

# Reliability Analysis of an Uncertain Single Degree of Freedom System Under Random Excitation

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**Abstract**—In practical engineering problems, uncertainty exists not only in external excitations but also in structural parameters. This study investigates the influence of structural geometry, elastic modulus, mass density, and section dimension uncertainty on the stochastic earthquake response of portal frames subjected to random ground motions. The North-South component of the El Centro earthquake in 1940 in California is selected as the ground excitation. Using the power spectral density function, the two-dimensional finite element model of the portal frame's base motion is modified to account for random ground motions. A probabilistic study of the portal frame structure using stochastic finite elements utilizing Monte Carlo simulation is presented using the finite element program ABAQUS. The dynamic reliability and probability of failure of stochastic and deterministic structures based on the first-passage failure were examined and evaluated. The results revealed that the probability of failure increases due to the randomness of stiffness and mass of the structure. The influence of uncertain parameters on reliability analysis depends on the extent of variance in structural parameters.

**Keywords**—reliability; Monte Carlo simulation; uncertain system; random excitation; stochastic; finite element analysis

## I. INTRODUCTION

In practical engineering problems, the external excitations, such as wind loading and seismic waves, and the parameters of a structure exhibit uncertainty. Structural parameter uncertainty may strongly influence structural response and reliability [1]. Earthquakes are the most disastrous natural phenomena. Therefore, the seismic response of many types of structures and buildings has been widely investigated. However, most modeling attempts of seismic random response analysis of structures belong to deterministic models in which all structural parameters were regarded as deterministic parameters. From another aspect, most engineering structures can be classified as random due to the variability in their geometric or material properties. Therefore, the problem of stochastic structures subject to stochastic seismic excitation is of great importance [2]. Considering that ground motion induced by an earthquake represents a type of random excitation, the theory and methods of random vibration should be applied to analyze the seismic

response of structures. The random excitation is usually specified regarding its Power Spectral Density (PSD) [3]. The random vibration theoretical framework has been well established. The dynamic analysis of systems with deterministic structural parameters to random excitations is available. However, the dynamic analysis of systems with stochastic structural parameters under random excitations has not been developed to the same extent [4]. A natural frequency represents one of the most influencing parameters on system response. Uncertainty in the natural frequency can arise from uncertainties in the stiffness or inertia properties of the structure. In this case, probabilistic-based analysis methods should be utilized. Thus, the considered statistical parameters associated with the distribution of random variables should be determined. In general, the stochastic finite element-based method is a probabilistic analysis method and is well suited to deal with random parameter problems [5]. Given the random nature of loading, material specifications, and implementation issues, probabilistic-based analysis should be utilized. Thus, considering the statistical parameters associated with the distribution of random variables should be determined. The reliability-based analysis is a new approach to structural analysis and design that takes uncertainty into account [6].

A survey of previous studies indicated that structural reliability methods have been mainly developed for rationally evaluating the safety of deterministic structures with excitation. However, the studies on nondeterministic structures to investigate statistical uncertainty characteristics are limited. The current study emphasizes on reliability analysis of a stochastic structure with uncertain parameters and excitation to assess the reliability and safety of this system. To achieve the goal of this study, a single degree of freedom system subject to seismic base excitation is examined using a probabilistic finite element ABAQUS code using Monte Carlo Simulation (MCS) and Python script to generate pseudo-random values for the considered random parameters.

