Assessment of Shear Strength Models of Reinforced Concrete Columns

Van-Hoa Nguyen
Department of Civil Engineering
Vinh University
Vinh, Vietnam
vanhoakxd@vinhuni.edu.vn

Ngoc-Minh Pham
Department of Civil Engineering
Vinh University
Vinh, Vietnam
minhkxd@vinhuni.edu.vn

Trong-Cuong Vo
Department of Civil Engineering
Vinh University
Vinh, Vietnam
cuongvqc@gmail.com

Duy-Duan Nguyen
Department of Civil Engineering
Vinh University
Vinh, Vietnam
duyduankxd@vinhuni.edu.vn

Received: 7 August 2022 | Revised: 21 August 2022 | Accepted: 26 August 2022

Abstract-Shear strength is a crucial parameter in designing Reinforced Concrete (RC) columns considering the effects of lateral loads such as wind or earthquakes. Numerous design codes and published studies have proposed equations for calculating the shear strength of RC columns. However, a discrepancy exists between the calculated models and the experimental results. The aim of this study is to evaluate the calculated models for the shear strength of rectangular RC columns based on 735 data sets, obtained from the literature. Six code-based and empirical models are investigated in this paper. The four code-based models include ACI 318 (2014), CSA (2014), Eurocode 8 (2005), and FEMA 273 (1997), and the two empirical models are proposed by Ascheim & Moehle (1992) and Sezen & Moehle (2004). The shear strengths of RC columns are calculated for the six models using inputs from the experimental database. Finally, the results are evaluated using statistical indicators, including coefficient of determination and root-mean-squared error. The results reveal that Eurocode 8 (2005) is the best model, followed by Sezen & Moehle (2004) and Canada CSA (2014) since the results of those models are close to the experimental ones and shown to be more conservative than the others.

Keywords-design code; empirical formula; experimental data; rectangular RC column; shear strength

I. INTRODUCTION

Reinforced Concrete (RC) columns are critical structural members in buildings and bridges. Failure of these members can lead to the partial or total collapse of structures. The load bearing capacity of columns depends on their geometric dimensions, materials, detailing, or applied loads [1-3]. There are three typical failure modes of RC columns under lateral loads, including flexure, shear, and flexure-shear failure modes. Flexure failure occurs when the lateral stiffness is reduced due to cracking and spalling of the concrete cover, yielding of the longitudinal reinforcements, and crushing of core concrete. Meanwhile, the shear failure mode is governed if diagonal cracks are predominant. Flexure-shear failure forms after yielding of rebars and is combined with shear failure. It should be noted that the shear failure mode is unexpected and it is avoided in designing columns, especially structures in earthquake-prone areas. Shear strength is the most important parameter in the design of RC columns, specifically when considering the effects of lateral loads such as earthquakes or wind. Currently, there are numerous design codes and published studies, which proposed equations for calculating shear strength of RC columns. Typical design codes are ACI 318 (2014) [4], CSA (2014) [5], Eurocode 8 (2005) [6], and FEMA 273 (1997) [7]. Additionally, some empirical models were developed by some authors such as Ascheim & Moehle (1992) [8] and Sezen & Moehle (2004) [9]. However, a discrepancy between calculated models and experimental results is existing. Therefore, it is necessary to evaluate the different calculated models based on a large experiment database.

The aim of this study is to assess the different calculated shear strength models of rectangular RC columns, in which code-based and empirical-based formulas are considered. For that, an extensive database including 735 experimental results is collected from the literature. Six calculated models are investigated, consisting of the four mentioned design codes [4-7] and [8-9]. It should be noted that the equation proposed in [8] is also used in ASCE/SEI-41-06 (2007) [10]. The shear strength of rectangular RC columns is calculated for the six models using the collected database. Finally, the calculated results are evaluated using statistical properties comprising of the coefficient of determination and the root-mean-squared error.

