Assessment of Shear Strength Models of Reinforced Concrete Columns

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Abstract — Shear strength is a crucial parameter in designing Reinforced Concrete (RC) columns considering the effects of lateral loads such as wind or earthquakes. Numerous design codes and published studies have proposed equations for calculating the shear strength of RC columns. However, a discrepancy exists between the calculated models and the experimental results. The aim of this study is to evaluate the calculated models for the shear strength of rectangular RC columns based on 735 data sets, obtained from the literature. Six code-based and empirical models are investigated in this paper. The four code-based models include ACI 318 (2014), CSA (2014), Eurocode 8 (2005), and FEMA 273 (1997), and the two empirical models are proposed by Ascheim & Moehle (1992) and Sezen & Moehle (2004). The shear strengths of RC columns are calculated for the six models using inputs from the experimental database. Finally, the results are evaluated using statistical indicators, including coefficient of determination and root-mean-squared error. The results reveal that Eurocode 8 (2005) is the best model, followed by Sezen & Moehle (2004) and Canada CSA (2014) since the results of those models are close to the experimental ones and shown to be more conservative than the others.

Keywords — design code; empirical formula; experimental data; rectangular RC column; shear strength

I. INTRODUCTION

Reinforced Concrete (RC) columns are critical structural members in buildings and bridges. Failure of these members can lead to the partial or total collapse of structures. The load bearing capacity of columns depends on their geometric dimensions, materials, detailing, or applied loads [1-3]. There are three typical failure modes of RC columns under lateral loads, including flexure, shear, and flexure-shear failure modes. Flexure failure occurs when the lateral stiffness is reduced due to cracking and spalling of the concrete cover, yielding of the longitudinal reinforcements, and crushing of core concrete. Meanwhile, the shear failure mode is governed if diagonal cracks are predominant. Flexure-shear failure forms after yielding of rebars and is combined with shear failure. It should be noted that the shear failure mode is unexpected and it is avoided in designing columns, especially structures in earthquake-prone areas. Shear strength is the most important parameter in the design of RC columns, specifically when considering the effects of lateral loads such as earthquakes or wind. Currently, there are numerous design codes and published studies, which proposed equations for calculating shear strength of RC columns. Typical design codes are ACI 318 (2014) [4], CSA (2014) [5], Eurocode 8 (2005) [6], and FEMA 273 (1997) [7]. Additionally, some empirical models were developed by some authors such as Ascheim & Moehle (1992) [8] and Sezen & Moehle (2004) [9]. However, a discrepancy between calculated models and experimental results is existing. Therefore, it is necessary to evaluate the different calculated models based on a large experiment database.

The aim of this study is to assess the different calculated shear strength models of rectangular RC columns, in which code-based and empirical-based formulas are considered. For that, an extensive database including 735 experimental results is collected from the literature. Six calculated models are investigated, consisting of the four mentioned design codes [4-7] and [8-9]. It should be noted that the equation proposed in [8] is also used in ASCE/SEI-41-06 (2007) [10]. The shear strength of rectangular RC columns is calculated for the six models using the collected database. Finally, the calculated results are evaluated using statistical properties comprising of the coefficient of determination and the root-mean-squared error.

II. CALCULATED MODELS OF SHEAR STRENGTH OF RECTANGULAR RC COLUMNS

In this study, we employed 6 typical equations for calculating the shear strength of RC columns, from current
design codes and well-known previous studies, as described in Table I.

