A Study of One-dimensional Weak Shock Propagation Under the Action of Axial and Azimuthal Magnetic Field: An Analytical Approach

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Abstract-The present paper presents an analytical study of the one-dimensional weak shock wave problem in a perfect gas under the action of a generalized magnetic field subjected to weak shock jump conditions (R-H conditions). The magnetic field is considered axial and azimuthal in cylindrically symmetric configuration. By considering a straightforward analytical approach, an explicit solution exhibiting time-space dependency for gas-dynamical flow parameters and total energy (generated during the propagation of the weak shock from the center of the explosion) has been obtained under the significant influence of generalized magnetic fields (axial and azimuthal) and the results are analyzed graphically. From the outcome, it is worth noticing that for an increasing value of Mach number under the generalized magnetic field, the decay process of physical parameters (density, pressure, and magnetic pressure) is a bit slower, whereas the velocity profile and total energy increase rapidly with respect to time. Moreover, for increasing values of Shock-Cowling number the total energy grows rapidly with respect to time.

Keywords-weak shock waves; analytical solution; Rankine-Hugoniot conditions; magnetogasdynamics

I. INTRODUCTION

In general, the complex physical phenomena occurring in nature are non-linear and are described by mathematical models in the form of Non-linear Partial Differential Equations (NPDEs). During the last few decades such nonlinear mathematical models are getting a vital role in the fields of natural, engineering, and medical sciences such as plasma physics, solid-state physics, fluid mechanics, optical fibers, geophysics, biomechanics, etc. and many efforts have been made for confronting the exact and numerical solutions of such physical systems [1-6]. Due to the high complexity, finding the exact (closed form) solution of such realistic models is thus a challenging and rigorous task because it comprises many physical and natural intricacies and only in certain cases can we explicitly unravel the solutions.

Mathematically, the nonlinear wave (shock wave) propagation phenomenon is formulated as a quasilinear hyperbolic system of partial differential equations. The shock wave occurs in gaseous media by a dynamical mechanism in which an immense amount of energy is abruptly released over a small interval of time during the propagation of wave discontinuity, e.g. prolonged rapid electrical discharges in the air such as thunder-strokes, explosions of long thin wires, etc. It is well known that shock processes usually occur due to the high temperature in which the gas ionizes, so the effect of the magnetic field also becomes significant. The understanding of the influence of the magnetic field on wave propagation phenomenon and the resulting flow field is of great importance as it involves many applications in the field of space science research, atmospheric sciences, nuclear sciences, etc. The study of shock wave propagation under the action of a magnetic field for a perfect gas are important for the interpretation of the phenomena encountered in astrophysics from the theoretical and experimental points of view, as the ideal magneto-gasdynamic flow involves a plasma in which the diffusion effects are negligible.

Many researchers [7-11] have worked to better understand the magnetic field effects on the dynamics of shock waves. In keeping with [7-11], authors in [12] discussed the self-similar solution of the blast wave problem with identical geometry under constant axial current. Authors in [13] studied one dimensional steady flow of a perfect gas and reported the first

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complete explicit shock solution in the aligned field magnetogasdynamics. In [14], the authors considered spherical shock propagation in a magnetized medium and found that the model has some limitations on real flare produced shocks, taking into account isothermal flow conditions in planar and identical geometry. Authors in [15] reported the existence of self-similar solutions for blast waves and observed that the magnetic field effects play a significant role in the theory of blast waves. Authors in [16] used the asymptotic approach and analyzed the decay behavior of sawtooth profile for planar and cylindrical gas dynamical flows headed by weak wave front under the action of axial magnetic field. Authors in [17] studied the evolution and formation of the 1-dimensional magneto-hydrodynamic shock wave in the approximation of the low plasma-to-magnetic pressure ratio and obtained an analytical expression for shock formation. A detailed theoretical, experimental, and computational review on magneto aerodynamics was presented in [18] and the features of shock structure in the presence of magnetic field were discussed.