II. CALCULATED MODELS OF SHEAR STRENGTH OF RECTANGULAR RC COLUMNS

In this study, we employed 6 typical equations for calculating the shear strength of RC columns, from current

Corresponding author: Duy-Duan Nguyen

design codes and well-known previous studies, as described in Table I.

TABLE I. SHEAR STRENGTH MODELS CONSIDERED IN THIS STUDY

Model	Equation	
ACI 318 [4]	$V = 0.166\left(1+\frac{P}{13.8A_g}\right)b_w d\sqrt{f_c'} + \frac{A_{sh}f_{yh}d}{s}$ P is the axial load, A_g is the gross cross-section area of the column, b_w is the effective shear width of column section, d is the effective depth of the column, f_c' is the compressive strength of concrete, A_{sh} and f_{yh} are the area and yield strength of transversal reinforcement, and s is the spacing of stirrups.	(1)
CSA [5]	$V = \min\left(\beta b_w d_v\sqrt{f_c'} + \frac{A_{sh}f_{yh}d}{s}\cot\theta;\ 0.25f_c' bd\right)$ $d_v = 0.9d$ b is the width of column section, β is a factor accounting for the shear resistance of cracked concrete, θ is the angle of inclination of diagonal compressive stresses to the longitudinal axis of the column.	(2)
Eurocode 8 [6]	$V = V_p + k(V_c + V_w)$ $V_c = 0.16max(0.5; 100\rho_l)\left(1 - 0.16min\left(5;\frac{a}{d}\right)\right)A_c\sqrt{f_c'}$ $V_w = \frac{A_{sw}}{s}(d - d')f_{yw}$ $V_p = \frac{D-x}{2a}\min(P; 0.55A_c f_c')$ $A_c = b_w d, (d = 0.8h)$ ρ_l is the longitudinal reinforcement ratio, a is the distance from the maximum moment section to the point of inflection (i.e. M/V)	(3)
FEMA 273 [7]	$V = 0.29\lambda\left(k+\frac{P}{13.8A_g}\right)bd\sqrt{f_c'} + \frac{A_{sh}f_{yh}d}{s}$ λ is a coefficient depending on concrete weigh (= 1.0). $k = 1.0$ for low ductility demand. $k = 0$ for moderate and high ductility demand.	(4)
Ascheim & Moehle [8]	$V = 0.3\left(k+\frac{P}{13.8A_g}\right)0.8A_g\sqrt{f_c'} + \frac{A_{sh}f_{yh}d}{s\tan(30^0)}$ $k = \frac{4-\mu}{3}$, μ is the displacement ductility $d = 0.8H$	(5)
Sezen & Moehle [9]	$V = k\left(\frac{0.5\sqrt{f_c'}}{a/d}\sqrt{1+\frac{P}{0.5A_g\sqrt{f_c'}}}\right)0.8A_g + k\frac{A_{sh}f_{yh}d}{s}$ $d = D - \text{cover}$ $k = 1$ for $\mu < 2.0$; $k = 0.7$ for $\mu > 6.0$; $0.7 \le k = 1.15 - 0.075\mu \le 1.0$ for $2.0 \le \mu \le 6.0$ a is the shear span, (i.e. the distance from the loading point to the boundary).	(6)

III. COLLECTED DATABASE

A significant database, which covers a wide range of scenarios, was collected to evaluate the calculated shear strength models. A total of 735 experimental data sets of rectangular RC columns were extensively collected from [11-36]. Figure 1 depicts the configurations and reinforcement properties of the rectangular RC column. It should be noted that L is the column height, B and H are the width and depth of the column section respectively, and ρ_l and ρ_h are the longitudinal and transversal reinforcement ratios respectively. The statistical properties of the experimental results are described in Table II. The frequency histograms of input parameters and failure modes of the 735 data samples are shown in Figure 2.

