Improved Optimization of the Charge Simulation Method for the Calculation of the Electric Field Around Overhead Transmission Lines Using Statistical Methods

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Abstract-In order to decide the appropriate arrangements of fictitious charges in the charge simulation method, the use of the Monte Carlo method is proposed for the estimation of the probability density function of two variables, the radius ratio, and the angle ratio. The scale and shape parameters of the Weibull's distribution are determined by the maximum likelihood estimator. The obtained results are used to calculate the electric field at arbitrary points in the neighborhood of high voltage transmission lines. The comparisons between the results computed by this method, the results calculated by the genetic algorithm, and those measured, confirm the effectiveness and accuracy of the proposed method.

Keywords-charge simulation method; Monte Carlo method; optimization; genetic algorithm; high voltage transmission lines; electric field calculation

I. INTRODUCTION

Designing any high voltage device and analyzing discharge phenomena requires complete knowledge of electric and magnetic field distribution [1]. The potential surface gradient is a critical design parameter for planning and designing overhead lines (insulation or discharge) [2, 3]. The electric fields can be calculated using several analytical and numerical methods. The most used is the Charge Simulation Method (CSM) [4-9]. The CSM was introduced in 1969 [4]. Its basic concept is to replace the distributed charge of conductors and the polarization charges on the dielectric interfaces with a large number of fictitious discrete charges. The magnitudes of these charges have to be calculated so that their integrated effect satisfies the boundary conditions precisely at a selected number of points on the boundary. The principle of this method is to simulate an actual field with a field formed by a finite number of simulation charges (point and line charges of infinite and semi-infinite length [11]) placed outside the region where the field is to be calculated. The values of the discrete charges are determined by satisfying the boundary conditions at a selected number of contour points:

$$[Qs] = [P]^{-1}[V_b] \quad (1)$$

where $[V_b]$ is the vector of contour point voltages, $[Qs]$ is the vector of unknown simulation charges, and $[P]$ is the matrix of potential coefficients calculated by contour points and simulation charges.

For overhead lines consisting of n parallel conductors placed above the ground, the elements of the matrix of potential coefficients are given by the following relation:

$$P_{ij} = \frac{1}{2\pi\varepsilon_0}\ln\left(\frac{D_{ij}}{d_{ij}}\right) \quad (2)$$

where ε_0 is the electric permittivity of vacuum ≈8.854 10^{-12}F/m, D_{ij}, d_{ij} are respectively the distance between the jth point charge and the image of the ith point charge and the distance between the jth point charge and the ith point charge.

Based on the Laplace equation (3), the superposition theorem and image charge theory, the components at an arbitrary point in the plane y-z plane produced by n point charges $M(y,z)$ can be calculated by (4) and (5):

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$$\Delta V = -\frac{\rho}{\varepsilon_0} \quad (3)$$

$$E_y(y,z) = \sum_{i=1}^{n} \frac{Q_i}{2\pi\varepsilon_0}\left(\frac{y-y_i}{R_i^2} + \Gamma\,\frac{y-y_i}{R_i^2}\right) \quad (4)$$

$$E_z(y,z) = \sum_{i=1}^{n} \frac{Q_i}{2\pi\varepsilon_0}\left(\frac{z-z_i}{R_i^2} + \Gamma\,\frac{z+z_i}{R_i^2}\right) \quad (5)$$

where Γ is the reflection coefficient of soil's surface, R_i is the distance between the arbitrary point $M(y,z)$ and the ith point charge, y_i and z_i are the coordinates of the ith contour point, and Q_i is the ith fictitious charge. The reflection coefficient of the soil can be approximated as $\Gamma = -1$.

The current paper aims to optimize CSM parameters using stochastic optimization employing the Monte Carlo Method (MCM). This optimization is based on estimating the Probability Density Functions (PDFs) of the polar coordinates of the simulation charges where the relative mean square error of the voltage on the conductor surface is less than a threshold. To ensure the accuracy of this method, a comparison is made with the measured values of the electric field at arbitrary points near high voltage transmission lines with standard dimensions of the tower on the 40kV line SS Sarajevo 10 –SS Sarajevo 20 [10].

