

# Application of Fminsearch Optimization to Minimize Total Maintenance Cost with the Aim of Reducing Environmental Degradation

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**Abstract**-This study examined a production system under deterioration and its impact on the degradation of the environment. The environment degrades as the production system reaches a certain level of deterioration. To reduce environmental degradation, the system's deterioration is monitored through scheduled inspections, after which preventive or corrective actions are taken. To achieve the optimum inspection dates that minimize the average total cost per time unit, the Fminsearch algorithm was applied to calculate the optimal inspection dates for two cases of sequential inspection: periodic and aperiodic. To validate the performance of the proposed Fminsearch algorithm, simulation results were compared with the Nelder-Mead method. The comparison results showed the superiority of the Fminsearch algorithm in optimizing inspection maintenance dates to reduce the environmental degradation ratio.

**Keywords**-environment degradation; fminsearch algorithm optimization; maintenance; Nelder-Mead; production system

## I. INTRODUCTION

Environmental protection is described by international standards to which manufacturers must comply. The degradation of industrial systems can impact the environment in many ways. For example, the process of meeting energy demands raises concerns about energy sustainability and environmental protection, in conjunction with the market and regulatory requirements [1, 2]. Many environmental taxes have been imposed in recent decades all over the world. In the United States, the use of pollution prevention activities has increased significantly in the last two decades. The Pollution Prevention (P2) program is considered one of the main ways to reduce pollution [3, 4]. To ensure the efficient operation of an industrial system and meet the requirements of the environmental protection standards, inspection and maintenance activities are carried out. Inspection aids in controlling the degradation process of a production system and collecting crucial data on its reliability to determine the maintenance measures to be taken. Since the difficulties faced in maintenance inspections are attracting a lot of attention in

the industry, many inspection strategies have been developed. In [5], the optimal inspection dates and treats for the sequential inspection strategy were calculated, while a similar model was used in [6]. In [7], the same model was proposed with the estimate of the delay of the inspections, integrating the penalty cost in the mathematical model. The optimal inspection period and the maintenance threshold of system degradation were calculated in [8, 9]. A mathematical model was developed in [10] for the overall cost, including the production cost along with the maintenance action cost.

This study analyzed an inspection strategy that covers the environmental impact of the degradation of a production system. This strategy decreases degradation through maintenance actions to reduce environmental impact [11-13]. The system under consideration is submissive to continuous growing degradation. In this system, failure is detected only when it occurs, while the level of degradation of the system is only known after periodic or aperiodic sequential inspections. After checking the level of degradation when it exceeds a threshold, preventive actions are planned after a fixed timeframe, while corrective actions are carried out immediately after a system failure [14]. The main task was to minimize environmental degradation by decreasing the overall maintenance cost. The total cost is made up of the cost of corrective and preventive operations and inspection actions and the cost of penalties due to the environmental impact of system degradation. The main contribution of this study is the resolving of the optimization problem based on the Fminsearch algorithm. This algorithm is a direct search method to reduce non-linear functions [15-17].

## II. DESCRIPTION OF THE PROBLEM

### A. Notations

The following notations are used:

- $C_{cm}$ : The average cost of corrective maintenance.
- $C_{ed}$ : The average cost per time unit of severe environmental degradation.

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- $C_{ins}$ : The average cost of an inspection.
- $C_{pm}$ : The average cost of preventive maintenance.
- $TF_p$ : The timeframe to perform preventive maintenance.
- $T_{th}$ : A random elapsed time from when the system degradation began until it reaches the threshold.
- $t$ : Variable of  $T_{th}$  on the time axis.
- $f_d$ : The probability density function of  $T_{th}$ .
- $X$ : A random elapsed time from instant  $t$  until failure (lifetime of the system after exceeding the threshold).
- $x$ : Variable of  $X$ , from the instant  $t$ .
- $g_d, G_d$ : probability density and distribution function of  $X$  respectively.
- $N_{ins}$ : A random number of examinations during a cycle.
- $\varphi^*(\varphi_1, \varphi_2, \dots, \varphi_i)$ : The vector of examination dates.
- $T_{rd}$ : A random time of excessive environmental degradation.
- $T_{cy}$ : A random cycle time measured between two consecutive maintenance actions: preventive or corrective.
- $P_{cm}$ : Probability that the cycle will end with corrective maintenance action.
- $P_{pm}$ : Probability that the cycle will end with preventive maintenance action.
- $C_{tot}$ : The total cost incurred by maintenance and examination operations and by environmental degradation during a cycle.

