Constellation and Mapping Optimization of APSK Modulations used in DVB-S2

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***Abstract*—This article represents the algorithms of APSK constellation and mapping optimization. The dependencies of the symbol error probability P_s on the parameters of the 16APSK and 32APSK constellations are examined and several options that satisfy the requirements to the minimum value of P_s are selected. Mapping optimization is carried out for the selected APSK constellations. BER characteristics of the satellite DVB-S2 channels are represented when using optimized and standard 16APSK and 32APSK constellations and a comparative analysis of the results achieved is made.**

Keywords-satellite DVB channel; M-ary APSK constellation and mapping; concateneted BCH-LDPC codes; DVB-S2 channel noise immunity; QEF reception

I. INTRODUCTION

In the contemporary Digital Video Broadcasting (DVB) systems it is necessary to provide Quasi-Error-Free (QEF) reception while the values of the Carrier to Noise Ratio (CNR) parameter are relatively low and the encoding and decoding equipment is not very complex [1]. The term "Quasi-Error-Free" means that the bit error rate (BER) at the input of the MPEG demultiplexer is less than 10^{-10} to 10^{-11}.

The requirement of contemporary communication systems to provide a higher rate of transmitted data requires the application of modulation methods with a greater spectral efficiency. When an M-ary modulation technique of higher order is used then both the spectral efficiency and the bit rate are increased but, in turn, the channel noise immunity is decreased.

In satellite systems, apart from noise and interference, the channel nonlinearity also causes a problem. The nonlinearity exists because the working mode of the power amplifier board is set near the saturation point in order to reach the maximum level for the transmitted signal. The nonlinear signal distortions, similarly to the noise and interference in the radio channel, are the reasons for the increase of error probability.

An essential requirement to the satellite DVB systems is ensuring high power efficiency without excessively penalizing the spectrum efficiency. This is achieved by using noise resistant types of modulation and effective channel codes. As known, the APSK modulation provides great resistance to nonlinear distortion in a radio channel [2, 3], which is the reason why this modulation is used in the second-generation satellite DVB systems. The 16APSK and 32APSK have been proposed in the DVB-S2 standard [4] and their performances have been widely investigated over the AWGN channel, by considering typical satellite scenarios, also in the presence of High Power Amplifiers (HPAs). In order to enhance the noise immunity of a DVB-S2 channel, a concatenated error protection of Bose-Chaudhuri-Hocquenghem (BCH) outer code and Low Density Parity Check (LDPC) inner code was chosen [4, 5].

The aim of this paper is to study the influence of the 16APSK and 32APSK constellation parameters and mapping on the noise immunity of satellite DVB channels.

II. ALGORITHM FOR APSK CONSTELLATION AND MAPPING OPTIMIZATION

The APSK constellation consists of a N number of concentric circles, where the *k-th* circle contains n_k signal points. Each of the circles in the constellation is characterized by a primary phase shift φ_k and radius r_k. In the general case, the APSK constellation can be described as follows:

$$\psi = \begin{Bmatrix} r_1.\exp\left[j\left(\varphi_1 + \dfrac{2\pi}{n_1}n\right)\right] & n = 0,1,...,n_1 - 1 \\ r_2.\exp\left[j\left(\varphi_2 + \dfrac{2\pi}{n_2}n\right)\right] & n = 0,1,...,n_2 - 1 \\ ... & ... \\ r_N.\exp\left[j\left(\varphi_N + \dfrac{2\pi}{n_N}n\right)\right] & n = 0,1,...,n_N - 1 \end{Bmatrix} \quad (1)$$

For convenience, instead of the radiuses, their ratios relative to the radius of the innermost circle – $\gamma_k = r_{(k+1)}/r_1$ may be used. The APSK modulation is usually denoted as n_1-n_2-…APSK.

Figure 1 and Figure 2 show 16APSK and 32APSK standard constellations that are used in the DVB-S2 systems. The parameters of these constellations are as follows: N=2, n_1=4,

n_2=12, φ_1=45°, φ_2=15° and $\gamma_1=r_2/r_1$=2.6 (for 16APSK) and N=3, n_1=4, n_2=12, n_3=16, φ_1=45°, φ_2=15°, φ_3=0°, $\gamma_1=r_2/r_1$=2.54 and $\gamma_2=r_3/r_1$=4.33 (for 32APSK). These constellations are determined to be optimal at an LDPC code rate of 8/9 and BCH code rate of 0.983.