## II. RANDOM EXCITATION

The stochastic earthquake analysis in this study is based on the stationary assumption, in which the statistical parameters'

mean and variance do not vary with time. A stationary model makes them less sophisticated, simplifies computations, and gives satisfactory results [7]. As a single record is insufficient for producing general conclusions, an ergodicity assumption is applied. Moreover, only one earthquake record from the local area can be utilized. The PSD function of acceleration seismic motion is assumed to be in the form of a filtered Gaussian white noise ground motion. The model was suggested in [8, 9] and was later modified in [10]. This model can be expressed as:

$$S_{\ddot{X}_g}(\omega)_{CP} = \frac{1+4\xi_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+4\xi_g^2(\omega/\omega_g)^2} \frac{(\omega/\omega_f)^2}{[1-(\omega/\omega_f)^2]^2+4\xi_f^2(\omega/\omega_f)^2} S_0 \quad (1)$$

where  $\xi_g$ ,  $\omega_g$ ,  $\xi_f$ , and  $\omega_f$  represent the damping ratio and natural frequency of the soil and high pass filter respectively, and  $S_0$  is the intensity of the white noise of ground motion. To estimate the filter parameters, the method of the spectral moment [11] is the key statistical parameter of the PSD function [12]. The  $i^{\text{th}}$  spectral moment  $\lambda_i$  is defined as:

$$\lambda_i = \int_0^\infty \omega^i G(\omega) d\omega \quad (2)$$

The variance of the excitation is the zero spectral moment:

$$\lambda_0 = \sigma_0^2 = \int_0^\infty G(\omega) d\omega \quad (3)$$

The central frequency,  $\omega_c$ , and the shape factor,  $\delta$ , of the random process can be directly evaluated from the first few spectral moments:

$$\omega_c = \sqrt{\lambda_2/\lambda_0}, \quad (4)$$

$$\delta = \sqrt{1 - (\lambda_1^2/\lambda_2\lambda_0)} \quad (5)$$

As the central frequency and shape factor are functions of the spectral moments ( $\lambda_0$ ,  $\lambda_1$ , and  $\lambda_2$ ), they are expressed in terms of the filter parameters, i.e.  $\omega_g$ ,  $\xi_g$ , and  $S_0$ . Hence, they can be computed by matching the variance of acceleration, the central frequency, and the shape factor of the actual and theoretical PSD.

### III. STATIONARY RANDOM VIBRATION ANALYSIS

The equation of motion for a Single Degree of Freedom (SDOF) structure subjected to random ground acceleration is:

$$m\ddot{X}(t) + c_0\dot{X}(t) + kX(t) = -m\ddot{X}_g(t) \quad (6)$$

where  $\ddot{X}_g$ ,  $m$ ,  $c_0$ , and  $k$  are the ground acceleration process, structural mass, viscous damping, and elastic stiffness respectively. The PSD of the displacement response may be represented as in (7) if the ground motion acceleration ( $\ddot{X}_g$ ) is considered a stationary Gaussian random process [10].

$$S_{yy}(\omega) = H(\omega) H(\omega)^* S_{\ddot{X}_g}(\omega) \quad (7)$$

where  $S_{\ddot{X}_g}(\omega)$  is the PSD function of the ( $\ddot{X}_g$ ),  $H(\omega)$  is the frequency response function as in (8), and (\*) stands for complex conjugate:

$$H(\omega) = \frac{1}{\omega_j^2 - \omega^2 + 2i\xi_j\omega_j\omega} \quad (8)$$

where  $\omega_j$  and  $\xi_j$  are the  $j^{\text{th}}$  order inherece frequency and mode damping of structure respectively. The mean square and the root mean square of the relative displacement can be expressed as in (9) and (10) respectively [10]:

$$\sigma_x^2 = \int_{-\infty}^\infty S_{yy}(\omega) \quad (9)$$

$$\sigma_x = \sqrt{\int_{-\infty}^\infty S_{yy}(\omega)} \quad (10)$$

### IV. MONTE CARLO SIMULATION

Although being a highly time-consuming computational method, the MCS method is considered one of the most powerful and accurate simulation tools to estimate numerically the reliability and failure probability of uncertain structures [18]. This method is used to calculate the response uncertainty and the numerical estimate of failure probability. It uses random sampling from random variable distributions. The "crude" or "direct" MCS, which is a pseudo-random sampling, is the basic version [13]. In MCS, the failure probability is described as:

$$\hat{p}_f = \frac{N_f}{N} \quad (11)$$

where  $N$  is the total number of samples and  $N_f$  is the number of samples in the failure domain.