II. RELATED WORK

Since 1969, when the CSM was used for the first time [4], it has been applied and developed in many cases. Some of the main contributions, in chronological order, are:

Authors in [12] used CSM combined with the Rosenbloom's method to solve the potential distribution of the rod-plane. Authors in [13] calculated the field distribution for multi-phase AC sources or in configurations including volume resistance. Authors in [5] simulated the sheathed three cores belted power cable using the complex fictitious charges. Authors in [14] combined CSM with the Genetic Algorithm (GA) to optimize the CSM for a 2D electrode system with an asymmetrical structure. Authors in [15] used CSM-GA to calculate the electric field of a 35kV Vacuum Interrupter (VI). GA has been utilized to compute the electric field [16], to model the horizontal sphere gap [17], and to model the horizontal sphere gap above the ground plane [18]. Authors in [19] calculated the electric field around the head of a transmission tower and its composite insulators by coupling CSM with BEM. Authors in [20] used an optimization strategy to arrange the simulated charges in the thin electrode. Authors in [21] combined CSM with GA to solve the inverse problem in electric-fields of high voltage insulators. Authors in [22] combined CSM with Hashing integrated Adaptive GA (HAGA) to the contour design of support insulators. Authors in [23] used CSM combined with the gold section method to calculate the conductors' surface electrical field of ±800kV UHVDC transmission lines. Authors in [24] used CSM-GA to enhance the computation precision of electric fields associated with plate-type electrostatic separators. An adapting Particle Swarm Optimization (PSO) combined with CSM was used for calculating the field distribution with non-axial symmetry resulting from a floating spherical conductor between the spheres in [25]. Authors in [26] improved the calculation accuracy of the electric fields associated with electrostatic plate separators by using CSM-GA. For the optimization of high voltage electrode surfaces, authors in [27] used CSM combined with a Biogeography-based algorithm. Authors in [28] used CSM-PSO for sphere-plane gaps. 3D calculation of electric field intensity under transmission lines with CSM-PSO and CSM-GA was conducted in [29]. Authors in [30] made a comparison between the performance of PSO, GA, and Grey Wolf Optimizer (GWO) in 3D quasi-static modeling of the electric field produced by High Voltage (HV) overhead power lines. To optimize the ion flow field calculation, authors in [31] used CSM combined with the Flux Tracing Method (FTM).

III. THE PROPOSED ALGORITHM

A. *Intoduction*

The proposed algorithm is based on Stochastic Optimization (SO) methods. The SO methods generate and use random variables [32]. They are used in many areas, including aerospace, medicine, transportation, finance, electrical engineering, and many more science and engineering fields. SO can rely on sampling methods such as MCM [33], Latin hypercube sampling [34], or the Quasi-Monte Carlo Method (QMCM) [35]. The algorithm aims to optimize the location of fictitious charges by generating a bivariate distribution of N×N random variables $\langle C_r, C_a\rangle$ which are respectively the ratio between r_c and r_b, θ_c and θ_b according to (6)-(8). As shown in Figure 1, the contour points are arranged at equal distances on the perimeter of the conductor and are determined by their polar coordinates r_c and $\theta^k{}_b$ according to (7)-(8). The simulation charges are also arranged at an equal distance on the perimeter of a virtual circle inside and are determined by their polar coordinates r_c and θ_c^k.

$$\theta_b^k = \frac{2\pi k}{n_c}(k-1)\ , \qquad k = 1\ to\ N_c \quad (6)$$

$$r_c = C_r r_b \quad (7)$$

$$\theta_c^k = \theta_b^k + C_a.\frac{2\pi}{N_c} \quad (8)$$

where r_b is the radius of the conductor, $\theta^k{}_b$ the angle of the k^{th} contour point, N_C the number of contour points, r_c the radius of the virtual circle that contains simulation charges, $\theta^k{}_c$ the angle of the k^{th} fictitious charge, and C_r and C_a the radius and angle ratios ranging between 0 and 1.