Within the above mentioned framework, several investigations have been recently performed for finding the analytical solution of ideal MHD equations. Authors in [19] obtained a particular solution of strong discontinuity waves in ideal magneto-gasdynamic flow via lie group transformation analysis and studied the effect of the applied magnetic field on the evolutionary behavior of reflected and transmitted waves which arises when a magnet-acoustic wave collides with a shock. The methodology proposed in [20, 21] gave the precise solution of the shock wave problem involving strong discontinuities in magneto-gas-dynamic regime. The convergence of the shock wave in the cylindrical symmetry was investigated in [22] and rendered analytical asymptotic results on the shock trajectory for small radii. By using the method of generalized wavefront expansion, authors in [23] derived nonlinear coupled evolution equations and assessed how the magnetic field, either axial or azimuthal, influences the formation of weak shock waves in an ideal gas. Authors in [24] presented the numerical description of the flow field in magnetogasdynamics and analyzed that the presence of magnetic field suppresses the instabilities in the point explosion problem. Authors in [25] reported an approximate analytical solution for propagation of cylindrical shock waves in isothermal flow conditions with azimuthal magnetic field. Authors in [26] initiated the analysis of weak shock propagation in a simplified van der Waals gas influenced by thermal radiation under optically thin limit. Authors in [27] studied the strong explosion problem in perfect gasdynamics with magnetic field effects using power series solution and recovered the result described by [28] in the absence of magnetic field. Recently, authors in [29] scrutinized the point explosion problem in perfect magneto-gasdynamic flow on stellar surface and reported an explicit exact solution of fluid characteristics (density, velocity, and pressure) which expounds time-position dependence. Authors in [30] highlighted the impact of dust-laden particles on the evolution of magnetic shock waves and analyzed the behavior of half N-wave. Authors in [31, 32] solved the Riemann problem for the hyperbolic system of conservation laws and examined the study of discontinuous solutions in non-ideal material media.

In this article, our goal is to construct a closed-form solution (which exhibits a space-time dependence) to the weak shock wave problem in an ideal gas for cylindrically symmetric flow under the influence of axial and azimuthal magnetic fields (modelled in the form of Shock-Cowling number, $C_0 = 2h_0/\rho_0\mathcal{G}^2$) and to investigate how the magnetic field and the Mach number affect the thermodynamic flow characteristics. It has been assumed that the mass density distribution obeys a power law of the radial distance from the centre of explosion. Furthermore, we derived an analytical expression for the total energy of the weak shock wave, influenced by the magnetic field. The way the behavior of physical parameters such as density, velocity, pressure, magnetic pressure and energy is affected by the Mach number and the presence of magnetic field, is also assessed.

II. MATHEMATICAL MODEL OF THE PROBLEM

To outline the approach, we consider a basic set of governing conservation equations, consisting one-dimensional compressible cylindrically symmetric unsteady adiabatic motion of a perfect gas under the influence of magnetic field in which viscous stress is negligible. Mathematically, this system of Euler's equations corresponding to mass balance, momentum balance, energy and magnetic pressure balance in non-conservative form is given by [33-35]:

$$\rho_t + u\rho_x + \rho u_x + (\rho u/x) = 0 \quad (1)$$

$$\rho(u_t + uu_x) + p_x + h_x + (2jh/x) = 0 \quad (2)$$

$$p_t + up_x + \rho c^2\{u_x + (u/x)\} = 0 \quad (3)$$

$$h_t + uh_x + 2h[u_x + \{(1-j)\,u/x\}] = 0 \quad (4)$$

In the above equations, $x \in \mathbb{R}$ and $t > 0$ stand for the spatial and time variables, $\rho(x,t)$ refers to the mass density, $p(x,t)$ is the pressure, $u(x,t)$ is the flow velocity, h the magnetic pressure defined as $h = \mu H^2/2$ with μ as the magnetic permeability, and H is the transverse magnetic field. The entity c accounts for the equilibrium speed of sound and is defined as $c^2 = \Gamma p/\rho$ where $\Gamma = c_p/c_v$ $(1 < \Gamma < 2)$ is the adiabatic exponent and thermodynamic constants c_p and c_v denote the specific heat of the gas at constant pressure and at constant volume respectively. Here and hereafter nonnumeric subscripts with respect to physical characteristics indicate partial differentiation unless stated otherwise. The constant $j = 0$ exhibits the axial magnetic field and $j = 1$ the azimuthal magnetic field. For motion in ideal gasdynamic medium, the governing dynamical equations (1)-(4) are supplemented with the following constitutive relation (or equation of state):

$$p = \rho R\mathcal{T} \quad (5)$$

where R is the specific gas constant and $\mathcal{T}$ is the temperature.