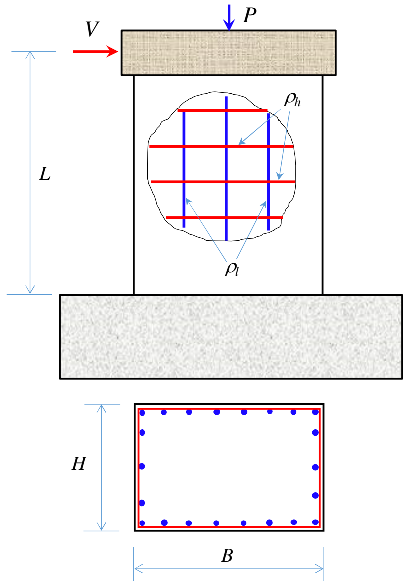

Fig. 1. Configurations and properties of rectangular RC columns.

TABLE II. SUMMARY OF THE DATABASE INPUT PARAMETERS

Input para-meter	L (mm)	B (mm)	H (mm)	s (mm)	f_c' (MPa)	f_{yl} (MPa)	f_{yh} (MPa)	ρ_l (%)	ρ_h (%)	P (kN)
Min	225	150	100	20	20	313	215	0.20	0.01	0.0
Mean	1286	284	301	101	49	448	496	2.15	0.94	1130
Max	3000	610	610	457	141	745	1470	4.50	4.00	5492
SD	647	109	115	77	27	77	222	0.69	0.94	1069
COV	0.53	0.38	0.38	0.76	0.55	0.17	0.45	0.32	0.99	0.95

IV. EVALUATION OF THE CALCULATED SHEAR STRENGTH OF THE RC COLUMNS

To quantitatively evaluate the shear strengths calculated from models in Table I, statistical parameters are employed including coefficient of determination (R^2) and Root-Mean-Squared Error ($RMSE$). R^2 value represents the percentage of data close to the regression line. The higher the R^2, the more the accuracy of the calculated model and vice versa. $RMSE$ is used for quantifying the difference (error) between the calculated and the experimental value. The smaller the $RMSE$, the more the accuracy of the calculated model and vice versa. The definitions of R^2 and $RMSE$ are described in (7) and (8).

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{n}(t_i - o_i)^2}{\sum_{i=1}^{n}(t_i - \bar{o})^2}\right) \quad (7)$$

$$RMSE = \sqrt{\left(\frac{1}{n}\right)\sum_{i=1}^{n}(t_i - o_i)^2} \quad (8)$$

where t_i and o_i are the test and calculated results of the i sample, n is number of the samples of the database, $\bar{o}$ is the mean of calculated results.

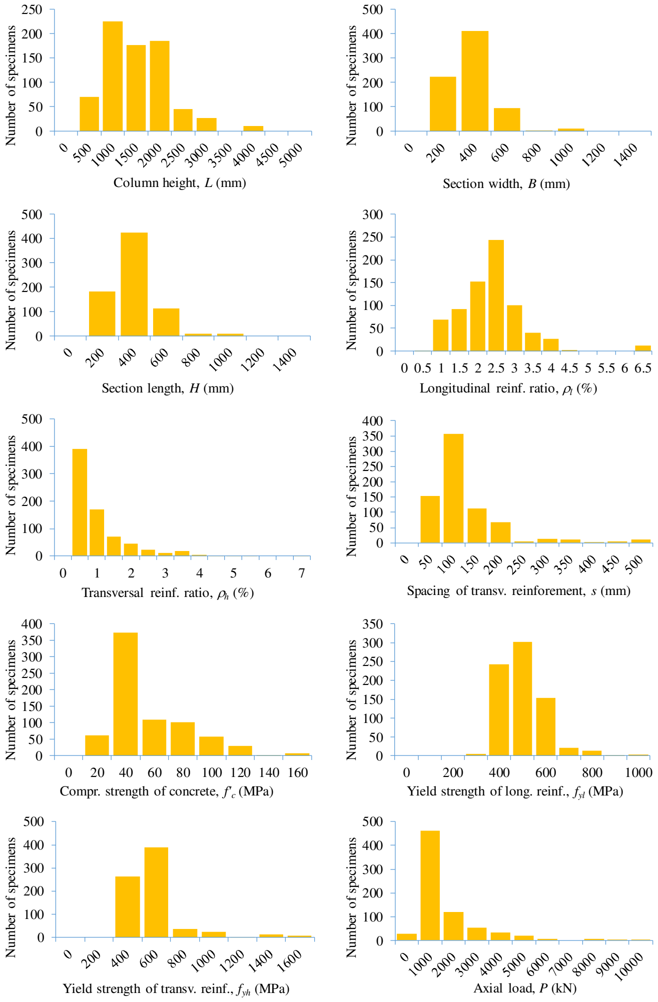

Fig. 2. Distributions of input database.

