sigma'_\mu)^2}$
Lognormal $f_X(x)=\frac{1}{\sqrt{2\pi}\xi x}exp\left[-\frac{1}{2}\left(\frac{\ln x-\lambda}{\xi}\right)^2\right]$	λ	Normal $f_\Lambda(\lambda)=\frac{1}{\sqrt{2\pi}\sigma}exp\left[-\frac{1}{2}\left(\frac{\lambda-\mu}{\sigma}\right)^2\right]$

TABLE II. MEAN AND VARIANCE OF THE PARAMETERS AND THEIR POSTERIOR STATISTICS [3]

Mean and variance of parameters	Posterior statistics
$E(\mu)=\mu_\mu$	$\mu''_\mu=\frac{\mu'_\mu(\sigma^2/n)+\bar{x}{\sigma'}^2_\mu}{\sigma^2/n+(\sigma'_\mu)^2}$
$Var(\mu)=\sigma^2_\mu$	$\sigma''_\mu=\sqrt{\frac{(\sigma'_\mu)^2(\sigma^2/n)}{(\sigma'_\mu)^2+\sigma^2/n}}$
$E(\lambda)=\mu$	$\mu''=\frac{u'(\xi^2/n)+\sigma^2\ln\bar{x}}{\xi^2/n+\sigma^2}$
$Var(\lambda)=\sigma^2$	$\sigma''=\sqrt{\frac{\sigma^2(\xi^2/n)}{\sigma^2+\xi^2/n}}$

III. STATISTICAL CHARACTERISTICS OF THE INDEPENDENT VARIABLES

This study considers the applied loads, cross-section dimensions, modulus of elasticity, girder span, plastic section modulus, and yield strength as random variables. The sample data of the independent variables were summarized and are presented in Table III. The prior statistical characteristics of these variables have been collected from the literature and can be seen in Table IV.

The posterior statistical data of the input variables have been calculated from the updating process of the prior data by the Bayesian method. The modulus of elasticity of steel E_s has been considered as an example of this process. Updating mean, standard deviation, and coefficient of variance have been estimated based on the relations in (1)-(3). The posterior statistical characteristics of the independent variables are summarized in Table V.

$$\mu''=\frac{\left[\frac{\bar{x}}{\left(\frac{\sigma}{\sqrt{n}}\right)^2}\right]+\left[\frac{\mu'}{(\sigma')^2}\right]}{\left[\frac{1}{\left(\frac{\sigma}{\sqrt{n}}\right)^2}\right]+\left[\frac{1}{(\sigma')^2}\right]}=\frac{\bar{x}(\sigma')^2+\mu'\left(\frac{\sigma^2}{n}\right)}{(\sigma')^2+\left(\frac{\sigma^2}{n}\right)}=\frac{(205)(16)+(200)\left(\frac{151.3}{5}\right)}{(16)+\left(\frac{151.29}{5}\right)}=202\text{GPa} \quad (1)$$

$$\sigma''=\sqrt{\frac{(\sigma')^2\left(\frac{\sigma^2}{n}\right)}{(\sigma')^2+\left(\frac{\sigma^2}{n}\right)}}=\sqrt{\frac{(16)\left(\frac{151.3}{5}\right)}{(16)+\left(\frac{151.3}{5}\right)}}=3.24\text{GPa} \quad (2)$$

$$cov''=\frac{\sigma''}{\mu''}=\frac{3.24}{202}=0.016 \quad (3)$$

TABLE III. SAMPLE STATISTICAL DATA OF THE VARIABLES

Variable	Mean / Nominal	COV	Distribution type	Reference
Dead load P_D	1.03	0.08	Normal	[6]
Live load P_L	1.00	0.1	Gumbel	[7]
Moment of inertia I_x	0.96	0.03	Normal	Local survey
Modulus of elasticity E_s	1.025	0.05	Normal	[8]
Modulus of elasticity E_c	0.98	0.07	Lognormal	[9]
Member span L	1.00	0.07	Lognormal	[10]
Plastic section modulus Z_x	1.04	0.05	Lognormal	[11]
Yield strength of steel F_y	1.10	0.07	Lognormal	[11]

TABLE IV. PRIOR STATISTICAL DATA OF THE VARIABLES

Variable	Mean / Nominal	COV	Distribution type	Reference
Dead load P_D	1.03	0.08	Normal	[6]
Live load P_L	1.00	0.1	Gumbel	[7]
Moment of inertia I_x	0.96	0.05	Normal	[12]
Modulus of elasticity E_s	1.00	0.02	Normal	[11]
Modulus of elasticity E_c	1.00	0.03	Lognormal	[13]
Member span L	1.00	0.004	Lognormal	[14]
Plastic section modulus Z_x	1.00	0.04	Lognormal	[15]
Yield strength of steel F_y	1.10	0.06	Lognormal	[12]