MCS is a most general approach for the Stochastic Finite Element Method (SFEM) [14]. The deterministic FEM and the MCS technique are merged in this methodology. SFEM can express randomness in one or more of the main components of the classic FEM, such as geometry, material properties, and external forces [15].

### V. DYNAMIC RELIABILITY

The reliability of a system is closely related to the concept of level crossing. This case is particularly true for first-passage failure, in which the system is considered to fail only when a particular stress process or displacement  $X(t)$  reaches a critical level  $b$  in the time interval  $[0, T]$ . When the structural response of a deterministic structural parameter is a stationary Gaussian process, the crossing time of the response  $x(t)$  and limit  $b$  submit to the Poisson process. The dynamic probability of failure of a SDOF structure can then be obtained from [16]:

$$p_f(t) \approx 1 - \exp\left\{-v_0 T \exp\left[-\frac{1}{2}\left(\frac{b}{\sigma_x}\right)^2\right]\right\} \quad (12)$$

$$R(t) \approx \exp\left\{-v_0 T \exp\left[-\frac{1}{2}\left(\frac{b}{\sigma_x}\right)^2\right]\right\} \quad (13)$$

where  $T$  is the duration of the stationary process,  $\sigma_x$  is a root mean square of the response, and  $v_0$  is zero mean cross rate expressed as follows:

$$v_0 = \frac{1}{2\pi} \sqrt{\frac{\lambda_2}{\lambda_0}} \quad (14)$$

where  $\lambda_0$  and  $\lambda_2$  are zero and the second spectral moment respectively, defined as follows:

$$\lambda_m = \int_0^{\omega_c} \omega^m G_s(\omega) d\omega \quad \text{For } m = 0,1,2 \quad (14)$$

When the structural parameters and the excitation are random, the system reliability may be evaluated by calculating the probability of an equivalent extreme-value event. Hence, the seismic excitation and structural response are assumed to have a zero mean.  $y_e$  and  $\sigma_x$  are the extreme value and standard deviation of structural response  $y(t)$  respectively. The dimensionless parameters are explained below [16].

$$\eta = \frac{y_e}{\sigma_x} \quad (15)$$

Assuming a Poisson process for the number of horizontal crossings, taking parameter uncertainty into account, the estimated mean of the extreme value is:

$$E(\eta) = \left( \sqrt{2 \ln v_0 T} + \frac{0.5772}{\sqrt{2 \ln v_0 T}} \right) \quad (16)$$

and the variance of  $\eta$  is:

$$\sigma^2(\eta) = \frac{\pi^2}{6} \frac{\sigma_x^2}{(2 \ln v_0 T)} \quad (17)$$

The extreme value of the stochastic process  $y(t)$  is expressed as:

$$y_e = E(\eta) \times \sigma_x \quad (18)$$

The limit state function of the inter-story drift system is expressed as:

$$G(\Delta) = R(\Delta) - Q(\Delta) \quad (19)$$

where  $R(\Delta)$  is the structure drift limit equal to 0.01, 0.015, and 0.02 from story height, and  $Q(\Delta)$  represents the extreme value of the structural drift because of the loading, including the uncertainties of the structural parameters. Limit state function  $G(\Delta) \leq 0$  is the failure state, and  $G(\Delta) > 0$  is a safe state. Table I shows the target reliability of the steel structure system.

TABLE I TARGET RELIABILITY INDICES [17]

Component type	Loading condition		
	D+(L or S)	D+L+W	D+L+E
Members	3.0	2.5	1.75
Connections	4.5	4.5	4.5

## VI. NUMERICAL EXAMPLE

A numerical example is presented and analyzed to demonstrate the reliability analysis for a case study of a simple frame structure, as shown in Figure 1. The stochastic response due to the uncertainty in structure physical properties and seismic excitation force is taken into account.