Figure 3 shows the comparison between the shear strengths of the RC columns calculated by the six models and the test results. The dash line represents the 1:1 line, while the red line is the regression. It can be seen that the models of EC8 , Sezen & Moehle, and CSA provide a smaller scattering than others, and results are mostly under the 1:1 line. In other words, the calculated results are smaller than those of experiments. Moreover, the results based on ACI 318, FEMA 273, and Ascheim & Moehle have a bigger scattering. These deviations may are caused by the ductility ratio and the diagonal compressive force in the columns.

Table III summarizes the calculated statistical parameters, i.e. R^2 and $RMSE$ for the investigated models. Additionally, the properties of the ratio $V_{calculate}/V_{test}$, are also obtained, in which minimum (Min), maximum (Max), Mean, Standard Deviation (SD), and Coefficient of Variation (CV) are determined. The results show that EC8 [6] is the most accurate model with the highest R^2 value (= 0.892) and the smallest $RMSE$ (= 163kN). Moreover, [9] and [5] are also good models with R^2 = 0.766 and 0.68, and $RMSE$ = 185kN and 220kN respectively. Besides, the mean values of $V_{calculate}/V_{test}$ of those models are smaller than 1.0, thus, the results calculated by [5, 6, 9] are conservative. It can be seen that the equations of [4, 7, 8] provide lower accuracy. Based on the results, we suggest the use of equations of the models of [5, 6, 9] for calculating the shear strength of rectangular RC columns.

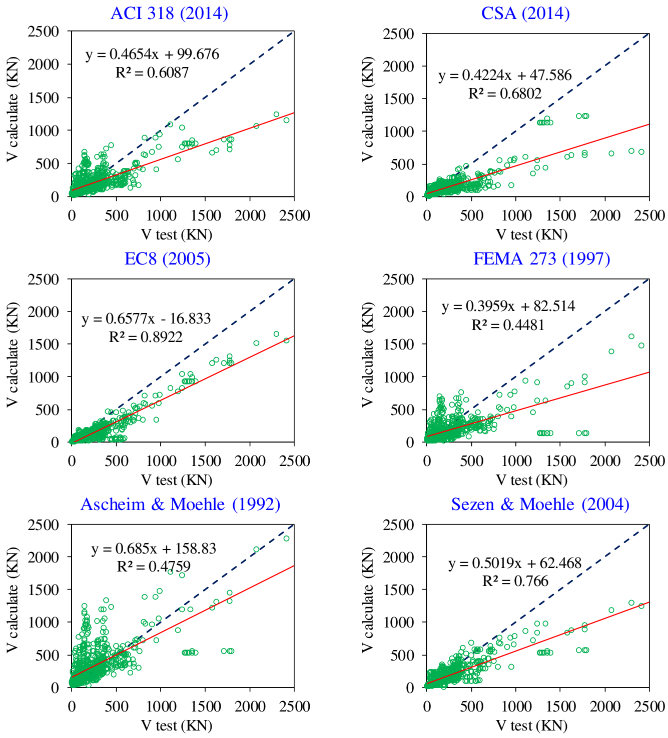

Fig. 3. Comparison between the six calculated models and the test results.