Let $\mathcal{R} = \mathcal{R}(t)$ be the shock-location at time t moving with the shock speed $\mathcal{G} = \frac{d\mathcal{R}}{dt}$ into the medium immediately ahead of the shock given by $u_0 = 0$, $p_0 =$ constant, $\rho_0 = \rho_0(x)$ and $h_0 = h_0(x)$, where the thermodynamic profiles with subscript 0 express the values in the pre-shock region. It has been also assumed here that weak shock propagation obeys a power law $\rho_0 = \rho_c \mathcal{R}^{\mathcal{J}}$, in which the density of the pre-shock (undisturbed) region ρ_0 varies with respect to the shock radius. Here, ρ_c is a dimensional constant and $\mathcal{J}$ a constant. The value of constant $\mathcal{J}$ is to be determined in the ongoing analysis.

It is well known that the energy produced during the propagation of a weak shock wave in any gas medium is equal to the sum of the kinetic energy and internal energy of the gas and is a constant. The balance equation for the total energy E_T in an ideal magneto-gas-dynamic flow is given by:

$$E_T = \int_0^{\mathcal{R}} \left[\frac{1}{2}u^2 + \frac{1}{(\Gamma-1)}\left(\frac{p}{\rho} - \frac{p_0}{\rho_0}\right) + \left(\frac{h}{\rho} - \frac{h_0}{\rho_0}\right)\right] \rho x \, dx \quad (6)$$

In view of the relation $\int_0^{\mathcal{R}} \frac{\rho}{\rho_0} x \, dx = \frac{\mathcal{R}^2}{2}$ obtained from Lagrangian equation of continuity, (6) yields the following expression for total energy:

$$E_T = \int_0^{\mathcal{R}} \left[\frac{\rho u^2}{2} + \frac{p}{(\Gamma-1)} + h\right] x \, dx - \frac{p_0 \mathcal{R}^2}{2(\Gamma-1)} - \frac{h_0 \mathcal{R}^2}{2} \quad (7)$$

III. FORMULATION OF SHOCK-JUMP CONDITIONS

At the outset, for the formulation of shock-jump conditions, we recast the fundamental set of balance equations (1) – (4) in conservation form which yields:

$$\rho_t + (\rho u)_x = -(\rho u/x) \quad (8)$$

$$(\rho u)_t + (\rho u^2 + p + h)_x = -(\rho u^2/x) - (2jh/x) \quad (9)$$

$$\left(\frac{\rho u^2}{2} + \frac{p + (\Gamma-1)h}{(\Gamma-1)}\right)_t + \left(\frac{\Gamma p u}{(\Gamma-1)} + \frac{\rho u^3 + 4uh}{2}\right)_x = \frac{\Gamma p u}{(1-\Gamma)x} - \frac{\rho u^3 + 4hu}{2x} \quad (10)$$

$$\left(h^{\frac{1}{2}}\right)_t + \left(u h^{\frac{1}{2}}\right)_x = \left\{h^{\frac{1}{2}}(j-1)u/x\right\} \quad (11)$$

The Rankine-Hugoniot conditions across the shock front that connects the flow ahead and behind the shock waves can be derived from (8)-(11) and are given by:

$$\rho u - \rho_0 u_0 = \mathcal{G}(\rho - \rho_0) \quad (12)$$

$$(\rho u^2 + p + h) - (\rho_0 {u_0}^2 + p_0 + h_0) = \mathcal{G}(\rho u - \rho_0 u_0) \quad (13)$$

$$\left(\frac{\Gamma p u}{\Gamma-1} + \frac{\rho u^3 + 4uh}{2}\right) - \left(\frac{\Gamma p_0 u_0}{\Gamma-1} + \frac{\rho_0 {u_0}^3 + 4u_0 h_0}{2}\right) = \mathcal{G}\left[\left(\frac{\rho u^2}{2} + \frac{p + h(\Gamma-1)}{(\Gamma-1)}\right) - \left(\frac{\rho_0 {u_0}^2}{2} + \frac{p_0 + h_0(\Gamma-1)}{(\Gamma-1)}\right)\right] \quad (14)$$

$$\left(u h^{\frac{1}{2}}\right) - \left(u_0 {h_0}^{\frac{1}{2}}\right) = \mathcal{G}\left(h^{\frac{1}{2}} - {h_0}^{\frac{1}{2}}\right) \quad (15)$$

Substituting the values of flow profiles from (12), (13), and (15) in (14) yields the following cubic equation in terms of density profile $\rho(x,t)$:

$$(\Gamma-2)C_0\mathcal{G}^2\rho^3 + \{(2p_0 + \rho_0 C_0 \mathcal{G}^2) + p_0(\Gamma-1)M^2\} \Gamma\rho_0\rho - \Gamma(\Gamma+1)p_0{\rho_0}^2 M^2 = 0 \quad (16)$$

where $M = \mathcal{G}/c_0$ represents the Mach number with $c_0 = (\Gamma p_0/\rho_0)^{1/2}$ and $C_0 = 2h_0/\rho_0\mathcal{G}^2$ denotes the Shock-Cowling number.