TABLE V. POSTERIOR STATISTICAL DATA OF THE VARIABLES

Variable	Mean / Nominal	COV	Distribution type
Dead load P_D	1.03	0.08	Normal
Live load P_L	1.00	0.1	Gumbel
Moment of inertia I_x	0.96	0.005	Normal
Modulus of elasticity E_s	1.01	0.016	Normal
Modulus of elasticity E_c	1.00	0.02	Lognormal
Member span L	1.00	0.003	Lognormal
Plastic section modulus Z_x	1.00	0.03	Lognormal
Yield strength of steel F_y	1.10	0.04	Lognormal

IV. PARAMETRIC STUDY

A steel single-story warehouse, as shown in Figure 1, with 45m total length and 15m width, consisting of 5 bays, has been considered in this parametric study. The superimposed dead load has been determined based on the assumption of a flooring system of a concrete slab with a thickness of 100mm and a metal deck of 7.5mm, supported on IPE 300 floor beams. The first inner frame shown in Figure 2 has been considered the most critical, as it includes the exterior face of the first interior support where the shear force is about 15% greater than the average value. The frame consists of IPE 600 steel girder and columns. The applied live loads on the girder have been determined based on the ASCE 7 specifications. The self-weight of the girder and columns have been determined depending on the cross-section dimensions and material density.

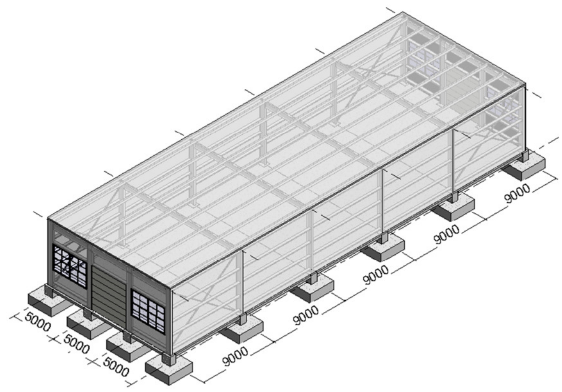


Fig. 1. Steel single-story warehouse building.

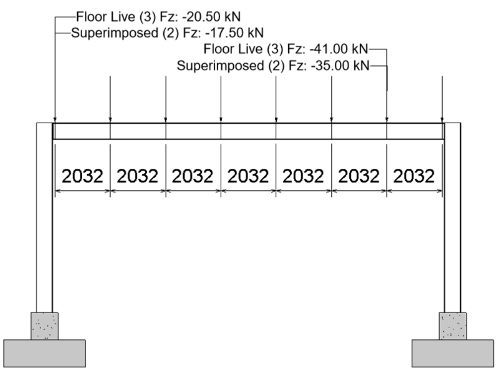


Fig. 2. Summary of the dead and live load on the interior supporting frame.

V. DERIVED AND SIMULATED STATISTICAL PROPERTIES FOR THE BUCKLING AND VIBRATION OF THE GIRDER

Two stages have been considered in the first-order approximation method and the MC simulation method. The randomness of the dependent variables, in the first stage, has been deduced based on the prior knowledge of Table IV, while in the second stage, it has been deduced based on the posterior knowledge of Table V.

A. *First-Order Approximation Method*

It is possible to expand the original model into an infinite Taylor Series (TS) around the mean values:

$$g(X) = g(\mu_X) + (X-\mu_X)\frac{dg}{dX} + \frac{1}{2}(X-\mu_X)^2\frac{d^2g}{dX^2} + \cdots + \frac{1}{n!}(X-\mu_X)^n\frac{d^ng}{dX^n} \quad (4)$$

where the function and derivatives are evaluated at μ_X. It is common to include only linear terms, assuming that random input variables are independent. A function $g(X)$ of N independent random variables can be approximated by linear terms of the TS, which are as follows [16]:

$$E(Y) \approx g(\mu_x) \quad (5)$$

and

$$Var(Y) \simeq Var(X-\mu_X)\left(\frac{dg}{dX}\right)^2 = Var(x)\left(\frac{dg}{dX}\right)^2 \quad (6)$$

1) *Buckling Analysis*

The buckling P_{cr} is a function of the span L, the modulus of elasticity E, and the moment of inertia I. By refereeing to (4) and (5) the mean and variance of the buckling can be estimated by:

$$E(P_{cr}) = \frac{1}{K^2}\frac{\pi^2 \times \bar{E} \times \bar{I}_{Major}}{(\bar{L})^2} \quad (7)$$

$$Var(P_{cr}) = Var(I)\left(\frac{\partial P_{cr}}{\partial I}\right)^2 + Var(L)\left(\frac{\partial P_{cr}}{\partial L}\right)^2 + Var(E)\left(\frac{\partial P_{cr}}{\partial E}\right)^2 \quad (8)$$

Based on the statistical characteristics of the independent variables illustrated in Tables IV and V, the prior and posterior statistical characteristics for P_{cr} have been estimated and are presented in Tables VI and VII.