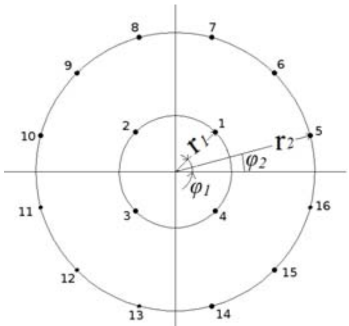

Fig. 1. Standard 4+12 APSK constellation

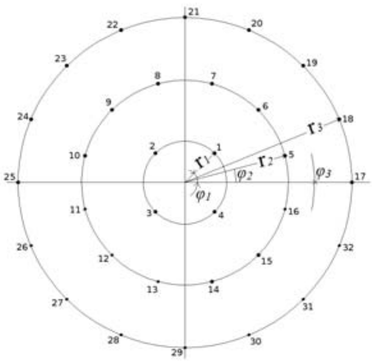

Fig. 2. 32APSK Constellation

There are several known algorithms for the optimization of APSK constellations [6, 7]. The aim of the optimization, presented in this paper, is to provide minimum symbol error probability. After processing the basic expressions, given in [8], the following dependence for the determination of the symbol error probability P_s was obtained:

$$P_s \le \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,j\ne i}^{M} P(s_i \to s_j) = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,j\ne i}^{M} \frac{1}{2}\operatorname{erfc}\left(\frac{d_{ij}}{2\sqrt{N_0}}\right) \quad (2)$$

In this expression, $P(s_i \to s_j)$ denotes the probability that instead of the i-th symbol the j-th one is accepted, M is the modulation order, N_0 is the noise power density and d_{ij} is the Euclidean distance between the i-th and j-th points of the constellation. The value of the error complementary function is obtained by [9]:

$$\operatorname{erfc}(x) \approx \frac{1}{x\sqrt{\pi}} \cdot \exp\left(-x^2\right) \quad (3)$$

In order to determine the Euclidean distance between two points which define the i-th and j-th positions of the APSK signal vector, we can use the cosines theorem, i.e.

$$d_{ij} = \sqrt{r_{p(i)}^2 + r_{q(j)}^2 - 2.r_{p(i)}.r_{q(j)}.\cos\Theta_{ij}} \quad (4),$$

where $r_{p(i)}$ and $r_{q(j)}$ are the radiuses of the circles where the two points are located and θ_{ij} is the angle between the signal vectors studied. The value of θ_{ij} is derived by [7]:

$$\Theta_{ij} = \left|\left(\varphi_p - \varphi_q\right) + 2\pi\left(\frac{i-1}{n_p} - \frac{j-1}{n_q}\right)\right| \quad (5),$$

where φ_p and φ_q denote the relative phase shifting of the signal points located on the p-th and q-th circles, n_p and n_q is the number of these points.

The relation between the radiuses of the circles in the APSK constellation and the energy per symbol E_s can be described by the following dependence:

$$E_s = \frac{n_1 r_1^2 + n_2 r_2^2 + \ldots + n_N r_N^2}{M} = \frac{r_1^2(n_1 + n_2\gamma_1^2 + \ldots + n_N\gamma_{N-1}^2)}{M} \quad (6)$$

where $\gamma_i = r_{(i+1)}/r_1$.

The mathematical model described allows the optimization of the following parameters of APSK constellations: the number of phase states n_k of the signal vector with amplitude r_k, the ratios between the amplitudes of the signal vector $\gamma_{(k-1)} = r_k/r_1$ and the relative phase shifting of the symbol points φ_k, where k=1, 2, … N. For the optimization of the APSK constellation parameters, we need to find the minimum value of the functional dependence which is presented by (1).

