Static Shear Strength of a Non-Prismatic Beam with Transverse Openings

Amjad Majeed Al-Hilali
Department of Civil Engineering
University of Baghdad
Baghdad, Iraq
amjadmajeedal@gmail.com

Amer Farouk Izzet
Department of Civil Engineering
University of Baghdad
Baghdad, Iraq
amer.f@coeng.uobaghdad.edu.iq

Nazar Oukaili
Department of Civil Engineering
University of Baghdad
Baghdad, Iraq
nazar.oukaili@coeng.uobaghdad.edu.iq

Received: 29 January 2022 | Revised: 10 February 2022 | Accepted: 13 February 2022

Abstract-In this study, a predicated formula is been proposed to find the shear strength of non-prismatic beams with or without openings. It depends on the contributions of concrete shear strength considering the beam depth variation and existing openings, shear steel reinforcements and defines the critical shear section, the effect of diagonal shear reinforcement, the effect of inclined tensile steel reinforcement, and the compression chord influence. The verification of the proposed formula has been conducted on the experimental test results of 26 non-prismatic beams with or without openings at the same loading conditions. The results reflect that the predicted formula finds the shear capacity of non-prismatic beams with openings, it is conservative and can be used for designing without the strength reduction factor.

Keywords-critical section; shear strength; opening; non-prismatic beam

I. INTRODUCTION

The primary benefit of openings is that mechanical equipment may enter through the beams instead of below them. This decreases the building's floor-to-floor height and total cost. On the other hand, the load-carrying capacity of a reinforced concrete beam is diminished by the openings (i.e. deterioration of flexural and shear rigidities to the parts containing the opening) [1, 2]. To analyze and design beams with openings, various mechanical and numerical models have been presented for estimating their shear strength. Most models adopt typical beams with a constant prismatic section along them. The Reinforced Concrete Haunched Beams (RCHBs) are widely used. This type of beams has been extensively preferred in industrial and framed buildings, bridges, and structural portal frames, due to their advantages [3, 4]. The weight of the structure can be reduced and larger spans can be achieved by the use of RCHBs instead of prismatic beams without clear deterioration in loading capacity [5].

Only a few works have been conducted on the shear behavior of RCHBs. It should be noted that since the effective depth of RCHBs is variable along the length of the beams, their structural analysis and mechanical behavior differ from the analysis and behavior of prismatic beams. The ACI-318-19 code gave only simple observations, while the ACI 318-14 debates the variable depth members. These sections consider the effect of inclined flexural compression in calculating the shear strength of concrete where the internal shear forces at any cross-section are increased or decreased by the vertical component of inclined flexural forces. Section 27.4.5.3 of the ACI 318-14 discusses the inclined shear crack in the variable depth beams and recommends measuring the depth at the crack mid-length. According to the explanations stated above, the code did not provide any formulas to calculate the critical section or to consider the effect of the inclination on the shear force capacity of variable depth beams [6].

Currently, only a few details are available for calculating the shear behavior of non-prismatic reinforced concrete beams with web openings. Previous works dealt with solid prismatic reinforced concrete beams (with web openings). Some of them considered the analysis of non-prismatic reinforced concrete beams without openings. The objective of the current paper includes introducing a new shear design model of RCHBs with multiple transverse web openings. The proposed model depends on the experimental results. The proposed empirical expression considers the same conditions, adopting the total shear resistance of the RC beam with openings as the resistance provided by both concrete and the steel bars intersecting the failure plane by an angle of 45° pass through the center of the opening.

Corresponding author: Amjad M. Al-Hilali

II. Previously Proposed Equations for Shear Resistance of RC Beams with Web Openings

A. Shear Design Model for a Prismatic Beam with Openings

Mansur [7] suggested that the beam may fail in two different modes. The first mode is similar to the failure seen in solid beams, where the failure plane passes through the opening's center. The second model includes the formation of two separate diagonal cracks, one in each member, linking the two solid beam segments, leading to failure. These kinds of failure were classified as beam-type failures and frame-type failures respectively. The developed equation for the beam-type failure depending on the failure mechanism is:

$$Vn = \frac{1}{6}\sqrt{fc'}\, bw(d - do) + \frac{Av\ fyv}{S}(dv - do) + Ad\ fyd\, sina \quad (1)$$

where fc' is the strength of concrete, bw is the web width, d is the effective depth, and do is the diameter of the opening. Av is the area of vertical legs of stirrups per spacing, fyd is the yield of steel stirrups, S is the spacing between stirrups, Ad is the cross-sectional area of the additional reinforcement within the failure section, and α is the angle of inclination of diagonal reinforcement.