On solving the cubic equation (16), one obtains the values of the thermo-dynamical characteristics u, p, and h in terms of the density parameter ρ. Thus, (16) along with (12), (13), and (15) allow us to obtain the Rankine-Hugoniot jump conditions as follows:

$$\rho = \frac{\Gamma+1}{\Gamma-1}\left\{1 + \frac{2}{(\Gamma-1)M^2}\right\}^{-1}\rho_0 \quad (17)$$

$$u = \frac{2}{\Gamma+1}\left(1 - \frac{1}{M^2}\right)\mathcal{G} \quad (18)$$

$$p = \left[\frac{2\left\{1 - \frac{\Gamma-1}{2\Gamma M^2}\right\}}{\Gamma+1} + \frac{C_0\left[\left\{\Gamma-1+\frac{2}{M^2}\right\}^2 - (\Gamma+1)^2\right]}{2\left\{\Gamma-1+\frac{2}{M^2}\right\}^2}\right]\rho_0\mathcal{G}^2 \quad (19)$$

$$h = \frac{C_0(\Gamma+1)^2}{2\left\{\Gamma-1+\frac{2}{M^2}\right\}^2}\rho_0\mathcal{G}^2 \quad (20)$$

IV. DERIVATION OF THE ANALYTICAL SOLUTION TO THE WEAK SHOCK WAVE PROBLEM

In this section we shall derive an exact solution of compressible Eulerian equations (1)-(4) governing the one-dimensional unsteady propagation of weak shock waves in generalized magnetogasdynamics by using an analytical approach proposed in [6]. In order to find an exact solution, subject to the Rankine-Hugoniot's ratios (17)-(20), we establish an expression immediately behind the shock front for the physical characteristics of pressure and magnetic pressure of the flow, in terms of the other thermo-dynamical variables, density and flow velocity, given as:

$$p = \frac{4(\Gamma-1)^2\left\{1-\frac{\Gamma-1}{2\Gamma M^2}\right\} + C_0(\Gamma+1)\left[(\Gamma-1)^2 - (\Gamma+1)^2\left\{1+\frac{2}{(\Gamma-1)}\frac{1}{M^2}\right\}^{-2}\right]}{8(\Gamma-1)\left(1-\frac{1}{M^2}\right)^2\left\{1+\frac{2}{(\Gamma-1)}\frac{1}{M^2}\right\}^{-1}}\rho u^2 \quad (21)$$

$$h = \frac{C_0(\Gamma+1)^3\left\{1+\frac{2}{(\Gamma-1)}\frac{1}{M^2}\right\}^{-1}}{8(\Gamma-1)\left(1-\frac{1}{M^2}\right)^2}\rho u^2 \quad (22)$$

An inspection of (21) and (22) leads the Eulerian nonlinear system (2)-(4) to the following form:

$$u_t + \frac{u u_x}{(1+\mathcal{F})^{-1}} - \frac{u\rho_t}{\rho\mathcal{F}^{-1}} + \frac{u^2}{x}\left[\frac{2jC_0\left(\Gamma-1+\frac{2}{M^2}\right)^{-1}}{8(\Gamma+1)^{-3}\left(1-\frac{1}{M^2}\right)^2} - \mathcal{F}\right] = 0 \quad (23)$$

where $\mathcal{F} = \frac{4\left\{1-\frac{(\Gamma-1)}{2\Gamma}\frac{1}{M^2}\right\} + C_0(\Gamma+1)}{8\left\{\Gamma-1+\frac{2}{M^2}\right\}^{-1}\left(1-\frac{1}{M^2}\right)^2}$ and:

$$u_t + u u_x + \frac{(\Gamma-1)}{2}u\left(u_x + \frac{u}{x}\right) = 0 \quad (24)$$