In order to evaluate the efficiency of the power used, the PAPR parameter (Peak to Average Power Ratio) is used. It is the ratio between the maximum and the average signal power $s(t)$ for a period T and is obtained using the expression [10]:

$$PAPR = \frac{\max_{t\in[0,T]}|s(t)|^2}{\frac{1}{T}\int_0^T |s(t)|^2\, dt} = \frac{M r_N^2}{\sum_{i=1}^{M} r_{p(i)}^2} \quad (7),$$

where r_N denotes the radius of the outermost circle, and $r_{p(i)}$ denotes the radius of the circle on which the i-th signal point is located.

In order to ensure the minimum bit error probability P_b, it is required to select an appropriate combination of $\log_2$M bits (mapping) for each signal point of the APSK constellation. The following dependence can be used when mapping optimization is carried out:

$$P_b \le \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,j\ne i}^{M}\frac{h_{ij}}{\log_2 M}P(s_i \to s_j) = \ldots$$

$$\ldots = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,j\ne i}^{M}\frac{h_{ij}}{2\log_2 M}\operatorname{erfc}\left(\frac{d_{ij}}{2\sqrt{N_0}}\right) \quad (8),$$

where h_{ij} is the Hamming distance between the i-th and j-th signal points.

III. PARAMETERS OF THE OPTIMIZED 16APSK CONSTELLATIONS

In order to determine the optimal parameters of the 16APSK constellation, different combinations of parameters n_1, n_2 and φ_1 were used and for each of them, the minimum of the functional dependence of the symbol error probability P_s on parameters φ_2 and γ_1 was established. The analysis of the results achieved shows that the initial phase shift φ_2 of the signal points of the outer circle does not impact considerably the P_s. Therefore, in this study it is assumed that $\varphi_2=0^o$.

Figure 3 shows the dependences of the symbol error probability on the ratio between the outer and inner circles of the 16APSK constellation γ_1 for four combinations of parameters n_1, n_2 and φ_1, for which the best results have been achieved. These dependences have been obtained at an energy per symbol to noise power density ratio of E_s/N_0=12 dB. When determining the value of E_s/N_0 parameter, the real Carrier to Noise Ratio (CNR) at the input of the satellite receiver has been considered (usually varies from 9 to 12 dB).

The values of the parameters of the four studied constellations at which a minimum symbol error probability is achieved are given in Table I. The table also shows the calculated values of the PAPR parameter for the evaluation of the power efficiency of these constellations. Obviously, the highest radio channel noise immunity can be ensured at using the fourth 16APSK constellation (CN4), but in that case the power efficiency of the system is the lowest. The best power efficiency is achieved at the first of the compared constellations (CN1), but it would provide the lowest radio channel noise immunity. So when choosing an APSK constellation, a compromise between the noise immunity and the power efficiency is to be made.

The mapping optimization carried out demonstrates that there is more than one possible combination of $m=\log_2(M)$ bits corresponding to the signal points, which provide a minimum of the functional dependence. The optimization is based on two constraints that define the permissible number of differing bits for two adjacent signal points ζ. The first one relates to the signal points of one and the same circle, and for them ζ=1. Only one breach of this rule is allowed, and then ζ=2. The second constraint requires that the value of the parameter ζ for the adjacent signal points from different circles is not higher than 2.

For each of the 16APSK constellations presented in Table I, one of the possible m=4 bits combinations at which a minimum value of bit error probability P_b is achieved is shown in Table II.

TABLE I. PARAMETERS OF THE OPTIMAL 16APSK CONSTELLATION.

Constellation Number (CN)	γ_1	n_1, n_2	φ_1	$P_{s\,min}$	PAPR
CN1	2.99	3, 13	60	0.1524	1.197
CN2	2.61	4, 12	45	0.1291	1.266
CN3	2.37	5, 11	36	0.1175	1.335
CN4	2.20	6, 10	30	0.1166	1.403

TABLE II. OPTIMAL SYMBOL MAPPING

Symbol Point №	CN1	CN2	CN3	CN4
1	0000	1100	0000	0000
2	1000	1110	1000	1000
3	1100	1111	1100	1100
4	0100	1101	1110	1110
5	0110	0100	1010	0110
6	0010	0000	0010	0100
7	0011	1000	0011	0011
8	0001	1010	0001	0010
9	1001	0010	1001	1010
10	1011	0110	1101	1011
11	1010	0111	0101	1001
12	1110	0011	0100	1101
13	1111	1011	0110	1111
14	1101	1001	0111	0111
15	0101	0001	1111	0101
16	0111	0101	1011	0001