B. Assumptions

This study was based on the following assumptions:

- After each inspection, two types of action are likely: to leave it as it is or perform preventive maintenance.
- Corrective maintenance is performed immediately after a system failure.
- The inspection actions are assumed to be perfect, i.e. the inspection reveals the actual level of the system's degradation without error.
- Corrective and preventive maintenance actions are meant to be perfect.
- The durations of inspection and maintenance actions are negligible.
- System degradation leads to environmental degradation.
- The system only fails if its degradation level exceeds the alarm threshold. Such a failure is supposed to be detected automatically (self-declared failure case).
- The system works as it should after corrective or preventive maintenance actions.

- The costs  $C_{cm}, C_{pm}, C_{ins}$  and  $C_{ed}$ , with the timeframe  $TF_p$  as well as the densities  $f_d$  and  $g_d$  are kept in the system's database.
- Preventive maintenance action is scheduled after a timeframe  $TF_p$ , if the inspection reveals that the level of degradation has exceeded the alarm threshold, as seen in Figure 1. Any inspections in this interval are canceled. As an example: in the interval  $[\varphi_4, \varphi_4 + TF_p]$ , the inspections are canceled.

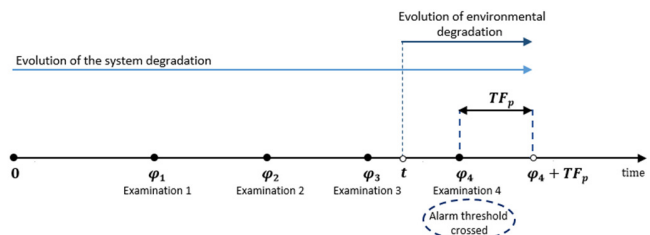


Fig. 1. Evolution of system degradation and distribution of examinations over time.

III. THE MATHEMATICAL MODEL OF MAINTENANCE COST

This section presents the mathematical model of the average total cost per time unit  $Q$  of maintenance.  $Q$  is defined as the ratio between the average global cost  $E(C_{tot})$  and the average cycle time  $E(T_{cy})$  [18]:

$$Q = E(C_{tot}) / E(T_{cy}) \quad (1)$$

This cost includes the cost of preventive and corrective actions, the inspection cost, and the penalty cost. In the case of corrective action, the average cost is  $(C_{cm} \times P_{cm})$ , while the expression  $(C_{pm} \times P_{pm})$  indicates the average cost in the case of preventive maintenance.  $E(N_{ins})$  is defined as the average number of inspections during the time cycle, and their average cost is given by  $C_{ins} \times E(N_{ins})$ .  $E(T_{ed})$  is the average time of the critical environmental degradation and the expression  $C_{ed} \times E(T_{ed})$  corresponds to the penalty cost due to severe environmental degradation. The following formula illustrates the average cost:

$$E(C_{tot}) = (C_{cm} \times P_{cm}) + (C_{pm} \times P_{pm}) + (C_{ins} \times E(N_{ins})) + (C_{ed} \times E(T_{ed})) \quad (2)$$

This formula uses the proposals developed in [4, 18]. The probability  $P_{cm}$  that a time cycle will end with corrective actions is given by:

$$P_{cm} = \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} G_d(\varphi_i + TF_p - t) f_d(t) dt \quad (3)$$