TABLE III. STATISTICAL PROPERTIES FOR THE EVALUATION OF THE SHEAR STRENGTH OF RC COLUMNS

Model	R^2	$RMSE$	Statistical properties of $V_{calculate}/V_{test}$				
			Min	Max	Mean	SD	CV
ACI 318 [4]	0.608	202	0.19	20.88	1.40	1.42	1.01
CSA [5]	0.680	220	0.18	9.88	0.93	0.74	0.80
EC8 [6]	0.892	163	0.07	4.84	0.64	0.37	0.57
FEMA 273 [7]	0.448	238	0.07	19.62	1.15	1.33	1.15
Ascheim & Moehle [8]	0.476	251	0.21	35.78	2.14	2.41	1.13
Sezen & Moehle [9]	0.766	185	0.09	8.51	1.02	0.71	0.69

V. CONCLUSIONS

This study presents and analyzes six code- and empirical-based models for the shear strength calculation of rectangular RC columns. A set consisting of 735 experimental data samples of rectangular RC columns was collected. The accuracy of the models is evaluated by statistical parameters. The following conclusions are drawn from the current study:

- EC8 [6] is the most accurate model for estimating the shear strength of rectangular RC columns, followed by Sezen & Moehle [9] and CSA [6].

- The equations of ACI 318 [4], FEMA 273 [7], and Ascheim & Moehle [8] have a lower accuracy.
- It should be noted that this study considered rectangular RC columns. For other RC column types, such as circular or hollow sections, further investigation is required.

REFERENCES

[1] L. Hamzaoui and T. Bouzid, "The Proposition of an EI Equation of Square and L–Shaped Slender Reinforced Concrete Columns under Combined Loading," *Engineering, Technology & Applied Science Research*, vol. 11, no. 3, pp. 7100–7106, Jun. 2021, https://doi.org/10.48084/etasr.4048.

[2] H. Ullah, M. Rizwan, M. Fahad, and S. a. A. Shah, "Seismic Evaluation of the RC Moment Frame Structure using the Shake Table," *Engineering, Technology & Applied Science Research*, vol. 11, no. 1, pp. 6674–6679, Feb. 2021, https://doi.org/10.48084/etasr.3959.

[3] A. R. Khoso, J. S. Khan, R. U. Faiz, M. A. Akhund, A. Ahmed, and F. Memon, "Identification of Building Failure Indicators," *Engineering, Technology & Applied Science Research*, vol. 9, no. 5, pp. 4591–4595, Oct. 2019, https://doi.org/10.48084/etasr.2872.

[4] *ACI318-(2008), Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary.* Farmington Hills, MI, USA: American Concrete Institute, 2008.

[5] *CSA-(2014), Design of concrete structures (CSA A23. 3-14).* Toronto, ON, Canada: Canadian Standards Association, 2014.

[6] *SS-EN 1998-1(2004), Eurocode 8: Design Of Structures For Earthquake Resistance - Part 1: General Rules, Seismic Actions And Rules For Buildings*. London, UK: British Standards Institution, 2004.

[7] *FEMA 273, NEHRP Guidelines For The Seismic Rehabilitation Of Buildings*. Washington, DC, USA: Building Seismic Safety Council for the Federal Emergency Management Agency, 1997.

[8] M. Ascheim and J. P. Moehle, "Shear Strength and Deformability of RC Bridge Columns Subjected to Inelastic Cyclic Displacements," University of California, Berkeley, CA, USA, UCB/EERC-92/04, Mar. 1992.

[9] H. Sezen and J. P. Moehle, "Shear Strength Model for Lightly Reinforced Concrete Columns," *Journal of Structural Engineering*, vol. 130, no. 11, pp. 1692–1703, Nov. 2004, https://doi.org/10.1061/(ASCE)0733-9445(2004)130:11(1692).

[10] *ASCE/SEI 41-06(2007), Seismic Rehabilitation of Existing Buildings*. Reston, VA, USA: American Society of Civil Engineers, 2007.

[11] W. Ghannoum *et al.*, "NEES: ACI 369 Rectangular Column Database," *DEEDS*, 2015. https://doi.org/10.4231/D36688J50.