### Table I. Shear Strength Models Considered in This Study

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318 [4]</td>
<td>$V = 0.166 \left(1 + \frac{P}{13.6A_p}\right)b_w d \sqrt{f_y} + \frac{A_{sh} f_{sh}}{s}$</td>
</tr>
<tr>
<td></td>
<td>$P$ is the axial load, $A_p$ is the gross cross-section area of the column, $B_w$ is the effective shear width of column section, $d$ is the effective depth of the column, $f_y$ is the compressive strength of concrete, $A_{sh}$ and $f_{sh}$ are the area and yield strength of transversal reinforcement, and $s$ is the spacing of stirrups.</td>
</tr>
<tr>
<td>CSA [5]</td>
<td>$V = V_c + k (V_c + V_w)$</td>
</tr>
<tr>
<td></td>
<td>$V_c = 0.166 \max (0.5; 100 \mu) \left(1 - 0.16 \min \left(5; \frac{s}{\mu}\right)\right) A_c \sqrt{f_y}$</td>
</tr>
<tr>
<td></td>
<td>$V_w = \frac{\mu}{2s} \min (P; 0.55 A_{sh} f_y)$</td>
</tr>
<tr>
<td></td>
<td>$\rho_s$ is the longitudinal reinforcement ratio, $a$ is the distance from the maximum moment section to the point of inflection (i.e. $M/V$)</td>
</tr>
<tr>
<td>Eurocode 8 [6]</td>
<td>$V = V_p + k (V_p + V_w)$</td>
</tr>
<tr>
<td></td>
<td>$V_p = 0.29 \left(1 + \frac{P}{13.6A_p}\right) b_d \sqrt{f_y} + \frac{A_{sh} f_{sh}}{s}$</td>
</tr>
<tr>
<td></td>
<td>$\lambda$ is a coefficient depending on concrete weigh ($= 1.0$), $\kappa = 1.0$ for low ductility demand, $k = 0$ for moderate and high ductility demand.</td>
</tr>
<tr>
<td>FEMA 273 [7]</td>
<td>$V = 0.3 \left(1 + \frac{P}{13.6A_p}\right) b_d \sqrt{f_y} + \frac{A_{sh} f_{sh}}{s}$</td>
</tr>
<tr>
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<td>$V_w = \frac{\mu}{2s} \min (P; 0.55 A_{sh} f_y)$</td>
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<td>$\rho_s$ is the longitudinal reinforcement ratio, $a$ is the distance from the maximum moment section to the point of inflection (i.e. $M/V$)</td>
</tr>
<tr>
<td>Asche &amp; Moehle [8]</td>
<td>$V = \frac{A_{sh} f_{sh}}{\sin \theta} \frac{B}{2}$</td>
</tr>
<tr>
<td></td>
<td>$k = \frac{A_{sh}}{s}$, $\mu$ is the displacement ductility</td>
</tr>
<tr>
<td>Sezen &amp; Moehle [9]</td>
<td>$V = \left(1 + \frac{P}{13.6A_p}\right) A_c \sqrt{f_y} + k \frac{A_{sh} f_{sh}}{s}$</td>
</tr>
<tr>
<td></td>
<td>$k = 1$ for $\mu &lt; 2.0$; $k = 0.7$ for $\mu &gt; 6.0$; $0.7 \leq k \leq 1 - 0.075\mu \leq 1.0$ for $2.0 \leq \mu \leq 6.0$</td>
</tr>
<tr>
<td></td>
<td>$a$ is the shear span, (i.e. the distance from the loading point to the boundary).</td>
</tr>
</tbody>
</table>

### III. Collected Database

A significant database, which covers a wide range of scenarios, was collected to evaluate the calculated shear strength models. A total of 735 experimental data sets of rectangular RC columns were extensively collected from [11-36]. Figure 1 depicts the configurations and reinforcement properties of the rectangular RC column. It should be noted that $L$ is the column height, $B$ and $H$ are the width and depth of the column section respectively, and $\rho_l$ and $\rho_s$ are the longitudinal and transversal reinforcement ratios respectively. The statistical properties of the experimental results are described in Table II. The frequency histograms of input parameters and failure modes of the 735 data samples are shown in Figure 2.

### IV. Evaluation of the Calculated Shear Strength of the RC Columns

To quantitatively evaluate the shear strengths calculated from models in Table I, statistical parameters are employed including coefficient of determination ($R^2$) and Root-Mean-Squared Error ($RMSE$). $R^2$ value represents the percentage of data close to the regression line. The higher the $R^2$, the more the accuracy of the calculated model and vice versa. $RMSE$ is used for quantifying the difference (error) between the calculated and the experimental value. The smaller the $RMSE$, the more the accuracy of the calculated model and vice versa. The definitions of $R^2$ and $RMSE$ are described in (7) and (8).