$$u_t + u u_x + \frac{u}{2}\left[u_x + \frac{(1-2j)u}{x}\right] = 0 \quad (25)$$

Plugging (23) and (24) and performing some algebraic manipulations, we get the resulting equation as:

$$\mathcal{S}(t) = \rho u^{(2-\mathcal{L})}(x)^{\mathcal{M}-\mathcal{L}} \quad (26)$$

where $\mathcal{S}$(t) denotes the single function of time and the values of the constants $\mathcal{L}$ and $\mathcal{M}$ are given as:

$$\mathcal{L} = \frac{(\Gamma-1)}{2\mathcal{F}} \text{ and } \mathcal{M} = 2j\frac{C_0(\Gamma+1)^3\left\{1+\frac{2}{(\Gamma-1)}\frac{1}{M^2}\right\}^{-1}}{8(\Gamma-1)\left(1-\frac{1}{M^2}\right)^2\mathcal{F}}$$

Using (26) and after some analytical steps, the mass-balance equation (1) of the compressible Euler system is eventually recast in the following form:

$$\frac{(2-\mathcal{L})}{u}u_t + (1-\mathcal{L})u_x - (\mathcal{L}-\mathcal{M}+1)\frac{u}{x} - \frac{1}{\mathcal{S}}\frac{d\mathcal{S}}{dt} = 0 \quad (27)$$

On solving (24) and (27) we have:

$$u = \frac{1}{[\mathcal{L}\Gamma+\mathcal{M}-2(\mathcal{L}+\Gamma)]}\frac{x}{\mathcal{S}}\frac{d\mathcal{S}}{dt} \quad (28)$$

By substituting the value of flow velocity from (28) in (27) and after simplification we have:

$$\mathcal{S}(t) = \mathcal{S}_0(t)^{-\mathcal{W}} \quad (29)$$

where $\mathcal{W} = \frac{2(\mathcal{L}+\Gamma)-\mathcal{L}\Gamma-\mathcal{M}}{\Gamma}$ and $\mathcal{S}_0$ is an arbitrary constant.

On using the Rankine-Hugoniot jump boundary conditions (17) and (18), we obtain explicit expressions for the radius of shock front $\mathcal{R}$ and constant $\mathcal{J}$ as:

$$\mathcal{R} = (t)^{\frac{(\Gamma+1)}{2\Gamma\left(1-\frac{1}{M^2}\right)}} \quad (30)$$

$$\mathcal{J} = \frac{2\Gamma(M^2-1)\{2-(\mathcal{L}+\mathcal{W})\}+(2\mathcal{L}-\mathcal{M}-2)(\Gamma+1)M^2}{(\Gamma+1)M^2} \quad (31)$$

Eventually, in reference of jump conditions (17)-(20), the analytical solution of the flow-field variables to the weak shock wave problem modelled in the prior section is given as:

$$\rho = \mathcal{S}_0(\Gamma)^{2-\mathcal{L}}\frac{(x)^{2\mathcal{L}-\mathcal{M}-2}}{(t)^{(\mathcal{L}+\mathcal{W})-2}} \quad (32)$$

$$u = \frac{\{2\mathcal{L}(1-\Gamma)+\Gamma(2+\mathcal{L})-\mathcal{M}\}}{\Gamma\{2(2\mathcal{L}+\Gamma)-\mathcal{L}(2+\Gamma)-\mathcal{M}\}}\frac{x}{t} \quad (33)$$

$$p = \frac{\mathcal{S}_0(x)^{2\mathcal{L}-\mathcal{M}}\left[\left(1-\frac{\Gamma-1}{2\Gamma M^2}\right)+\frac{1-\frac{(\Gamma+1)^2}{\left(\Gamma-1+\frac{2}{M^2}\right)^2}}{4\{C_0(\Gamma+1)\}^{-1}}\right]}{2(\Gamma)^{\mathcal{L}}(t)^{(\mathcal{L}+\mathcal{W})}\left(1-\frac{1}{M^2}\right)^2\left(\Gamma-1+\frac{2}{M^2}\right)^{-1}} \quad (34)$$

$$h = \frac{\mathcal{S}_0 C_0(x)^{2\mathcal{L}-\mathcal{M}}\left(\Gamma-1+\frac{2}{M^2}\right)^{-1}}{8(\Gamma)^{\mathcal{L}}(t)^{(\mathcal{L}+\mathcal{W})}(\Gamma+1)^{-3}\left(1-\frac{1}{M^2}\right)^2} \quad (35)$$