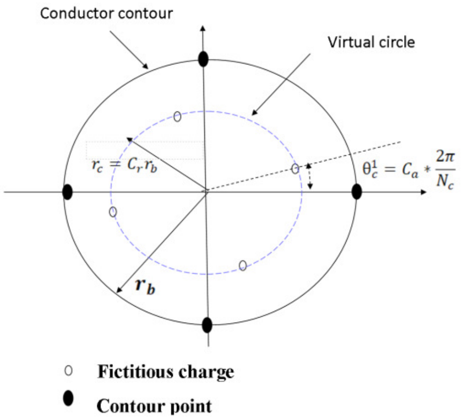

Fig. 1. Arrangement of contour points, fictitious charges, and test point.

B. The Algorithm

For each iteration, we have a set of coordinates of fictitious charges, denoted by (C_r^i, C_a^i) such that:

The set of all coordinates is: $\mathcal{E} = \langle C_r, C_a \rangle$ and the set of acceptable coordinates is $\tilde{\mathcal{E}} = \langle \tilde{C}_r, \tilde{C}_a \rangle$. The work is carried out in two steps.

1) Step 1: Extraction of the Set of Solutions $\tilde{\mathcal{E}}$

Data: The electrical and geometric parameters of the line such as the potential of point contour V_b, the number of conductor n_c, the number of fictitious charges N_c, the objective function threshold $Fobj_{threshold}$, the number of iterations N which generate N random (C_r, C_a) pairs.

From i=1 to N

Do

Calculate N_c*n_c coordinates of fictitious charges with (6) and (7).

Calculate the potential created by these fictitious charges with (2).

Calculate the fictitious charges with (1).

Calculate the potential created by these fictitious charges:

$$[V_t] = [P_t][Qs]$$

Calculate the objective function *Fobj:*

$$FObj = \frac{1}{n}\sum (V_t - V_b)^2$$

Compare *Fobj* with $Fobj_{threshold}$

If $Fobj > Fobj_{threshold}$ then (C_r, C_a) is rejected.

Else add (C_r, C_a) to the $\tilde{\mathcal{E}}$

2) Step 2: Statistical Study

The followed steps are:

1. Establish the histograms of C_r and C_a
2. Estimate the Weibull law parameters *A* and *B* with the Maximum Likelihood Estimator (MLE).
3. Calculate the mean and standard deviation of C_r and C_a

The above algorithm is executed for a simple geometry problem (Figure 2) where n_c=2, N_c=3, N=100, $Fobj_{threshold}$=4×10^{-12}, h=11m, d=2m, r_b=7cm, and V=400kV. After the iterations are completed, there are 26 accepted bivariates (C_r, C_a) and their histograms are shown in Figures 3 and 4. It is quite obvious that the greatest PDF is concentrated around 0.95 for C_r and 0.49 for C_a. From the obtained results, it should be noted that the shapes of the two histograms are asymmetrical. The obtained data of the first histogram are grouped near the upper limit and incline to the left towards the lower values. On the other hand, in the second histogram, the data are grouped towards the center, which leads to estimating the two parameters of the Weibull distribution as follows.

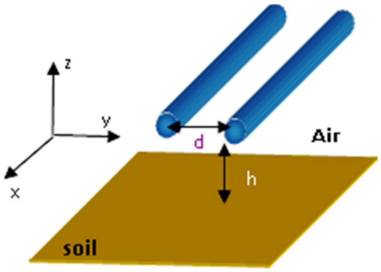

Fig. 2. Histogram geometry problem.

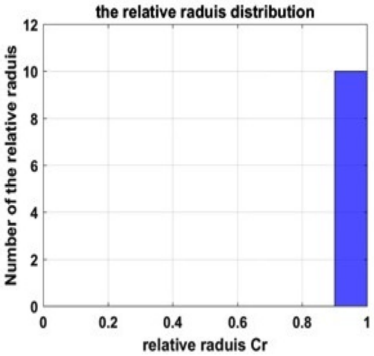

Fig. 3. Histogram of C_r.