The probability that corresponds to the case where the time cycle ends with preventive action is given by:

$$P_{pm} = 1 - P_{cm} \quad (4)$$

The average number  $E(N_{ins})$  of examinations during a time cycle is given by:

$$E(N_{ins}) = \sum_{i=1}^n i \left( \int_0^{\varphi_{i+1}} G_d(\varphi_{i+1} - t) f_d(t) dt \times \int_0^{\varphi_i} G_d(\varphi_i - t) f_d(t) dt \right) \quad (5)$$

The average time  $E(T_{rd})$  of excessive environmental degradation through a time cycle is given by:

$$E(T_{rd}) = \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt \quad (6)$$

The expression of the numerator of the function presented by (1) is concluded from (2) - (6) as:

$$E(C_{tot}) = ((P_{cm} - P_{pm}) \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} G_d(\varphi_i + TF_p - t) f_d(t) dt + P_{pm} + C_{ins} \sum_{i=1}^n i \left( \int_0^{\varphi_{i+1}} G(\varphi_{i+1} - t) f_d(t) dt \int_0^{\varphi_i} G(\varphi_i - t) f_d(t) dt \right) + C_{ed} \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt \quad (7)$$

The denominator of (1) is given by:

$$E(T_{cy}) = \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( t + \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt \quad (8)$$

From (7) and (8), the general expression of the mathematical model of the average total cost per time unit of the maintenance function defined by (1) is deduced as:

$$Q(\varphi_i) = \frac{\left( (P_{cm} - P_{pm}) \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} G_d(\varphi_i + TF_p - t) f_d(t) dt + P_{pm} + C_{ins} \sum_{i=1}^n i \left( \int_0^{\varphi_{i+1}} G(\varphi_{i+1} - t) f_d(t) dt \int_0^{\varphi_i} G(\varphi_i - t) f_d(t) dt \right) + C_{ed} \sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt \right)}{\sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( t + \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt} \quad (9)$$

The minimum value of the average total cost per time unit  $\min Q(\varphi)$  is considered an objective function. The Fminsearch optimization was applied to calculate the optimal inspections vector to reduce the environmental degradation ratio and minimize the maintenance cost value  $Q(\varphi)$ .

#### IV. OPTIMIZATION PROCEDURE

A periodic and an aperiodic sequential examination plan were applied to solve the problem without any constraint on the examination periods. In the aperiodic plan, the interval between examinations is independent. This problem is a multidimensional nonlinear problem [19]. The mathematical model exposed in Section III was exploited to solve this problem. The objective was to find the optimal examination dates which minimize the average total cost per time unit  $Q(\varphi_i)$  given by (9). The Fminsearch algorithm was used to find the minimum of an unconstrained multivariable function using a derivative-free method. This algorithm has been used repeatedly to resolve nonlinear functions and is well known for its robustness and ease of implementation [14].

##### A. Analytical Procedure

The flowchart shown in Figure 2 corresponds to the analytical approach and was developed to determine the optimal sequential examination dates that minimize the average global cost per time unit. At first, the input data:  $f_d, g_d, C_{cm}, C_{pm}, C_{ed}, C_{ins}$  and  $TF_p$  are given. Afterward, the Fminsearch algorithm is applied to calculate the optimal vector of examination dates  $(\varphi^*(\varphi_1, \varphi_2, \dots, \varphi_n))$ . Then, the iteration number is tested: If the inequality  $n > N$  is unattained,  $n$  increases. If it was attained, iterations stop. Finally, the solution vector of examination dates  $\varphi_i$  that match up to the minimum value  $Q(\varphi_i)$  is selected.

```

Begin
% inputs
f_d : % the probability density function of T_{th}
g_d : % the probability density function of X
C_{cm} : % the average cost of corrective maintenance
C_{pm} : % the average cost of preventive maintenance
C_{ed} : % the average cost of environmental degradation
C_{ins} : % the average cost of examination (inspection)
TF_p : % timeframe to perform preventive maintenance
N : % iteration number
i = 1;
While i ≤ N;
    fun = Q(φ_i); /* function cost*/
    (φ_i^*, valQ(φ_i^*)) = fminsearch(fun, φ_i); % optimize the maintenance function cost
End while
valQ(φ^*)_{min} = min(valQ(φ_1^*), valQ(φ_2^*), valQ(φ_N^*)); % optimal value of function cost
sol(φ^*) = φ^*; % solution of optimal examination vector
End.
    
```

Fig. 2. Pseudo code of the Fminsearch algorithm.