By applying the values of the physical flow parameters from (32)-(35), the explicit cumbersome solution for total energy E_T is given as:

$$E_T = \Lambda.(t)^{\frac{(\Gamma+1)M^2\{2(\mathcal{L}+1)-\mathcal{M}\}-2\Gamma(\mathcal{L}+\mathcal{W})(M^2-1)}{2\Gamma(M^2-1)}} \quad (36)$$

where:

$$\Lambda = \frac{\mathcal{S}_0\left[\frac{C_0(\Gamma+1)^3}{\left\{\Gamma-1+\frac{2}{M^2}\right\}}+4\left(1-\frac{1}{M^2}\right)^2-\frac{2(\Gamma+1)\left\{\frac{1}{2\Gamma(\Gamma-1)M^2}+\frac{C_0}{4}\right\}}{\{2(\mathcal{L}+1)-\mathcal{M}\}^{-1}\left\{\Gamma-1+\frac{2}{M^2}\right\}^{-1}}+\frac{4\left\{1-\frac{(\Gamma-1)}{2\Gamma}\frac{1}{M^2}\right\}+C_0(\Gamma+1)\left[1-(\Gamma+1)^2\left\{\Gamma-1+\frac{2}{M^2}\right\}^{-2}\right]}{\left\{1+\frac{2}{(\Gamma-1)}\frac{1}{M^2}\right\}^{-1}}\right]}{8\{2(\mathcal{L}+1)-\mathcal{M}\}(\Gamma)^{\mathcal{L}}\left(1-\frac{1}{M^2}\right)^2}$$

V. RESULTS AND DISCUSSION

The formulation of precise (closed form) and analytical solutions is of utmost importance in the field of natural and engineering sciences as these solutions are very helpful to understand and classify the involved physical phenomena. The expressions (32)-(36) depict explicitly the analytical solution of physical flow characteristics (such as density, velocity, pressure, and magnetic pressure) and total energy, to the cylindrical symmetric weak shock wave problem in an ideal supersonic magnetogasdynamic flow. It should be noted that the exact solutions obtained for the density, pressure, and magnetic pressure flow are greatly affected by the generalized magnetic field and shock Mach number. However, velocity profile and shock radius remain unchanged under the influence of the magnetic field, whereas the effect of shock Mach number appears significantly.

Figures 1-3 illustrate the behavior of flow characteristics density, pressure and magnetic pressure, for the propagation of cylindrical weak shock wave in azimuthal magnetic field with the variation in Mach number M with respect to time t. The values of the constants utilized for numerical computation have been taken as: $\mathcal{S}_0 = 1$, $\Gamma = 1.67$, and $C_0 = 0.05$. For the sake of convenience, the effect of magnetic field h has been entered through Shock-Cowling number C_0.

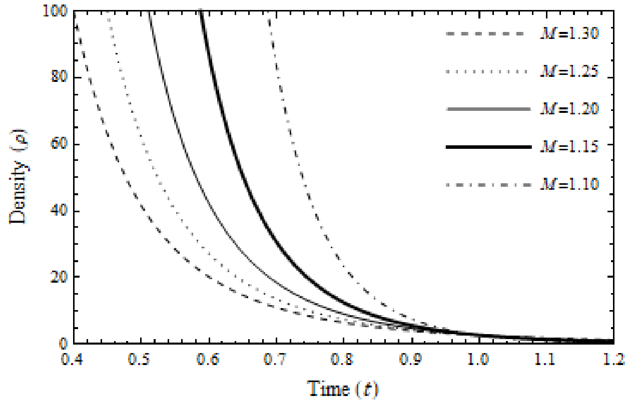

Fig. 1. Dispersal of density profile with time at various values of Mach number.

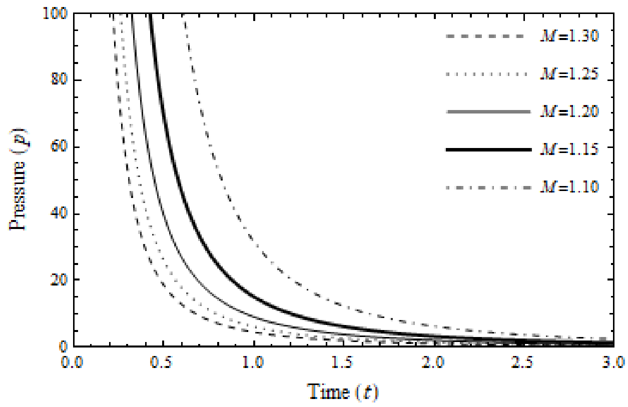

Fig. 2. Dispersal of pressure profile with time at various values of Mach number.