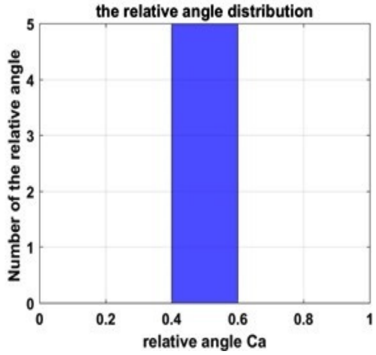

Fig. 4. Histogram of C_a.

The Weibull distribution is used in reliability studies, for example, to study the voltage breakage of electric circuits [36]. The Weibull distribution has two parameters, denoted in the following equation:

$$f(x|A,B) = B.A^{-B}x^{B-1}e^{-\left(\frac{x}{A}\right)^B} \quad (10)$$

where $A>0$ is the scale parameter and $B>0$ is the shape parameter of the distribution.

The Maximum Likelihood Estimator (MLE) [37] estimates the Weibull parameters *A* and *B*. The results are given in Table I and the estimate distributions are shown in Figures 5 and 6.

TABLE I. ESTIMATED WEIBULL PARAMETERS

Data	*A*	*B*
C_r Relative radius	0.968882	31.02033
C_a Relative angle	0.507651	27.0408

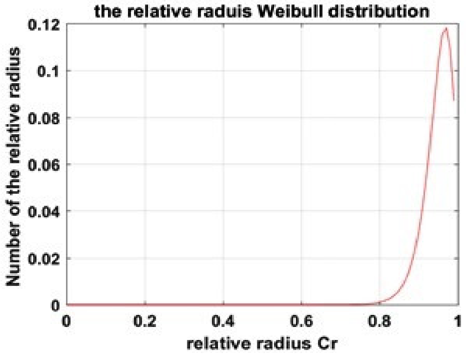

Fig. 5. Estimate distribution of C_r.

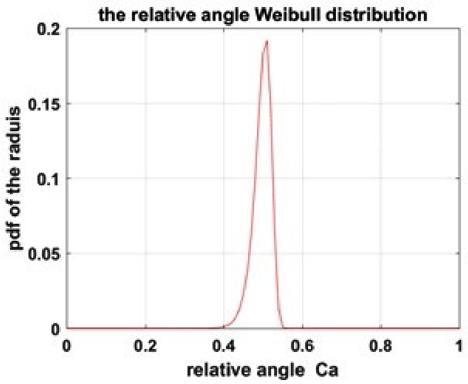

Fig. 6. Estimate distribution of C_a.

The estimated distributions shown in Figures 6 and 7 agree with the histograms obtained in Figures 3 and 4. The estimated distributions have mean and variance μ_r=0.9519, σ_r=0.0015 for the relative radii C_r, and μ_r=0.9519, σ_r=0.0015, μ_a=0.4975, σ_a=5.2887 10^{-4} for the relative angular distribution C_a. The optimum position of the coordinates of the fictitious charges obtained are $Cr \in [\mu_r-\sigma_r, \mu_r+\sigma_r]$ and $C_a \in [\mu_a-\sigma_a, \mu_a+\sigma_a]$ with respective mean [0.9505; 0.9534] and [0.4970; 0.4980]. For example, for C_r=0.95 and C_a=0.49 the optimum arrangement of the fictitious charges is shown in Figure 7.

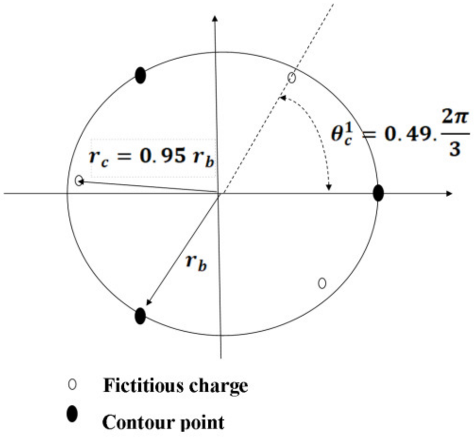

Fig. 7. Optimum arrangement C_r=0.95 and C_a=0.49.