In the case of frame-type failure, Mansur [7] considered the two chords to behave independently like a framed structure. Therefore both chord members demand independent treatment. Accordingly, a free-body diagram at the web opening was suggested. The applied shear, Vu, may be distributed between the two chords in proportion to their cross-sectional areas. Accordingly, the shear forces can be estimated by:

$$(Vu)t = Vu\frac{At}{At + Ab} \quad (2)$$

$$(Vu)b = Vu - (Vu)t \quad (3)$$

The Architectural Institute of Japan (AIJ) (1988) [8] adopted the following expression to predict the shear capacity Vn of beams that contain a small opening.

$$Vn = \left[\frac{0.092Ku.Kp(fc' + 17.7}{\frac{M}{Vd} + 0.12}\left(1 - \frac{1.61do}{h}\right) + 0.846\sqrt{\rho'w.fyv}\right] bd \quad (4)$$

where $Kp = 0.82\ (100A_s/\ bd)^{0.23}$, do is the diameter of the circular opening or, in the case of a square opening the diameter of the equivalent circle, which should be less than or equal to $h/3$, h is the total depth of the beam, and $\frac{M}{Vd}$ is taken as less than or equal to 3. The term Ku is a function of the effective depth d to account for size effects in shear and has a value from 0.72 to 1.0, fyv is the yield strength of stirrup reinforcement, dv is the distance between the top and bottom longitudinal bars, the term $\rho'w$ refers to the ratio of stirrup reinforcement with an area Av placed within a longitudinal distance $dv/2$ from the center of the opening and defined as:

$$\rho'w = \frac{Av(\sin\alpha + \cos\alpha)}{b.dv} \quad (5)$$

Mansur [7] modified the maximum admissible shear force $(Vu)_{max}$ formula of (ACI - 318) for RC beams with openings:

$$(Vu)max = 5 * 0.85\left[0.17\sqrt{fc'}\,.b.(d - do)\right] \quad (6)$$

B. Shear Design Models for RCHBs without Openings

Based on the ACI 318 code's equation intended for the evaluation of shear strength of prismatic concrete sections, authors in [9] proposed an expression for the shear strength which includes a modification factor that considers the depth variation along RCHBs. This factor is presented by the term (1+1.7 $tan\ \alpha$). The proposed expression considers also the effect of the inclined flexural reinforcement. It takes the following shape:

$$\frac{v}{b.ds} = \left(0.16\sqrt{fc'} + 17\rho s\frac{Vu.d}{Mu}\right) \times$$

$$(1 + 1.7\tan\propto) + \rho v.fy + 0.25\rho s.\sin\alpha \quad (7)$$

The German code DIN 1045-01 [10] addressed the shear resistance mechanism of RCHBs in detail. It introduced (8) to design the shear resistance of RCHBs:

$$V_{Ed} = V_{Edo} - V_{ccd} - V_{td} \leq V_{ED}^{\alpha} \quad (8)$$

$$V_{ED}^{\alpha} = 0.1.k.(100\rho'.f_{ck})^{0.333}.b.d.k$$

$$= 1 + \sqrt{\frac{200}{d}} \leq d, \rho \leq 0.02' \quad (9)$$

$$V_{ccd} = \frac{M_{Ed}}{0.9d}\tan\alpha \quad (10)$$

where V_{Ed} is the concrete shear resistance, V_{Edo} is the shear force, V_{ccd} is the shear resistance due to the inclination of the compression chord, V_{td} is the shear resistance component of the inclined longitudinal tension reinforcements, and V_{ED}^{α} is the design value of the shear bearing capacity of non-prismatic beams at design section.

C. The Critical Effective Section for RCHBs

The main challenge in predicting the shear strength of RCHBs is the determination of the critical effective section due to the variation of depth along them. In this regard, some researchers aimed to give an appropriate formula for this topic. Authors in [8] proposed an expression for estimating the effective depth at the critical section, defined in (11).

$$d_{cr} = do + (C_h - S).\tan\alpha \quad (11)$$

$$C_h = do + \frac{do(1 + \tan\alpha) - S\tan\alpha}{0.68 - \tan\alpha} \approx 2.7do \quad (12)$$

Equation (13) was proposed in [12] to predict the critical effective depth (d_{cr}):

$$d_{cr} = d_{min} = [1 + 1.35\tan\alpha]$$

$$\leq \left[\left(\frac{h_{max}.h_{min} - h_{max}^2}{2l_h} + h_{max}\right) - r\right] \quad (13)$$

where h_{max} is the maximum depth of the beam, h_{min} is the minimum depth of the beam, r is the concrete cover, d_{min} is the effective depth at maximum depth, and l_h is the non-prismatic length.