[12] M. A. Belkacem, H. Bechtoula, N. Bourahla, and A. A. Belkacem, "Effect of axial load and transverse reinforcements on the seismic performance of reinforced concrete columns," *Frontiers of Structural and Civil Engineering*, vol. 13, no. 4, pp. 831–851, Aug. 2019, https://doi.org/10.1007/s11709-018-0513-3.

[13] D. Wang, H.-N. Li, and G. Li, "Experimental study on dynamic mechanical properties of reinforced concrete column," *Journal of Reinforced Plastics and Composites*, vol. 32, no. 23, pp. 1793–1806, Dec. 2013, https://doi.org/10.1177/0731684413492451.

[14] J. Xiao and Ch. Zhang, "Seismic behavior of RC columns with circular, square and diamond sections," *Construction and Building Materials*, vol. 22, no. 5, pp. 801–810, May 2008, https://doi.org/10.1016/j.conbuildmat.2007.01.010.

[15] H. Rodrigues, A. Furtado, and A. Arede, "Behavior of Rectangular Reinforced-Concrete Columns under Biaxial Cyclic Loading and Variable Axial Loads," *Journal of Structural Engineering*, vol. 142, no. 1, Jan. 2016, Art. no. 04015085, https://doi.org/10.1061/(ASCE)ST.1943-541X.0001345.

[16] J. Melo, H. Varum, and T. Rossetto, "Experimental cyclic behaviour of RC columns with plain bars and proposal for Eurocode 8 formula improvement," *Engineering Structures*, vol. 88, pp. 22–36, Apr. 2015, https://doi.org/10.1016/j.engstruct.2015.01.033.

[17] J. C. M. Ho, "Experimental Tests on High-Strength Concrete Columns Subjected to Combined Medium Axial Load and Flexure," *Advances in Structural Engineering*, vol. 15, no. 8, pp. 1359–1374, Aug. 2012, https://doi.org/10.1260/1369-4332.15.8.1359.

[18] D. Wu, Y. Ding, J. Su, Z.-X. Li, L. Zong, and K. Feng, "Effects of tie detailing configurations on reinforcement buckling and seismic performance of high-strength RC columns," *Soil Dynamics and Earthquake Engineering*, vol. 147, Aug. 2021, Art. no. 106791, https://doi.org/10.1016/j.soildyn.2021.106791.

[19] C. T. N. Tran, "Experimental and analytical studies on the seismic behavior of reinforced concrete columns with light transverse reinforcement," Ph.D. dissertation, Nanyang Technological University, Singapore, 2010.

[20] Y. C. Ou, D. P. Kurniawan, and N. Handika, "Shear behavior of reinforced concrete columns with high-strength steel and concrete under low axial load," in *Reinforced Concrete Columns with High Strength Concrete and Steel Reinforcement*, Toronto, ON, Canada, Oct. 2012, vol. 293, pp. 145–160.

[21] Y.-A. Li, Y.-T. Huang, and S.-J. Hwang, "Seismic Response of Reinforced Concrete Short Columns Failed in Shear," *Structural Journal*, vol. 111, no. 4, pp. 945–954, Jul. 2014, https://doi.org/10.14359/51686780.

[22] V. Popa, D. Cotofana, and R. Vacareanu, "Effective stiffness and displacement capacity of short reinforced concrete columns with low concrete quality," *Bulletin of Earthquake Engineering*, vol. 12, no. 6, pp. 2705–2721, Dec. 2014, https://doi.org/10.1007/s10518-014-9618-9.

[23] C. Jin, Z. Pan, S. Meng, and Z. Qiao, "Seismic Behavior of Shear-Critical Reinforced High-Strength Concrete Columns," *Journal of Structural Engineering*, vol. 141, no. 8, Aug. 2015, Art. no. 04014198, https://doi.org/10.1061/(ASCE)ST.1943-541X.0001167.

[24] M. M. EL-Attar, H. Z. El-Karmoty, and A. A. EL-Moneim, "The behavior of ultra-high-strength reinforced concrete columns under axial and cyclic lateral loads," *HBRC Journal*, vol. 12, no. 3, pp. 284–295, Dec. 2016, https://doi.org/10.1016/j.hbrcj.2014.10.003.