To verify this approach, an example of the electric field calculation around an overhead- transmission line, already treated in the literature [10] is examined below.

IV. RESULT COMPARISON

The considered example is a 400kV line with a horizontal conductor configuration, as shown in Figure 2. Measurements were taken at 1m height according to recommendations [38], and were performed in the middle of the range between the two adjacent transmission line towers. The electric field is calculated without considering the effect of conductor end, arc sag, and the influence of the tower. In addition, the electromagnetic fields caused by the overhead transmission lines can be approximated by quasi-static fields [39], where quasi-static field displacement current and changes in the magnetic flux are negligible, so the electric field has exactly the same characteristics as the static one. It is assumed that the component of the electric field vector in the x direction is equal to zero, and the electric field vector in an arbitrary point, caused by the n point charges, can be calculated using (7) and (8).

The application of CSM with a relative radius C_r=0.95 and relative angle C_a=0.497 normally leads to optimum results close to the measured results, but to ensure its effectiveness it must be compared with another method already used with CSM. The choice fell on the GA because it is the most used with CSM as shown above. Also, it has been widely used in the field of electrical engineering [40-43]. Figure 8 illustrates the graphs of the electric fields calculated by this method, by CSM-GA, and the measured values (the values are listed in Table II). It can be seen that the field calculated with the proposed method is closer to the measured field than the field calculated with CSM-GA.

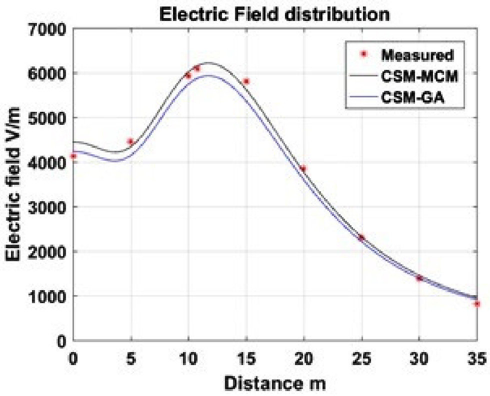

Fig. 8. Distribution of the electric field.

TABLE II. COMPARISON OF MEASURED AND CALCULATED RESULTS

Distance (m)	Measured (kV/m)	CSM-MCM	CSM-GA
0	4.13	17.69%	02.50%
5	4.45	02.50%	06.86%
10	5.93	01.44%	03.17%
10.8	6.09	01.11%	03.50%
15	5.81	03.20%	07.66%
20	3.84	01.75%	06.26%
25	2.30	00.99%	03.61%
30	1.39	04.69%	00.04%
35	0.81	16.11%	10.90%
Mean		04.38%	04.95%

V. CONCLUSION

A new approach for optimizing the charge simulation method using the MCM has been presented in this paper. The proposed algorithm aims to determine the PDFs of the classes of polar coordinates for which the error is minimal. The two PDFs follow Weibull's law. The PDF of relative radius (C_r) is asymmetric and concentrated near 1, while the PDF of the relative angle (C_a) is symmetric and is slightly centered at 0.49.

The proposed algorithm offers excellent flexibility and accuracy in determining the optimal locations of simulation charges. Accurate results are achieved for the electric field calculation around the overhead transmission lines. In addition, the solution is not a single element (C_a,C_r) like the results of other methods, but a range distributed according to the Weibull distribution whose parameters are calculated. This work aims at an optimal calculation of the electric field by CMS by arranging the fictitious charges so that they are very close to the edges of the conductor, and each imaginary charge mediates two consecutive contour points. The main contribution of this work is direct optimization without going through optimization methods.

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