Authors in [13] formalized (14) to predict the effective depth at the critical section by the consideration of the effective depth of the RCHBs on support. This equation was introduced in the experimental part of the study. The proposed range of the inclination angle lies between -14.62° and +14.62°.

$$h_c = \lambda \times h_s \quad (14)$$

$$\lambda = (1 - 3.04 \tan \propto)^{-0.608} \leq 1.55 \quad (15)$$

where h_s is the cross-sectional depth at support, h_c is the critical depth, and α is the inclination angle.

Hassan [14] suggested the effective section at approximately a quarter of the length of RCHBs for beams without openings. Two mechanical models for failure were proposed in [13] depending on the position of the major diagonal shear crack. The proposed models differ from each other according to the contribution of the internal stresses.

III. THE PROPOSED FORMULA

The proposed equation consists of five discrete components as stated in (16). These components represent the contribution of the internal stresses to the proposed mechanical models as follows:

$$V_u = \emptyset[Vc + V_v + V_d + V_F + V_N] \quad (16)$$

where $\emptyset$ is the shear strength reduction factor, which equals to 0.75, V_u is the ultimate shear strength resulting from the contributions of each concrete Vc, V_v is the shear reinforcement, V_d is the diagonal reinforcement, V_F the inclined flexural reinforcement, and V_N is the compression chord contribution.

A. *Contribution of Concrete (Vc)*

The ACI-318 approach predicts the shear strength of the reinforced concrete prismatic beams without shear reinforcement. This approach has been modified in [7] involving the subtraction of the depth of the opening from the effective depth. This information was modulated to (17), in which the effective depth d was replaced by the effective depth at a critical depth dc (from 14)) and do is the diameter or depth of the opening:

$$Vc = \left[0.17\sqrt{fc'}\right] bw.(dc - do) \quad (17)$$

For prestressed RCHBs with openings, the tension chord (lower chord) has been cracked at the ultimate stress, while the compression chord carried all the shear (upper chord). This is a safe approach because even after cracking, the lower chord will carry shear. The shear capacity of the compression chord can be found using the formula proposed in [15] for beams having prismatic sections, in which the balanced normal forces of lower chord tension and the compressive upper chord took into consideration the inclination of the upper chord ($\propto$) in calculating the axial force.

$$Vc = 2\left(1 + \frac{Nu \cos\propto}{2000Ag}\right)\sqrt{fc'}bw.d < 0.17\sqrt{fc'}bw.d \quad (18)$$

where

$$u{=}As\ fy + Aps\, fps \quad (19)$$

where bw, d, and Ag are the width, depth, and the cross-sectional area of the compression chord respectively, fc' is the concrete strength, Nu is the axial compression force in the upper chord, and Vc is the nominal shear strength provided by concrete.

B. *Contribution of Shear Reinforcement (V_v)*

Shear reinforcement has a significant effect on the beam capacity. Therefore, the corresponding equation of ACI 318 was used to calculate the contribution of shear reinforcement (V_v). Mansur [7] found that obtainable stirrups to support shear across the failure plane are those at the sides of the opening within a distance $(dv - do)$, where dv is the distance between the top and bottom longitudinal rebars, and do is the opening's diameter (or depth). On the other hand, for non-prismatic beams, (19) has been adopted for RCBHs with openings. The dv is replaced by the dvc referring to the distance between the centroids of extreme tension and compression reinforcement layers at the critical section according to (14) and the parameters Av, S and, fyv represent the area, spacing, and yield strengths of the steel stirrups respectively (Figure 1).

$$V_v = \frac{Av\, fyv}{s}(dvc - do) \quad (20)$$

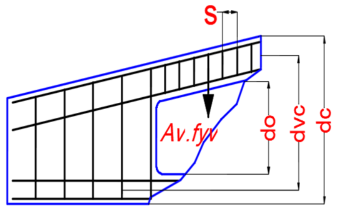

Fig. 1. Shear resistance components.