TABLE VI. PRIOR STATISTICAL CHARACTERISTICS OF THE BUCKLING OF THE GIRDER

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
P_{cr}	44198	42072	2172	0.051

TABLE VII. POSTERIOR STATISTICAL CHARACTERISTICS OF THE BUCKLING OF THE GIRDER

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
P_{cr}	44198	42493	750	0.017

2) *Vibration Analysis*

The vibration f_n is a function of the load *P*, span *L*, modulus of elasticity *E*, and moment of inertia *I*. The mean and variance of the vibration are calculated by:

$$E(f_n) = 0.18\frac{1}{\sqrt{f_k}}\sqrt{\frac{g}{\Delta_{girder}}} \Longrightarrow 1.247\sqrt{\frac{g}{f_k}}\sqrt{\frac{\bar{E}\bar{I}}{\bar{P}\bar{L}^3}} \quad (9)$$

$$Var(f_n) = Var(I)\left(\frac{\partial f_n}{\partial I}\right)^2 + Var(L)\left(\frac{\partial f_n}{\partial L}\right)^2 + Var(E)\left(\frac{\partial f_n}{\partial E}\right)^2 + Var(P)\left(\frac{\partial f_n}{\partial P}\right)^2 \quad (10)$$

Based on Tables IV and V, the prior and posterior statistical characteristics for vibration are presented in Tables VIII and IX.

TABLE VIII. PRIOR STATISTICAL CHARACTERISTICS

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
f_n	3.32	7.23	0.214	0.029

TABLE IX. POSTERIOR STATISTICAL CHARACTERISTICS

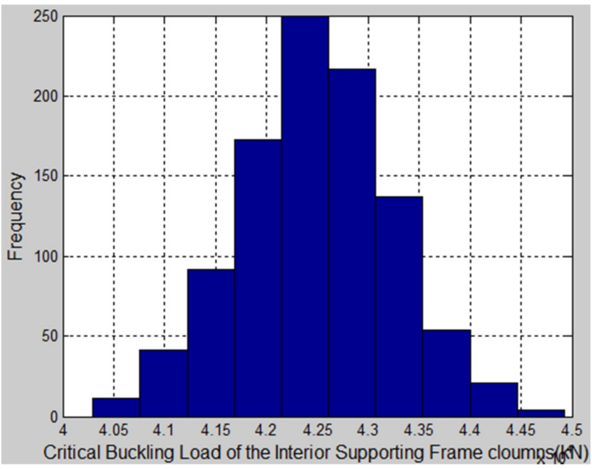
Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
f_n	3.32	7.267	0.103	0.014

B. Monte Carlo Simulation

The MC method is considered one of the most powerful and accurate simulation tools and can be applied to many practical problems allowing the direct consideration of any type of probability distribution for the random variables [17]. The MC method has been adopted to achieve the simulation processes of the buckling, and vibration for the interior supporting frame. MATLAB code has been used to generate pseudo-random sampling with a size of 1000 for each input variable.

1) Buckling Analysis

The prior statistical characteristics of the buckling are presented in Figure 3 and are summarized in Table X.



P_{cr}

Fig. 3. Histogram of the prior buckling of a typical interior girder.

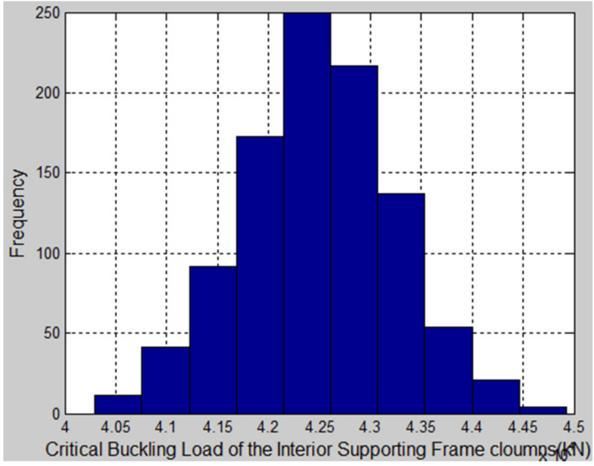
TABLE X. PRIOR STATISTICAL CHARACTERISTICS OF THE BUCKLING OF THE GIRDER

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
P_{cr}	41999	2279	0.054	Normal

The posterior statistical characteristics of the buckling were estimated and are presented in Figure 4 and Table XI.