Two techniques were proposed to calculate the optimal inspection dates of the production system: periodic and aperiodic sequential inspection. The results of the Fminsearch algorithm were compared to the Nelder-Mead optimization method [18, 20].

##### B. Numerical Applications

The above procedure was used to estimate the optimal examination dates. The input data were provided: probability densities  $f_d$  and  $g_d$ , the costs  $C_{cm}, C_{pm}, C_{ed}, C_{ins}$  and time flow  $TF_p$ . Time was expressed in time units, while cost was expressed in monetary units.  $\lambda$  is the ratio between the mean time  $E(T_{rd})$  of severe environmental degradation and the average time  $E(T_{cy})$  of the time cycle:

$$\lambda = \frac{E(T_{rd})}{E(T_{cy})} \quad (10)$$

Therefore:

$$\lambda = \frac{\sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt}{\sum_{i=1}^n \int_{\varphi_{i-1}}^{\varphi_i} \left( t + \int_0^{\varphi_i+TF_p-t} [1 - G_d(x)] dx \right) f_d(t) dt} \quad (11)$$

##### 1) Periodic Sequential Examination

In this case, inspection is performed periodically. The inspection period  $\Delta\varphi$  is defined as:

$$\Delta\varphi = (\varphi_i) - (\varphi_{i-1}) = Cte, \quad i = 1, 2 \dots n \quad (12)$$

The probability densities  $f_d$  and  $g_d$  correlate jointly to the random variables  $T_{cy}$  and  $X$ . The probability density  $f_d$  is given by:

$$f_d(\tau) = \frac{1}{\sigma f \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\tau - \mu f}{\sigma f} \right)^2} \quad (13)$$

with  $\mu f = 1100$  and  $\sigma f = 150$ . The probability density  $g_d$  is given by:

$$g_d(\tau) = \frac{1}{\sigma g \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\tau - \mu g}{\sigma g} \right)^2} \quad (14)$$

with  $\mu g = 130$  and  $\sigma g = 40$ . The average costs are fixed as  $C_{cm} = 16108, C_{pm} = 988,$  and  $C_{ins} = 150.$   $C_{ed}$  and  $TF_p$  are introduced as parameters.

a) Application 1:

To examine the impact of cost  $C_{ed}$  over the cost  $Q(\Delta\varphi^*)$  and the ratio  $\lambda$ , the analytical Fminsearch algorithm was implemented in Matlab. The delay  $TF_p$  was considered negligible, so the preventive operations are executed immediately. The cost  $Q(\Delta\varphi^*)$  and the ratio  $\lambda$  are presented in Table I.

TABLE I. THE IMPACT OF COST  $C_{ed}$  OVER  $Q(\Delta\varphi^*)$  AND RATIO  $\lambda$  WITH  $TF_p = 0$ , IN THE CASE OF A PERIODIC INSPECTION STRATEGY

$C_{ed}$	$\Delta\varphi^*$	$E(N_{ins})$	$E(T_{rd})$	$C_{tot}$	$E(T_{cy})$	$Q(\Delta\varphi^*)$	$\lambda$ [%]
0	22	3.48	15.02	987.36	726	1.36	2.07
10	19	2.01	8.50	985.53	541.5	1.82	1.57
20	18	2.20	5.97	986.58	486	2.03	1.23
30	15	3.54	3.51	985.50	337.5	2.92	1.04
40	13	2.12	2.53	986.11	253.5	3.89	1.00
50	12	1.62	1.96	987.12	216	4.57	0.91

As shown in Table I, when  $C_{ed}$  increases,  $\lambda$  decreases, reducing the rate of environmental degradation.  $E(T_{ed})$  decreases by increasing the penalty cost  $C_{ed}$ , therefore the rate of environmental deterioration decreases. Figure 3 presents the variation of  $Q(\Delta\varphi^*)$  as a function of  $C_{ed}$ . The results obtained by the Fminsearch algorithm show a decrease in the value of  $Q(\Delta\varphi^*)$ . This value varies from 55.11% to 2.76% compared to the Nelder-Mead method.