### A. Proposed Structural System

Uncertainty and reliability analysis have been executed for an interior frame of the shear building system, as shown in Figure 1. The floor system consists of a concrete slab 200mm thick supported by 3 steel girders with a W12x190 cross-section and the girders are supported by steel columns with a W10x33 cross-section, as shown in Figure 2. In addition to the own weight, uniformly distributed pressures of 2 and 1kPa were adopted for the superimposed and live loads respectively.

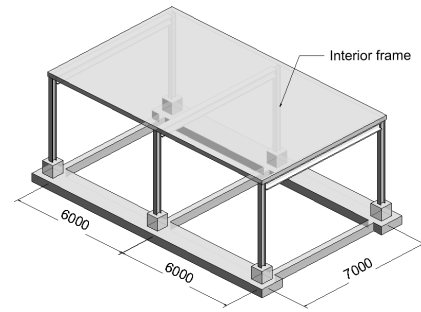


Fig. 1. One-story two-bay frame.

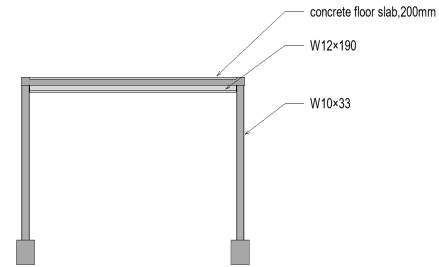


Fig. 2. Interior frame.

### B. Random Ground Motion

In this study, the North-South component of the 1940 El Centro earthquake was chosen as the ground motion. Using the spectral moment method, the acceleration spectral density function parameters of filtered white noise ground motion have been estimated, as shown in Figure 3. The calculated values of natural frequency and the damping ratio for the first and second filters were  $\omega_g = 18.5 \text{ rad/s}$ ,  $\xi_g = 0.43$ , and  $\omega_f = 1.5 \text{ rad/s}$ , and  $\xi_f = 0.6$  respectively. The intensity factor of the earthquake was  $S_0 = 0.00282 \text{ m}^2/\text{s}^3$ .

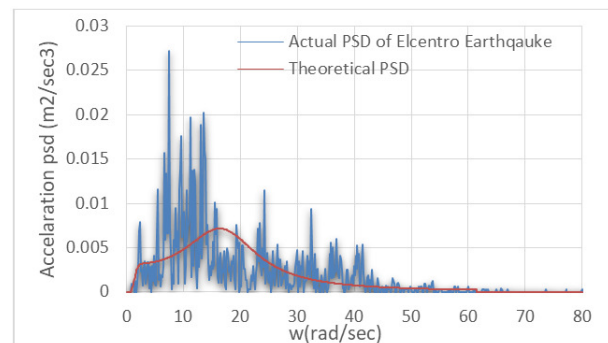


Fig. 3. Actual and theoretical PSD of the El-Centro earthquake.

### C. Stochastic Earthquake Analysis with Deterministic Structural Parameters

The dynamic characteristics of the deterministic portal frame structures have been analyzed with ABAQUS finite element software [18]. The two-dimension portal frame has been modeled with wire part B21, as shown in Figure 4. The inertia mass has been selected based on the effective weight of the concrete slab, superimposed, and a quarter of the live load according to ASCE7-16. The girder and columns were

discretized using a mesh size of 100mm. A rigid body constraint has been adopted for the girder. In addition, the density of the column has been reduced to achieve the assumption of a shear frame of the rigid girder supported by weightless columns with fixed supports. The structure's natural frequency was obtained by modal analysis, and two values were extracted, as shown in Figure 5 and Table II.

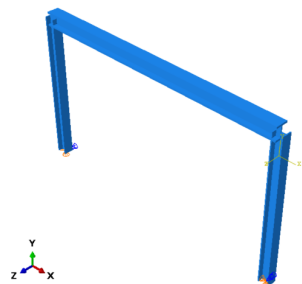


Fig. 4. 3D portal frame view.