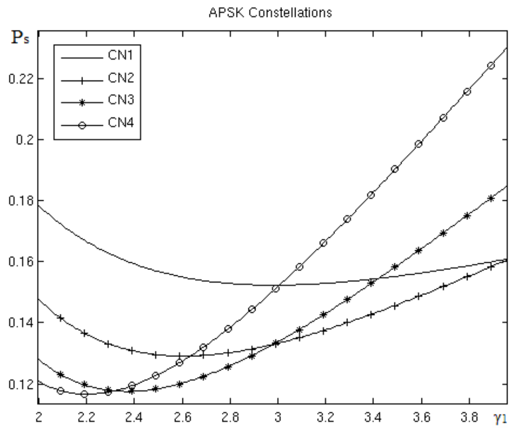

Fig. 3. Dependence of P_s on γ_1 at E_s/N_0=12 dB and $\varphi_2=0^o$

IV. PARAMETERS OF THE OPTIMIZED 32APSK CONSTELLATIONS

At determining the optimum parameters of the 32APSK constellation, different combinations of parameters n_1, n_2, n_3, φ_1, φ_2 and φ_3 have been used, and for each of them the values of γ_1 and γ_2 for which the symbol error probability P_s was minimal were determined. Figure 4 shows the dependence of P_s on γ_1 and γ_2 of the 4+12+16 APSK constellation, for which the most favorable results are achieved. The parameters of this constellation, denoted by CN1, as well as the calculated values of P_s and PAPR are given in Table III.

Similar studies have been carried out for other combinations of the 32APSK constellation parameters and after an analysis of the results achieved another four options have been selected and marked as CN2, CN3, CN4 and CN5. Figure 5 and 6 show the dependences of the symbol error probability P_s on parameters γ_1 and γ_2 of the 4+8+20 APSK (CN2) and 5+11+16 APSK (CN3) constellations. It is assumed that E_s/N_0=15 dB for these dependences. As seen from Table III, the 5+11+16 APSK constellation would provide the minimum symbol error probability, and the 4+8+20 APSK the most efficient use of the transmitter's power.

The mapping optimization carried out shows that a minimum of the functional dependence can be achieved for various bit combinations in the 32 symbol points of the studied APSK constellations. The combinations, for which the lowest values of the P_b parameter are obtained, are given in Table IV.

V. NOISE IMMUNITY OF THE DVB-S2 CHANNEL

In order to assess the noise immunity of the DVB-S2 channel, the dependence of bit error probability P_b, respectively BER at the output of the channel decoder, on the energy per symbol to noise power density ratio E_b/N_0 has been used. The expressions for determining the P_b are given in [11], and the values of E_b/N_0 parameter are calculated by:

$$\frac{E_s}{N_0} = \frac{E_b}{N_0} + 10.\lg(m) + 10.\lg(R_{\mathrm{LDPC}}) + 10.\lg(R_{\mathrm{BCH}}) \quad (9),$$

where $m=\log_2(M)$ denotes the number of bites in one symbol, and R_{LDPC} and R_{BCH} are the rates of the channel codes used.

Figure 7 shows the dependences of BER on the E_b/N_0 of DVB-S2 channels at using the four studied constellations and the standard 16APSK one. These dependences have been obtained at channel code rates R_{LDPC}=8/9 and R_{BCH}=0.983. The values of the parameter E_b/N_0, for which the bit error rate is 10^{-11}, are given in Table V. As it is evident in Table V, the best radio channel noise immunity can be provided when the fourth APSK constellation is used. In comparison with the APSK constellation which is standard for DVB-S2, the achieved benefit is 0.1 dB.

The assessment of the noise immunity of DVB-S2 channels at using the studied constellations and the standard 32APSK one can be derived from Figure 8. In this case also, it is assumed that the channel code rates are RLDPC=8/9 and RBCH=0.983. The values of the parameter Eb /N0, for which the bit error rate is 10^{-11}, are given in Table VI.