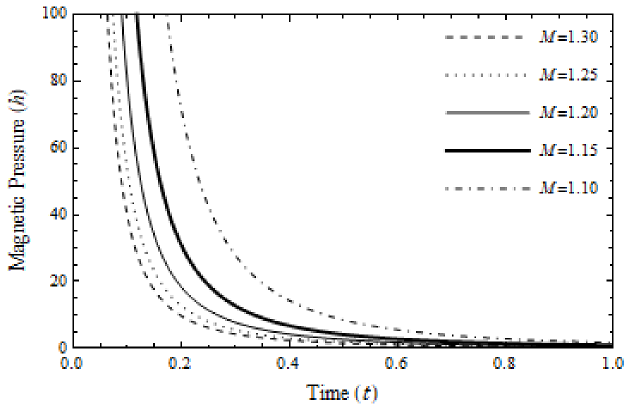

Fig. 3. Dispersal of pressure profile with time at various values of Mach number.

From Figures 1-3, it is evident that for increasing value of shock-Mach number, the decay process of physical parameters (density, velocity, and pressure) gets a bit slower with respect to time. Further, it is observed that for the case of axial magnetic field $(j = 0)$, and various values of the ratio of specific heats (Γ), the behavior of density, pressure and magnetic pressure remains unchanged, however, for smaller values of Shock-Cowling number, the discussed flow profiles decay rapidly with respect to time as compared to larger values of C_0 which closely coincide with the results obtained in [36, 37] for non-magnetic flow.

Figure 4 exhibits the behavior of velocity profile for cylindrical weak shock wave in ideal magneto-gas-dynamic flow with the variation in M with respect to t. It is observed that when increasing the value of *M*, the velocity profile of the cylindrical weak shock waves at the shock front under the influence of the magnetic field increases rapidly with time. Additionally we perceive that as compared to $\Gamma = 1.67$, the fluid velocity characteristics enhances faster for the adibatic index $\Gamma = 1.4$. Figures 5 and 6 demonstrate the behavior of the energy carried by the cylindrical weak shock wave in magneto-gas-dynamic regime with respect to t for varying values of Mach number and Shock-Cowling number respectively.

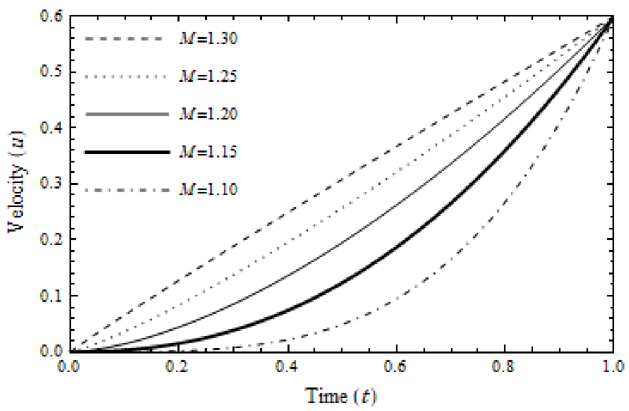

Fig. 4. Dispersal of velocity profile with time at various values of Mach number.

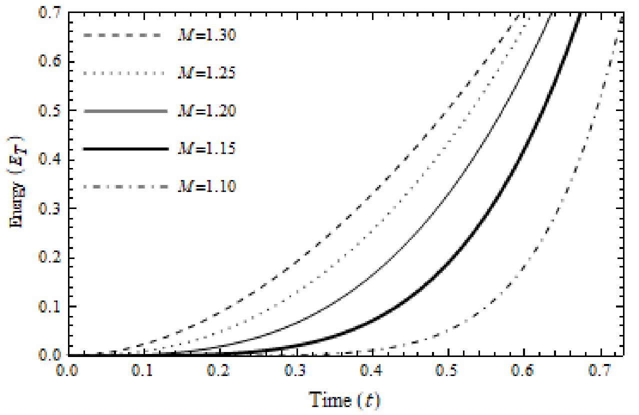

Fig. 5. Dispersal of total energy with time at various values of Mach number.