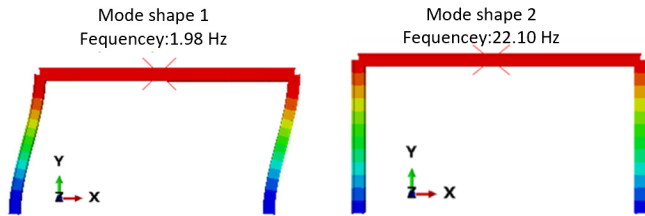


Fig. 5. First two mode shapes.

TABLE II NATURAL FREQUENCY AND MASS PARTICIPATION FACTOR

Mode	Frequency (Hz)	Frequency (rad/sec)	Mass participation factor	Direction
1	1.98	12.46	99%	Along x-direction
2	22.10	138.85	99%	Along y-direction

Random vibration analysis was conducted with a base motion of the portal frame in the x-direction. The frequency range of interest and the response have been set. Hence, the PSD of the relative displacement was obtained as shown in Figure 6. One peak detected in the vibration responses refers to the resonance occurrence at the first natural frequency of the investigated system.

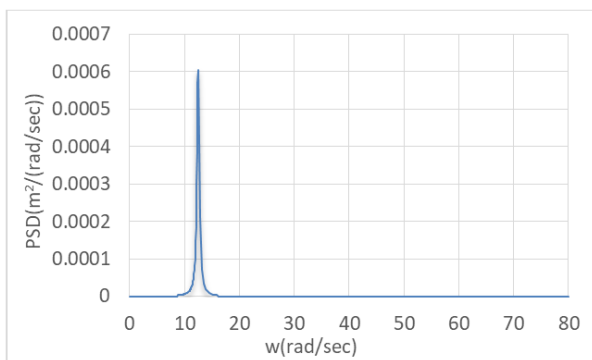


Fig. 6. Response PSD of relative displacement.

By integrating the response PSD, the mean square and Root Mean Square (RMS), of relative displacement were  $7.48 \times 10^{-5} \text{m}^2$  and  $8.65 \times 10^{-3} \text{m}$  respectively. The displacement resistance limit of the structure has been taken from ASCE 7–16 structural design codes, where the allowable story drift is related to the risk categories and the structural system. Thus, this research intends to investigate the different allowable limits, i.e. 1.0%, 1.5%, and 2%, of story height. The probability of failure of the system has been estimated for 3 response intervals of 10, 15, and 20s, as shown in Figure 7. Based on this estimate, the reliability index was obtained and is presented in Table II. The results showed that the probability of failure slightly increased when the response time increased. Therefore, the reliability index of the system was not significantly affected. From another aspect, comparing the reliability index with the target reliability shown in Table I reveals that the structure meets the specified safety level.

TABLE III FAILURE PROBABILITY AND RELIABILITY INDEX FOR A DETERMINISTIC SYSTEM

Threshold	10s		15s		20s	
	pf	$\beta$	pf	$\beta$	pf	$\beta$
0.01H	$1.11 \times 10^{-3}$	3.06	$1.66 \times 10^{-3}$	2.94	$2.21 \times 10^{-3}$	2.85
0.015H	$5.32 \times 10^{-9}$	5.72	$7.98 \times 10^{-9}$	5.65	$1.06 \times 10^{-8}$	5.60
0.02H	$\approx 0$	–	$\approx 0$	–	$\approx 0$	–

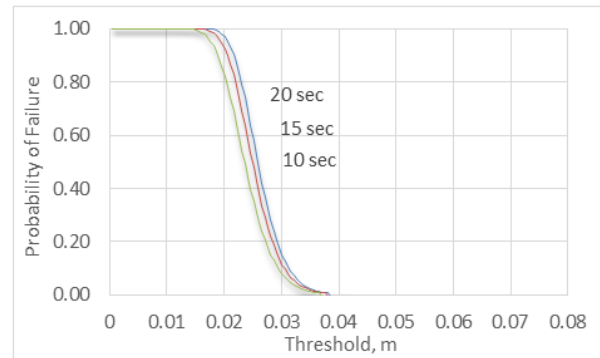


Fig. 7. First-passage failure for the three-time response interval.