Obviously, the best radio channel noise immunity can be provided when the first 32APSK constellation is used. In comparison with the 32APSK constellation which is standard for DVB-S2, the achieved benefit is 0.085 dB.

VI. CONCLUSION

The analytic and graphic dependences presented in this paper make it possible for the parameters and mapping of 16APSK and 32APSK constellations to be determined. These dependences can be used in any other communication system in which such types of modulations are applied. The researches on the noise immunity of DVB-S2 channels, for the formation of which the studied 16APSK and 32APSK constellations have been used, show that there are several options for achieving similar results to those set out in the standard.

ACKNOWLEDGMENT

The research described in this paper is supported by the Bulgarian National Science Fund under the contract No DDVU 02/74/2010.

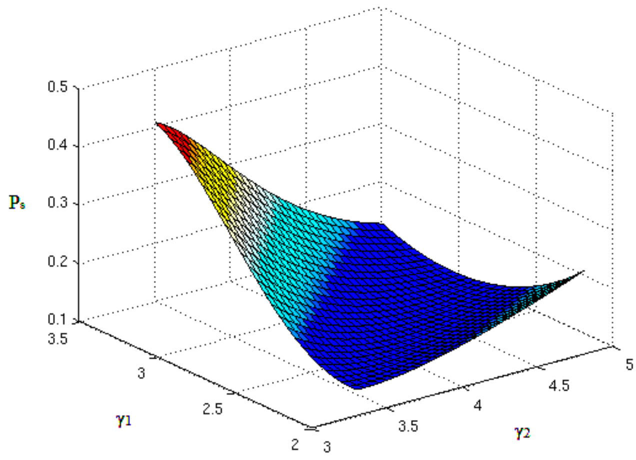

Fig. 4. Dependence of P_s on γ_1 and γ_2 of 4+12+16 APSK constellation at E_s/N_0=15 dB

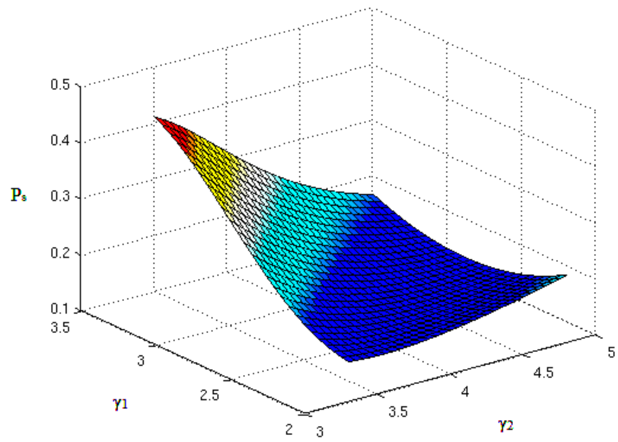

Fig. 5. Dependence of P_s on γ_1 and γ_2 of 4+8+20 APSK constellation

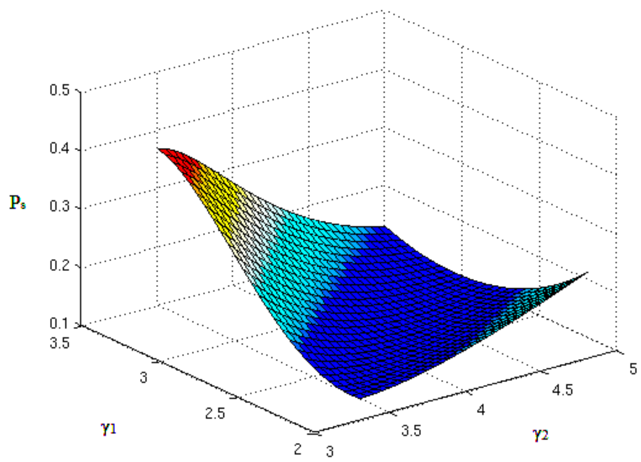

Fig. 6. Dependence of P_s on γ_1 and γ_2 of 5+11+16 APSK constellation

TABLE III. PARAMETERS OF THE OPTIMAL 32APSK CONSTELLATION.