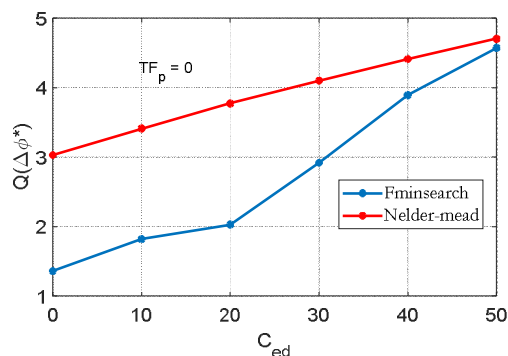


Fig. 3. Influence of penalty cost  $C_{ed}$  over  $Q(\Delta\varphi^*)$  in the case of a periodic examination strategy.

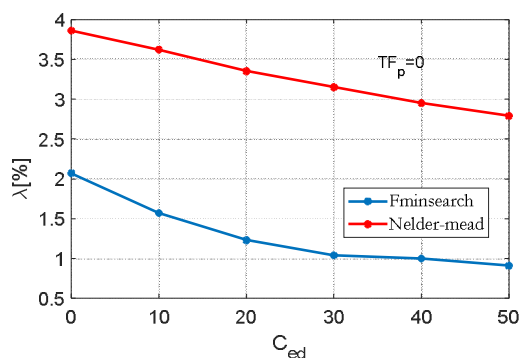


Fig. 4. Impact of penalty cost  $C_{ed}$  on the ratio  $\lambda$ , in the case of periodic examination strategy.

Figure 4 shows the variations of  $\lambda$ . The Fminsearch algorithm gives lower values that vary from 46.37% to 67.38%

compared to the Nelder-Mead, for an increasing value of  $C_{ed}$ . On the other hand, the ratio  $\lambda$  decreases by increasing the penalty cost, i.e. the degradation of the system as well as that of the environment are reduced.

b) Application 2

The penalty cost  $C_{ed}$  was fixed at 20 monetary units to study the impact of the duration  $TF_p$  over  $Q(\Delta\varphi^*)$  and  $\lambda$ . Table II shows the optimal period between inspections  $\Delta\varphi^*$  for different values of the parameter  $TF_p$ , by applying the Fminsearch algorithm. From Table II, it is noted that for  $TF_p = 0$ ,  $\lambda$  and  $Q(\Delta\varphi^*)$  have a minimal value of 1.20% and 1.64, respectively. Therefore, preventive operations should be carried out as soon as possible, i.e. the delay  $TF_p$ , should be minimal. Figure 5 presents the variation of the optimal value of the cost  $Q(\Delta\varphi^*)$  as a function of  $TF_p$ . The Fminsearch algorithm gives better cost optimization for low values of  $TF_p$ , (up to 56.50%).

TABLE II. THE IMPACT OF  $TF_p$  OVER  $Q(\Delta\varphi^*)$  AND  $\lambda$  WITH  $C_{ed} = 20$ , IN THE CASE OF A PERIODIC EXAMINATION STRATEGY.