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{n}(t_i - \hat{a}_i)^2}{\sum_{i=1}^{n}(t_i - \bar{t}_i)^2}\right)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (t_i - \hat{a}_i)^2}$$

where $t_i$ and $\hat{a}_i$ are the test and calculated results of the $i$ sample, $n$ is number of the samples of the database, $\bar{t}$ is the mean of calculated results.
respectively. Besides, the mean values of the properties of the ratio \( q_{Gnr} \) with the highest determined. The results show that EC8 [6] is the most accurate Deviation (SD), and Coefficient of Variation (CV) are which minimum (Min), maximum (Max), Mean, Standard Deviation (SD), and Coefficient of Variation (CV) are determined. The results show that the models of EC8, Sezen & Moehle, and CSA provide a smaller scattering than others, and results are mostly under the 1:1 line. In other words, the calculated results are smaller than those of experiments. Moreover, the results based on ACI 318, FEMA 273, and Ascheim & Moehle have a bigger scattering. These deviations may be caused by the ductility ratio and the diagonal compressive force in the columns.

Table III summarizes the calculated statistical parameters, i.e. \( R^2 \) and RMSE for the investigated models. Additionally, the properties of the ratio \( V_{\text{calculate}}/V_{\text{test}} \), are also obtained, in which minimum (Min), maximum (Max), Mean, Standard Deviation (SD), and Coefficient of Variation (CV) are determined. The results show that EC8 [6] is the most accurate model with the highest \( R^2 \) value (0.892) and the smallest RMSE (= 163kN). Moreover, [9] and [5] are also good models with \( R^2 = 0.766 \) and 0.68, and RMSE = 185kN and 220kN respectively. Besides, the mean values of \( V_{\text{calculate}}/V_{\text{test}} \) of those models are smaller than 1.0, thus, the results calculated by [5, 6, 9] are conservative. It can be seen that the equations of [4, 7, 8] provide lower accuracy. Based on the results, we suggest the use of equations of the models of [5, 6, 9] for calculating the shear strength of rectangular RC columns.

Figure 3 shows the comparison between the shear strengths of the RC columns calculated by the six models and the test results. The dash line represents the 1:1 line, while the red line is the regression. It can be seen that the models of EC8, Sezen & Moehle, and CSA provide a smaller scattering than others, and results are mostly under the 1:1 line. In other words, the calculated results are smaller than those of experiments. Moreover, the results based on ACI 318, FEMA 273, and Ascheim & Moehle have a bigger scattering. These deviations may be caused by the ductility ratio and the diagonal compressive force in the columns.

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<table>
<thead>
<tr>
<th>Model</th>
<th>( R^2 )</th>
<th>RMSE</th>
<th>Statistical properties of ( V_{\text{calculate}}/V_{\text{test}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318 [4]</td>
<td>0.608</td>
<td>202</td>
<td>Min 0.19 Max 20.88 Mean 1.40 SD 1.42 CV 1.01</td>
</tr>
<tr>
<td>CSA [5]</td>
<td>0.680</td>
<td>220</td>
<td>Min 0.18 Max 9.88 Mean 0.93 SD 0.74 CV 0.80</td>
</tr>
<tr>
<td>EC8 [6]</td>
<td>0.892</td>
<td>163</td>
<td>Min 0.07 Max 4.84 Mean 0.64 SD 0.37 CV 0.57</td>
</tr>
<tr>
<td>FEMA 273 [7]</td>
<td>0.448</td>
<td>238</td>
<td>Min 0.07 Max 19.62 Mean 1.18 SD 1.33 CV 1.15</td>
</tr>
<tr>
<td>Ascheim &amp; Moehle [8]</td>
<td>0.476</td>
<td>251</td>
<td>Min 0.21 Max 35.78 Mean 2.14 SD 2.41 CV 1.13</td>
</tr>
<tr>
<td>Sezen &amp; Moehle [9]</td>
<td>0.766</td>
<td>185</td>
<td>Min 0.09 Max 8.51 Mean 1.02 SD 0.71 CV 0.69</td>
</tr>
</tbody>
</table>

V. Conclusions

This study presents and analyzes six code- and empirical-based models for the shear strength calculation of rectangular RC columns. A set consisting of 735 experimental data samples of rectangular RC columns was collected. The accuracy of the models is evaluated by statistical parameters. The following conclusions are drawn from the current study:

- EC8 [6] is the most accurate model for estimating the shear strength of rectangular RC columns, followed by Sezen & Moehle [9] and CSA [6].

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The equations of ACI 318 [4], FEMA 273 [7], and Ascheme & Moehe [8] have a lower accuracy. It should be noted that this study considered rectangular RC columns. For other RC column types, such as circular or hollow sections, further investigation is required.

REFERENCES

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