C. *Contribution of Diagonal Reinforcement (V_d)*

When diagonal reinforcement is used in an RC beam, energy absorption is improved and ensures that the beam with openings will reach its ultimate flexural capacity which is referred to as the flexural failure. Mansur [7] converted the contribution of the additional diagonal steel reinforcement by adding the slope angle of reinforcement. The contribution of diagonal reinforcement may be calculated by:

$$V_d = Ad\ fyd \sin \propto \quad (21)$$

D. *Contribution of Inclined Flexural Reinforcements (V_F)*

In the case of non-prismatic section with a horizontal upper chord and an inclined lower one, the nominal shear contribution of concrete and of the longitudinal reinforcement is affected by the non-prismatic inclination flexural reinforcements concerning each prestressing or mild steel due to the vertical component of forces and the truss action of the inclined steel bars, as shown in Figure 2. Equation (22) is developed to calculate the contribution of the inclined flexural reinforcements by adding the angle of inclined flexural strength which affects the ultimate shear strength.

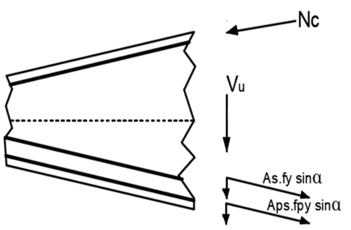

Fig. 2. Force actions of the varied-depth concrete members.

$$V_{FS} = As\ fy \sin \propto$$
$$V_{FP} = Aps\ fpy \sin \propto \quad (21)$$
$$V_F = V_{FS} + V_{FP}$$

where V_{FP}, V_{FS} are the shear resistance due to prestressed and mild steel forces respectively, and, from equalizing the tensile and compressive forces, V_F is the shear resistance due to the inclination of the compression chord of the beam.

E. Contribution of Compression Chord (V_N)

All the previous design methods to define the nominal shear strength of RCHBs neglect the compression chord influence. The new proposed formula includes the contribution of compression chord (NC) which has been found from the flexural equilibrium. At a vertical section along the beam as shown in Figure 3, the horizontal forces can be represented by two components: the horizontal component of the compressive force (NCh) equalized to the lower tensile force (Nt). These components have the values (from equilibrium):

$$Nch = Nt = (As\ fy + Aps\ fps) \quad (22)$$

The compression resultant NC acts at an angle α to the horizontal. The vertical component of the compressive resultant Ncv has the value seen in (23).

$$tan\alpha = \frac{Ncv}{Nch}, Ncv = Nt\ tan\alpha,$$
$$V_N = Ncv = (As\ fy + Aps\ fps) tan\alpha \quad (23)$$

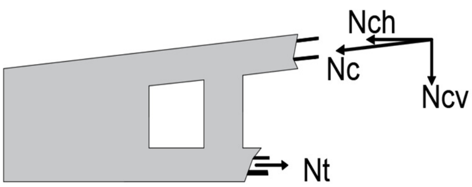

Fig. 3. Compression chord influence.

IV. EXPERIMENTAL DATABASE

The experimental database, consisting of the test results of 26 RCHBs, was compiled from [14, 16]. All tested beams were simply supported, had a rectangular cross-section with a width of 100mm, and varying height from 400mm at the center to 250mm at ends. All beams have a total length of 3000mm and a clear span of 2800mm. The one-point loading system was used. The experimental program conducted in [14] consisted of casting and testing 13 beams, including a reference beam without openings (solid). The details of reinforcement are the same for all the tested beams and consisted of 4-Ø6mm coordinated in two layers in the compression chord and 2-Ø6+2-Ø12mm coordinated in two layers in the tension chord. Closed shear stirrups, manufactured out of 4mm bars, were supplied over the entire span of the upper and lower chords of the tested specimens with openings at a constant spacing of 50mm. Table I shows the parametric details of the tested beams. Al-Khafaji [16] progressed the study conducted by Hassan [14] by using prestressing reinforcement, keeping the same experimental parameters.

V. MODEL VERIFICATION

A. Statistical Correlations

The proposed formula has been verified with the experimental results [14, 16]. It should be mentioned that from the five shear strength contributions of the predicated formula in (16), only three of them have been verified because no diagonal steel reinforcement nor inclined tensile reinforcing were used. Table II shows the comparison of the experimental results and the predicted shear strength values resulting from the proposed formula. The correlations of the results are represented by the Coefficient Of Variation (COV). The correlation factors for the experimental results of [14] to the proposed formula are: the average $\frac{Vexp}{Vpre}$ = 0.866, the standard deviation is 0.102 and the COV is 0.102. The correlation factors for the experimental results of [16] to the proposed formula are the average $\frac{Vexp}{Vpre}$ = 0.889, the standard deviation is 0.08, and the COV is 0.089. The statistical parameters stated above proved the good performance of (16) in [14, 16]. These results reflect that using the predicted formula to find the shear strength of non-prismatic beams with openings is conservative and can be used for design and analysis.