Constellation Number (CN)	CN1	CN2	CN3	CN4	CN5
n_1, n_2, n_3	4,12,16	4,8,20	5,11,16	5,10,17	6,10,16
γ_1	2.54	2.32	2.3	2.26	2.13
γ_2	4.23	4.07	3.75	3.73	3.41
φ_1	45	45	36	36	30
φ_2	0	22.5	16.36	18	0
φ_3	11.25	0	11.25	0	0
P_s	0.1365	0.1644	0.1331	0.1368	0.1348
PAPR	1.557	1.401	1.561	1.522	1.567

TABLE IV. OPTIMAL SYMBOL MAPPING

Symbol Point №	CN1	CN2	CN3	CN4	CN5
1	00000	00000	00000	00000	00000
2	10000	10000	10000	10000	10000
3	11000	11000	11000	11000	11000
4	01000	01000	11100	11100	11100
5	00001	00001	10100	01100	01100
6	00101	00101	00100	01001	01000
7	00100	00100	00110	00001	01001
8	10100	10100	00010	10001	00001
9	10101	11100	10010	11001	10001
10	10001	11101	11010	11011	11001
11	11001	11001	01010	11010	11011
12	11101	01001	01000	11110	11010
13	11100	00011	01100	01110	11110
14	01100	10011	01110	01010	01110
15	01101	10001	11110	01000	01010
16	01001	10101	10110	01101	01011
17	00011	10111	00101	01111	01111
18	00111	00111	00111	00111	01101
19	00110	00110	00011	00101	00101
20	10110	00010	00001	10101	10101
21	10111	10010	10001	11101	11101
22	10011	10110	10011	11111	11111
23	10010	11110	11011	10111	10111
24	11010	01110	01011	10011	10011
25	11011	01100	01001	10010	10010
26	11111	01101	11001	10110	10110
27	11110	01111	11101	10100	10100
28	01110	11111	01101	00100	00100
29	01111	11011	01111	00110	00110
30	01011	11010	11111	00010	00010
31	01010	01010	10111	00011	00011
32	00010	01011	10101	01011	00111

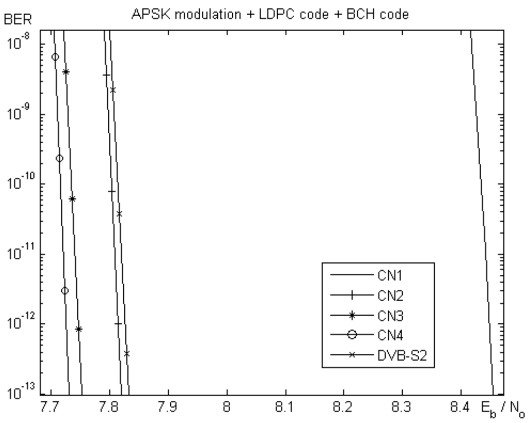

Fig. 7. BER characteristics of 16APSK channels

TABLE V. VALUES OF E_B/N_0 THAT PROVIDED BER=10^{-11} (16APSK)

CN1	CN2	CN3	CN4	DVB-S2
8.443 dB	7.809 dB	7.741 dB	7.721 dB	7.821 dB

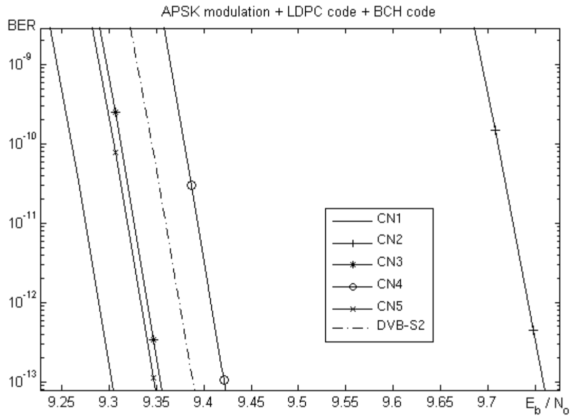

Fig. 8. BER characteristics of 32APSK channels

TABLE VI. VALUES OF E_B/N_0 THAT PROVIDED BER=10^{-11} (32APSK)

CN1	CN2	CN3	CN4	CN5	DVB-S2
9.275 dB	9.727 dB	9.326 dB	9.393 dB	9.319 dB	9.36 dB

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