TABLE XI. POSTERIOR STATISTICAL CHARACTERISTICS OF THE BUCKLING OF THE GIRDER

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
P_{cr}	42481	756	0.017	Normal

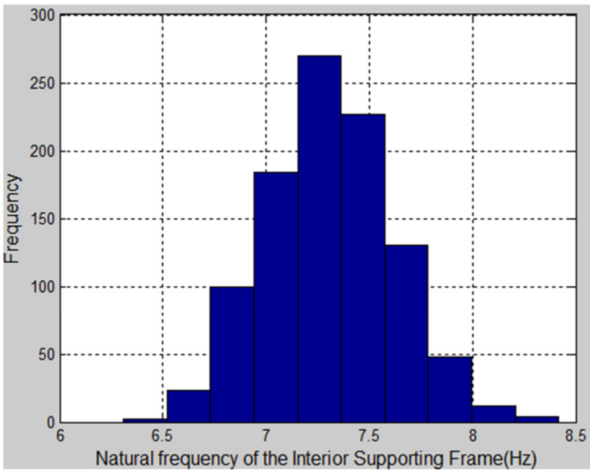


P_{cr}

Fig. 4. Histogram of the posterior buckling of a typical interior girder.

2) Vibration Analysis

The prior statistical characteristics of the natural frequency are presented in Figure 5 and Table XII.



f_n

Fig. 5. Histogram of the prior natural frequency of a typical interior girder.

TABLE XII. PRIOR STATISTICAL CHARACTERISTICS OF THE NATURAL FREQUENCY OF THE GIRDER

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
f_n	7.30	0.311	0.042	Normal

The estimated posterior statistical characteristics of the natural frequency are presented in Figure 6 and Table XIII.

TABLE XIII. POSTERIOR STATISTICAL CHARACTERISTICS OF THE NATURAL FREQUENCY OF THE GIRDER

Random variable	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
f_n	7.36	0.26	0.035	Normal

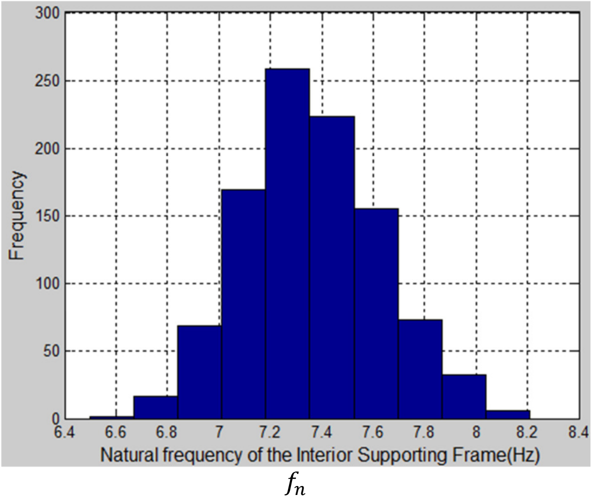


Fig. 6. Histogram of the posterior natural frequency of a typical interior girder.

VI. CONCLUSION

In this paper, buckling and vibration analysis was performed for the girder of the first inner frame for the prior and posterior statistical characteristics. Through the application of the Bayesian method, and from the results of the prior and posterior analysis, it was shown that the greater the knowledge gained about the independent parameters, the less randomness is in the dependent parameters, and thus the analysis and design of the system is enhanced. In the first order analysis, the covariance of the critical buckling load decreased from 0.051 to 0.017, and for the natural frequency from 0.029 to 0.014, while in Monte Carlo simulation, the covariance results were from 0.054 to 0.017 for the critical buckling load, and from 0.042 to 0.035 for the natural frequency. The decrement of the covariance of the dependent parameters is due to the decrement of the values of the covariance of the independent parameters because of the increased knowledge of the independent parameters.

REFERENCES

[1] N. L. Tran and T. H. Nguyen, "Reliability Assessment of Steel Plane Frame's Buckling Strength Considering Semi-rigid Connections," *Engineering, Technology & Applied Science Research*, vol. 10, no. 1, pp. 5099–5103, Feb. 2020, https://doi.org/10.48084/etasr.3231.