D. Stochastic Earthquake Analysis with Nondeterministic Structural Parameters

To illustrate the effect of the randomness of structural parameters, including stiffness  $k$  and mass  $m$  on the natural frequency and random seismic response, MCS was performed to update the random variables of interest for each Finite Element Analysis (FEA) trial. Python programming was used to develop the deterministic FE model, and then, the random input variables of interest were updated based on the idea of parameter updating functionality. In this study, the cross-section dimensions, modulus of elasticity, column length, and the applied load were considered as random variables. Table IV shows the statistical characteristics of these parameters.

Probabilistic modal analyses of random structural parameters of the interior portal frame were estimated using SFEM with python script coding. Matlab function was used to generate 5000 pseudo-random samples of cross-section dimensions, modulus of elasticity, column length, and

structural effective mass. Figure 8 depicts the natural frequency result obtained from the sample data. Mean value, standard deviation, and coefficient of variance were 12.34rad/sec, 1.3576, and 0.109 respectively. Notably, the mean value for the natural frequency is very close to that shown in Table II for the deterministic analysis, indicating the validity of the dynamic analysis with random properties and excitation. Due to the randomness in the natural frequency, the system response was affected.

Figure 9 shows the RMS of relative displacement. Mean, standard deviation, and Cov were 0.0105m, 0.0016, and 0.1483 respectively. The results for the coefficient of variance indicate the effectiveness of randomness in the stochastic structure properties and excitation on the response properties.

TABLE IV STATISTICAL CHARACTERISTICS OF VARIABLES

Random variables		Mean/nominal	COV	Distribution type	Ref.
Cross section dimension	Depth of the web	1.0009	0.004	Normal	[20]
	Width of the flange	1.0139	0.009	Normal	
	Thickness of the flange	0.9927	0.044	Normal	
	Thickness of the web	1.054	0.037	Normal	
Modulus of elasticity		0.993	0.034	Normal	[21]
Column length		1	0.07	Lognormal	[22]
Load	Weight of a girder	1.03	0.1	Normal	[23]
	Weight of a slab	1.05	0.1	Normal	
	Superimposed load	1.03	0.1	Normal	
	Live load	1	0.1	Gumbel	

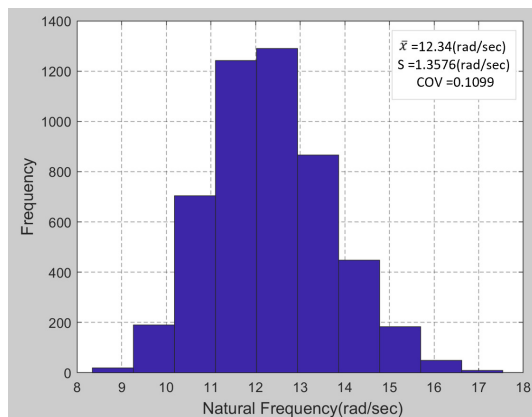


Fig. 8. Histogram of the first natural frequency.

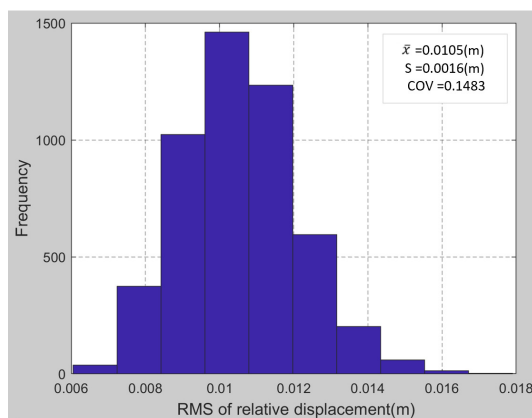


Fig. 9. Histogram of the RMS of the relative displacement.