$TF_p$	$\Delta\varphi^*$	$E(N_{ins})$	$E(T_{rd})$	$C_{tot}$	$E(T_{cy})$	$Q(\Delta\varphi^*)$	$\lambda$ [%]
0	20	2.41	7.2	984	600	1.64	1.20
10	17	1.35	8.97	984.04	433.5	2.27	2.07
20	14	2.75	9.73	987.84	294	3.36	3.31
30	13	2.12	10.74	981.04	253.5	3.89	4.24
40	11	1.23	8.67	987.36	181.5	5.44	4.78
50	10	2.44	6.19	797.04	121.5	6.58	5.10

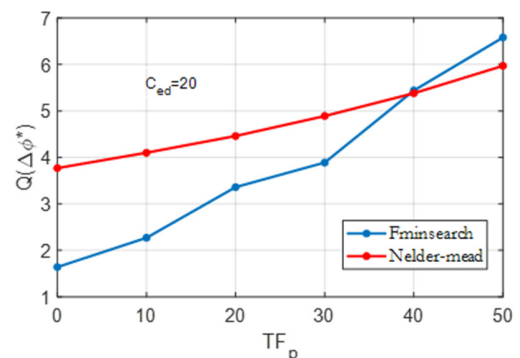


Fig. 5. Impact of timeframe  $TF_p$  over  $Q(\Delta\varphi^*)$ , in the case of periodic examination strategy.

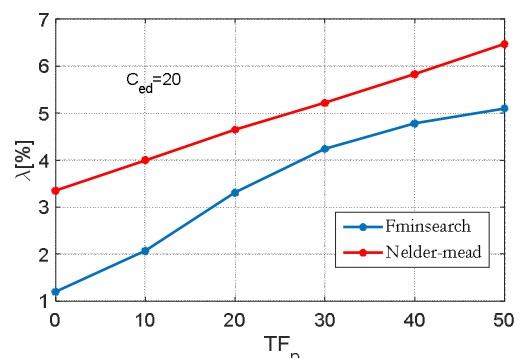


Fig. 6. Influence of timeframe  $TF_p$  over  $\lambda$ , in the case of a periodic inspection sequential strategy.

Figure 6 shows that the ratio  $\lambda$  increases with the lengthening of the time to carry out preventive operations. The Fminsearch algorithm gives an improved  $\lambda$  compared to the Nelder-Mead method.

2) Aperiodic Sequential Examination

The probability densities  $f_d$  and  $g_d$  correspond to the random variables  $T_{th}$  and  $X$ , respectively. It is assumed that each of the two densities follows the Weibull law whose scale parameter is expressed in time units, while the form parameter has no unit. The probability density  $f_d$  is given by:

$$f_d(t) = \left(\frac{\beta_f}{\alpha_f}\right) \left(\frac{t}{\alpha_f}\right)^{\beta_f-1} e^{-\left(\frac{t}{\alpha_f}\right)^{\beta_f}} \quad (15)$$

where the scale parameter is  $\alpha_f = 1164$  and the shape parameter is  $\beta_f = 8.7$ . The probability density  $g_d$  of the random time  $X$  elapsed from the instant  $t$  until the occurrence of the failure is given by:

$$g_d(x) = \left(\frac{\beta_g}{\alpha_g}\right) \left(\frac{x}{\alpha_g}\right)^{\beta_g-1} e^{-\left(\frac{x}{\alpha_g}\right)^{\beta_g}} \quad (16)$$

where the scale parameter is  $\alpha_g = 144$  and the form parameter is  $\beta_g = 3.6$ . The average costs are respectively fixed as  $C_{cm} = 6180$ ,  $C_{pm} = 4170$ , and  $C_{ins} = 492$ , while  $C_{ed}$  and  $TF_p$  are introduced as parameters.

a) Application 3:

The effect of  $C_{ed}$  on  $Q(\varphi^*)$  and  $\lambda$  was examined using the Fminsearch algorithm, when  $TF_p$  is fixed at zero, i.e. when preventive maintenance actions are not delayed. Table III shows the optimal  $Q$  for each value of the optimal vector  $\varphi^*$ . When  $C_{ed} = 0$ ,  $Q(\Delta\varphi^*)$  has a minimum and  $\lambda$  has a maximum value of 3.23 and 3%, respectively. According to Table III, the increase in the  $C_{ed}$  penalty cost induces an increase in the number of inspections. These additional inspections are necessary to detect, as quickly as possible, the exceeding of the degradation threshold to reduce the resulting cost, i.e. the reduction of  $(C_{ed} \times E(T_{rd}))$ .