[25] T.-S. Eom, S.-M. Kang, H.-G. Park, T.-W. Choi, and J.-M. Jin, "Cyclic loading test for reinforced concrete columns with continuous rectangular and polygonal hoops," *Engineering Structures*, vol. 67, pp. 39–49, May 2014, https://doi.org/10.1016/j.engstruct.2014.02.023.

[26] E. A. Opabola, K. J. Elwood, and S. Oliver, "Deformation capacity of reinforced concrete columns with smooth reinforcement," *Bulletin of Earthquake Engineering*, vol. 17, no. 5, pp. 2509–2532, May 2019, https://doi.org/10.1007/s10518-018-00540-w.

[27] C. Goksu, H. Yilmaz, S. R. Chowdhury, K. Orakcal, and A. Ilki, "The Effect of Lap Splice Length on the Cyclic Lateral Load Behavior of RC Members with Low-Strength Concrete and Plain Bars," *Advances in Structural Engineering*, vol. 17, no. 5, pp. 639–658, May 2014, https://doi.org/10.1260/1369-4332.17.5.639.

[28] Y. Zhang, S. Zheng, X. Rong, L. Dong, and H. Zheng, "Seismic Performance of Reinforced Concrete Short Columns Subjected to Freeze–Thaw Cycles," *Applied Sciences*, vol. 9, no. 13, Jan. 2019, Art. no. 2708, https://doi.org/10.3390/app9132708.

[29] S. Bousias, A.-L. Spathis, and M. N. Fardis, "Seismic Retrofitting of Columns with Lap Spliced Smooth Bars Through FRP or Concrete Jackets," *Journal of Earthquake Engineering*, vol. 11, no. 5, pp. 653–674, Oct. 2007, https://doi.org/10.1080/13632460601125714.

[30] K. K. Arani, M. D. Ludovico, M. S. Marefat, A. Prota, and G. Manfredi, "Lateral Response Evaluation of Old Type Reinforced Concrete Columns with Smooth Bars," *Structural Journal*, vol. 111, no. 4, pp. 827–838, Jul. 2014, https://doi.org/10.14359/51686734.

[31] T. P. Pham and B. Li, "Seismic Performance of Reinforced Concrete Columns with Plain Longitudinal Reinforcing Bars (with Appendix)," *Structural Journal*, vol. 111, no. 3, pp. 561–572, May 2014, https://doi.org/10.14359/51686572.

[32] J. Zhang, R. Cai, C. Li, and X. Liu, "Seismic behavior of high-strength concrete columns reinforced with high-strength steel bars," *Engineering Structures*, vol. 218, Sep. 2020, Art. no. 110861, https://doi.org/10.1016/j.engstruct.2020.110861.

[33] N. H. Dinh, S. Park, and K.-K. Choi, "Seismic performance of reinforced concrete columns retrofitted by textile-reinforced mortar jackets," *Structure and Infrastructure Engineering*, vol. 16, no. 10, pp. 1364–1381, Oct. 2020, https://doi.org/10.1080/15732479.2019.1708958.

[34] C.-G. Kim, H.-G. Park, and T.-S. Eom, "Effects of Type of Bar Lap Splice on Reinforced Concrete Columns Subjected to Cyclic Loading," *Structural Journal*, vol. 116, no. 2, pp. 183–194, Mar. 2019, https://doi.org/10.14359/51711142.

[35] H.-J. Hwang, J.-O. Noh, and H.-G. Park, "Structural capacity of reinforced concrete columns with U-shaped transverse bars," *Engineering Structures*, vol. 216, Aug. 2020, Art. no. 110686, https://doi.org/10.1016/j.engstruct.2020.110686.

[36] K.-K. Choi, G. T. Truong, and J.-C. Kim, "Seismic performance of lightly shear reinforced RC columns," *Engineering Structures*, vol. 126, pp. 490–504, Nov. 2016, https://doi.org/10.1016/j.engstruct.2016.07.060.