[2] D. Straub and I. Papaioannou, "Bayesian Updating with Structural Reliability Methods," *Journal of Engineering Mechanics*, vol. 141, no. 3, Mar. 2015, Art. No. 04014134, https://doi.org/10.1061/(ASCE)EM.1943-7889.0000839.

[3] A. H.-S. Ang and W. H. Tang, *Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering, 2e Instructor Site: Emphasis on Applications to Civil and Environmental Engineering*, 2nd ed. New York, NY, USA: Wiley, 2006.

[4] A. Belay, E. OBrien, and D. Kroese, "Truck fleet model for design and assessment of flexible pavements," *Journal of Sound and Vibration*, vol. 311, no. 3, pp. 1161–1174, Apr. 2008, https://doi.org/10.1016/j.jsv.2007.10.019.

[5] A. E. Mansour, H. Y. Jan, C. I. Zigelman, Y. N. Chen, and S. J. harding, "Implementation of Reliability Methods to Marine Structures," *Transactions - Society of Naval Architects and Marine Engineers*, vol. 92, pp. 353–382, 1984.

[6] M. S. Darmawan, A. N. Refani, M. Irmawan, R. Bayuaji, and R. B. Anugraha, "Time Dependent Reliability Analysis of Steel I Bridge Girder Designed Based on SNI T-02-2005 and SNI T-3-2005 Subjected to Corrosion," *Procedia Engineering*, vol. 54, pp. 270–285, Jan. 2013, https://doi.org/10.1016/j.proeng.2013.03.025.

[7] S. G. Buonopane and B. W. Schafer, "Reliability of Steel Frames Designed with Advanced Analysis," *Journal of Structural Engineering*, vol. 132, no. 2, pp. 267–276, Feb. 2006, https://doi.org/10.1061/(ASCE)0733-9445(2006)132:2(267).

[8] A. Ghabussi, J. Asgari Marnani, and M. S. Rohanimanesh, "Improving seismic performance of portal frame structures with steel curved dampers," *Structures*, vol. 24, pp. 27–40, Apr. 2020, https://doi.org/10.1016/j.istruc.2019.12.025.

[9] P. Geyskens, A. D. Kiureghian, and P. Monteiro, "Bayesian Prediction of Elastic Modulus of Concrete," *Journal of Structural Engineering*, vol. 124, no. 1, pp. 89–95, Jan. 1998, https://doi.org/10.1061/(ASCE)0733-9445(1998)124:1(89).

[10] H. Q. Jebur and S. R. Al-Zaidee, "Non-deterministic Approach for Reliability Evaluation of Steel Portal Frame," *Civil Engineering Journal*, vol. 5, no. 8, pp. 1684–1697, Aug. 2019, https://doi.org/10.28991/cej-2019-03091363.

[11] K. Atua, I. Assakkaf, and B. M. Ayyub, "Statistical Characteristics of Strength and Load Random Variables of Ship Structures," in *Proceedings of the Seventh Specialty Conference on Probabilistic Mechanics & Structural Reliability*, Worcester, MA, USA, 1996, pp. 106–109, Accessed: Nov. 19, 2022.

[12] H. Zhang, H. Liu, B. R. Ellingwood, and K. J. R. Rasmussen, "System Reliabilities of Planar Gravity Steel Frames Designed by the Inelastic Method in AISC 360-10," *Journal of Structural Engineering*, vol. 144, no. 3, Mar. 2018, Art. no. 04018011, https://doi.org/10.1061/(ASCE)ST.1943-541X.0001991.

[13] ACI Committee 318, *ACI 318-19 Building Code Requirements for Structural Concrete*. Farmington Hills, MI, USA: American Concrete Institute, 2019.

[14] *IS 7215: Tolerances for fabrication of steel structures*. Bureau of Indian Standards, 1974.

[15] P. E. Hess, D. Bruchman, I. A. Assakkaf, and B. M. Ayyub, "Uncertainties in Material and Geometric Strength and Load Variables," *Naval Engineers Journal*, vol. 114, no. 2, pp. 139–166, 2002, https://doi.org/10.1111/j.1559-3584.2002.tb00128.x.

[16] D. Mohammed and S. R. Al-Zaidee, "Deflection Reliability Analysis for Composite Steel Bridges," *Engineering, Technology & Applied Science Research*, vol. 12, no. 5, pp. 9155–9159, Oct. 2022, https://doi.org/10.48084/etasr.5146.

[17] M. S. M. Noori and R. M. Abbas, "Reliability Analysis of an Uncertain Single Degree of Freedom System Under Random Excitation," *Engineering, Technology & Applied Science Research*, vol. 12, no. 5, pp. 9252–9257, Oct. 2022, https://doi.org/10.48084/etasr.5193.