Figures 8 and 9 show the failure probability estimates for the threshold level of 1%, and Table IV presents the summary. Notably, the Monte Carlo estimate for 1.5% and 2% levels is not shown in the Figure because the sample size is not large enough to provide sufficiently accurate estimates for the probability of failure corresponding to this threshold level. The results revealed that randomness in the system's stiffness and mass influences the system's reliability. Moreover, generally, the probability of failure increased due to the randomness in the stiffness and mass of the structure. This conclusion is confirmed by comparing the results presented in Tables III and V for deterministic and stochastic systems. The Probability Density Function (PDF) in Figure 10 shows that the distribution ranges of the equivalent extreme value have a trend of moving toward the right-hand side with increasing response time intervals. This case accords to the trends of increasing the probability of failure. The cumulative density function in Figure 11 shows the boundary between the safe and failure domains, which is described by the limit of 0.04m on the x-axis.

TABLE V FAILURE PROBABILITY AND RELIABILITY INDEX FOR A STOCHASTIC SYSTEM

Threshold	10s		15s		20s	
	pf	$\beta$	pf	$\beta$	pf	$\beta$
0.01H	$2.8 \times 10^{-3}$	2.77	$9.4 \times 10^{-3}$	2.34	$21.4 \times 10^{-3}$	2.02

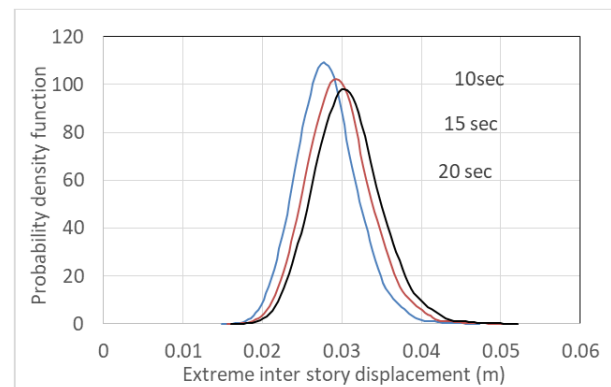


Fig. 10. Probability density function for the extreme value of drift.

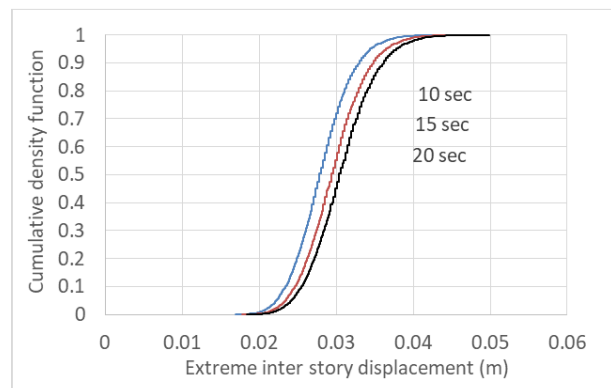


Fig. 11. Cumulative density function for the extreme value of drift.

## VII. CONCLUSIONS

In this study, a reliability analysis of portal frames excited by random ground motion with deterministic and stochastic structural parameters was performed. The following conclusions can be drawn:

- The probability of failure and the reliability index of the deterministic structure were affected slightly by the excitation time interval. The probability of failure increased and the reliability index decreased with increasing time interval.
- The reliability index of the deterministic structure was greater than the target reliability index for members subject to seismic base motion.
- The results for the mean values of the dynamic response for the SDOF system with random properties and excitation correlate well with the deterministic analysis results.
- Randomness in the system's stiffness and mass influences the system's reliability and probability of failure. Generally, the probability of failure increased due to the randomness in stiffness and mass of the structure.

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