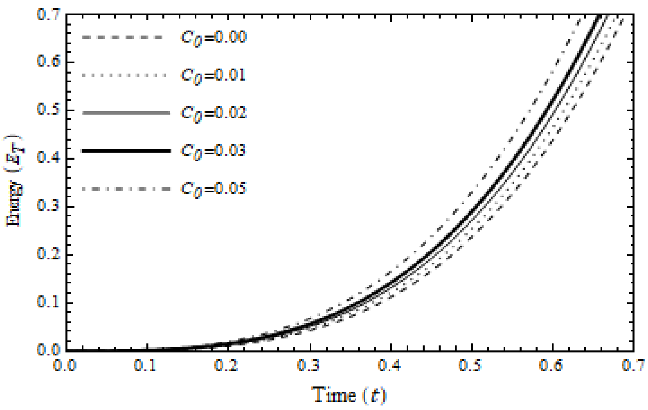

Fig. 6. Dispersal of total energy with time at various values of Shock-Cowling number.

It is evident from Figure 5 that when the value of M increases, the energy of the cylindrical weak shock waves under the influence of the magnetic field increases much faster with time. Moreover, it can be noted, that for the case of axial magnetic field $(j = 0)$, the values of the total energy with respect to time for varying M get a bit slower as compared to the azimuthal magnetic field $(j = 1)$, which confirms the experimental considerations. In Figure 6, the behavior produced due to the varying Shock-Cowling number reveals that increasing the values of C_0 causes a rapid upsurge in the total energy with respect to time. Furthermore, it should be noted, that for the case of axial magnetic field, for increasing values of C_0, the values of the total energy with respect to time are much closer, which means that the effect of azimuthal magnetic field has a more considerable impact on the behavior of energy with respect to time.

VI. CONCLUSION

In the present work, we performed an analytical investigation of the problem of propagation of quasi-one-dimensional unsteady state compressible non-viscous adiabatic cylindrical weak shock wave in a perfect gas-dynamical regime, under the influence of axial and azimuthal magnetic fields. For this purpose, the simple and efficient analytical technique proposed in [20] has been used and the exact solutions for the physical flow characteristics, such as density, velocity, pressure, and magnetic pressure, were obtained in the form of space-coordinates and time relations. Also, the cumbersome expression for total energy (i.e. the sum of kinetic and potential energy) carried by weak shock flow in the considered plasma influenced with magnetic field (axial and azimuthal) having dependency of space-time is determined. To describe the effect of Mach number and magnetic field (modeled in the form of the Shock-Cowling number) on flow-field characteristics and total energy carried are graphically presented. All the computations for this purpose have been done with the computational software package MATHEMATICA 9.0.

Since the shock proliferation processes are associated with a high-temperature gas dynamics phenomenon, thermal radiation has a significant impact on the wave phenomenon due to its coupling with the magnetic field and its various applications in terms of theoretical and industrial aspects. In addition, for such physical processes, the consideration that the medium is perfect is no longer valid. Therefore, an analytical investigation of the current study into realistic gas-dynamic regimes, and thermal radiation effects, is a potentially interesting area for future work.

Based on the results of the present study, the following conclusions are made:

- For increasing value of shock-Mach number, the decay process of physical parameters (flow velocity, density, and pressure) gets a bit slower with respect to time for a fixed value of C_0, which clearly resembles the pattern of non-magnetic flow.
- An increase in the value of Mach number causes a rapid increase to the flow velocity of weakly nonlinear waves with respect to time. Consequently, we perceive that as compared to the specific heat ratio $\Gamma = 1.67$, the velocity characteristics grows faster for the adibatic index $\Gamma = 1.4$.
- The total energy of the cylindrical weak shock waves under the influence of the azimuthal magnetic field increases much faster with respect to time for an increasing value of Mach number. However, as compared to the azimuthal magnetic field $(j = 1)$, in the of the axial magnetic field $(j = 0)$, the total energy with respect to time enhances a bit slower for varying Mach number, which validates the theoretical considerations.
- In azimuthal magnetic field, for increasing value of Shock-Cowling number (C_0) and with fixed values of Mach number, the total energy grows rapidly with respect to time. Moreover, for the case of axial magnetic field, the behavior remains unchanged although the values are much closer, which makes leads to the conclusion that as compared to the axial magnetic field, the effect of the azimuthal magnetic field has considerable impact on the behavior of energy with respect to time.

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