TABLE III. THE IMPACT OF PENALTY COST  $C_{ed}$  OVER  $Q(\varphi^*)$  AND  $\lambda$ , IN CASE OF A PERIODIC SEQUENTIAL INSPECTION STRATEGY

$C_{ed}$	$\varphi^*$	$Q(\varphi^*)$	$\lambda\%$
0	1.33, 5.99, 48.59	3.23	3.00
10	2.18, 6.15, 10.32, 12.96, 54.21	3.48	2.67
20	2.50, 5.89, 9.40, 15.04, 62.60	3.93	2.40
30	1.32, 4.58, 8.74, 9.24, 23.99, 62.49	4.94	2.22
40	1.32, 3.36, 8.74, 10.24, 24.03, 62.50	5.95	2.20
50	1.40, 4.58, 8.60, 9.80, 25.30, 61.49	6.96	2.17

Figure 7 shows the variation of the optimal value of the average total cost per time unit  $Q(\Delta\varphi^*)$  according to the penalty cost  $C_{ed}$ . The evolution of  $Q(\Delta\varphi^*)$  is directly proportional to the  $C_{ed}$  cost. The obtained results show that Fminsearch gives a better optimization of the cost  $Q(\Delta\varphi^*)$  compared to the Nelder-Mead algorithm, as there is a decrease in the value of  $Q(\Delta\varphi^*)$  up to 30.08%. The variation of the rate of degradation  $\lambda$  as a function of  $C_{ed}$  is presented in Figure 8. The evolution of the ratio  $\lambda$  is inversely proportional to  $C_{ed}$ , which implies that excessive environmental degradation is

reduced. Figure 8 confirms the superiority of the Fminsearch algorithm compared to the Nelder-Mead in minimizing the degradation ratio. The Fminsearch algorithm has a maximum reduction of 60.05% compared to the Nelder-Mead at  $C_{ed} = 0$

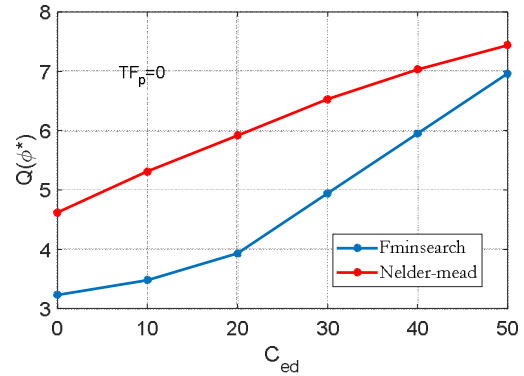


Fig. 7. Influence of the penalty cost  $C_{ed}$  on  $Q(\varphi^*)$  in the case of an aperiodic sequential inspection strategy.

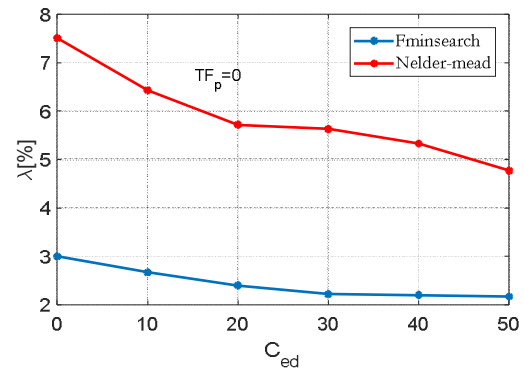


Fig. 8. Influence of the penalty cost  $C_{ed}$  on degradation rate  $\lambda$  in case of an aperiodic sequential inspection strategy.

b) Application 4:

To evaluate the impact of  $TF_p$  over  $Q(\varphi^*)$  and  $\lambda$ , the cost  $C_{ed}$  was set to 30 currency units, while several values are assigned to the duration  $TF_p$ . Table IV shows the average total cost per time unit  $Q(\varphi^*)$  with the corresponding ratio  $\lambda$  for each vector  $\varphi^*$ .

TABLE IV. INFLUENCE OF THE DURATION  $F_p$  OVER  $Q(\varphi^*)$  AND  $\lambda$  IN CASE OF AN APERIODIC SEQUENTIAL EXAMINATION STRATEGY

$TF_p$	$\varphi^*$	$Q(\varphi^*)$	$\lambda\%$
0	2.27, 3.73, 6.41, 7.79, 10.48, 63.39	4.00	2.59
10	4.23, 6.35, 7.56, 19.00, 59.88	5.24	3.53
20	3.43, 5.68, 21.51, 60.97	6.30	4.31
30	3.70, 5.76, 19.18, 56.98	7.32	5.14
40	1.28, 8.37, 54.14	8.38	5.92
50	1.24, 8.79, 55.29	9.36	6.26

According to Table IV, in the case where the duration  $TF_p$  is set to zero, the ratio  $\lambda$  is minimal and equal to 2.59%, and the optimal value of the average total cost per time unit  $Q(\varphi^*)$  is 4.00. Figure 9 shows the optimal value of the average total cost per time unit  $Q(\varphi^*)$  concerning  $TF_p$ . The Fminsearch



algorithm gives a better optimization of the cost, up to 53.04%, for low values of  $TF_p$ . Figure 10 shows the degradation rate  $\lambda$  as a function of  $TF_p$ . The Fminsearch algorithm improves the  $\lambda$  up to 53.99% compared to the Nelder-Mead algorithm.

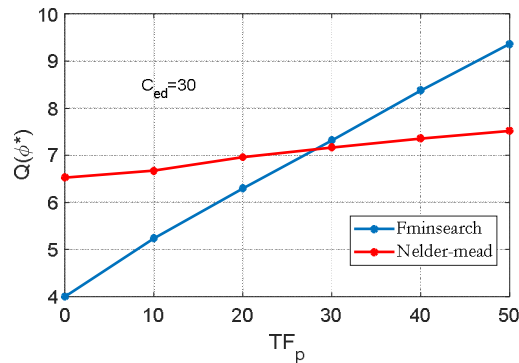


Fig. 9. Influence of the duration  $TF_p$  over  $Q(\phi^*)$  in the case of an aperiodic sequential inspection strategy.

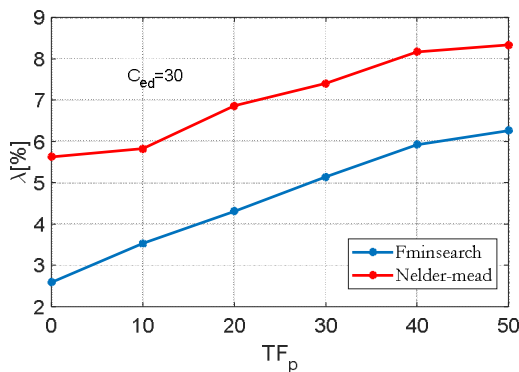


Fig. 10. Influence of the duration  $TF_p$  on  $\lambda$  in the case of an aperiodic sequential inspection strategy.

## V. CONCLUSION

This study aimed to maintain a production system whose declining performance causes environmental degradation by applying the Fminsearch algorithm to optimize periodic and aperiodic inspection maintenance dates. This method improved the impact of the environmental degradation cost  $C_{ed}$  and the duration  $TF_p$  on the total average maintenance cost and the ratio of environmental degradation. Moreover, the results of this method were compared to those of the Nelder-Mead method. The simulation results showed the superior performance of the Fminsearch algorithm in the optimization of inspection maintenance dates to reduce the ratio of environmental degradation.

Future work could examine the modification of certain assumptions, such as the duration of inspections and maintenance actions. These hypotheses could enable the study of the production system availability. Other optimization methods such as metaheuristics or simulation methods could be applied, while the industrial production could be described with real data.

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