B. Analysis Example

We analyze the web openings for GT6 (3m) span, with fc' of 35mPa, critical section from (14) λ = (1-3.04tan 5.906)$^{-0.608}$= 1.258 ≤ 1.55, and h_c =315mm at a distance of 625mm from the support pass through the first opening near the support.

Contribution of concrete (Vc):

$$Vc = \left[0.17\sqrt{35}\right] 100.(295 - 120) = 17.5\ kN$$

Contribution of shear reinforcement (Vs):

$$Vv = \frac{25\times370}{50}(275 - 120) = 28.5kN$$

Contribution of compression chord (VN):

$$V_N = (226 \times 600 + 56.5 \times 550) tan5.9 = 17.25kN.$$

So, we get:

$$Vu_{predicated.} = 0.75(17.5+28.5+17.25) = 47.43\text{kN}$$

VI. CONCLUSIONS

- The common failure mode for non-prismatic beams with openings is diagonal shear failure which does not exceed beam-type failure or frame-type failure.
- The shear strength contributions of the non-prismatic beam with openings consists of concrete (V_C), shear reinforcement (V_v), diagonal reinforcement (V_d), inclined flexural reinforcements (V_F), and compression chord (V_N). Each has a different effect on the shear strength.
- The results reflect that the predicted formula to find the shear capacity of non-prismatic beams with openings is conservative and can be used for design with the difference between the experimental results reaching 13% with 11.8% COV.

TABLE I. DETAILS OF THE TESTED BEAMS BY [14]

Group	Beam mark	Shape of openings	Number of openings	Total area of openings (mm^2)	Width of openings (mm)	Height of upper chord (mm)	Height of lower chord (mm)
A	GB	------	------	0	------	------	------
A	GT6	Trapezoidal	6	180000	200	100	100
A	GT8	Trapezoidal	8	174000	150	100	100
A	GT10	Trapezoidal	10	144000	100	100	100
B	GTH6	Trapezoidal	6	240000	200	75	75
B	GTH8	Trapezoidal	8	234000	150	75	75
B	GTH10	Trapezoidal	10	195000	100	75	75
C	GP6	Trapezoidal with inclined posts	6	154000	200	100	100
C	GP8	Trapezoidal with inclined posts	8	151000	150	100	100
C	GP10	Trapezoidal with inclined posts	10	138000	100	100	100
D	GC1	Circular	8	184200	D	75	75
D	GC2	Circular	8	128000	0.83D	100	100
D	GC3	Circular	8	82000	0.67D	120	120

TABLE II. ANALYSIS AND EXPERIMENTAL RESULTS OF TESTED BEAMS

Beam 's labeling [14]	$Vexp$ kN	$Vpre$ kN	$\frac{Vexp}{Vpre}$	Beam 's labeling [16]	$Vexp$ kN	$Vpre$ kN	$\frac{Vexp}{Vpre}$
GB	45	57	0.789	PGB	74.2	82.5	0.89
GT6	38.9	47.43	0.82	PGT6	66.1	74.25	0.89
GT8	40.1	47.66	0.84	PGT8	70	74.77	0.936
GT10	41	47.77	0.85	PGT10	71	75.75	0.937
GTH6	36.95	37.5	0.98	PGTH6	47.45	67.5	0.702
GTH8	38.4	38.25	1.00	PGTH8	52.3	67.87	0.770
GTH10	40.25	38.4	1.048	PGTH10	56	68.25	0.820
GP6	41.25	49.57	0.83	PGP6	69	73.5	0.938
GP8	41.9	50.7	0.826	PGP8	70.85	74.25	0.954
GP10	42.2	58.2	0.725	PGP10	71.6	78.75	0.909
GC1	40.75	41.32	0.98	PGC1	68.7	71.25	0.964
GC2	43	51.45	0.835	PGC2	71.9	75	0.958
GC3	43.95	59.7	0.736	PGC3	72.35	80.25	0.901
Average			0.866	Average			0.889
Standard deviation			0.102	Standard deviation			0.08
COV			11.8%	COV			8.9%

REFERENCES

[1] M. A. J. Hassan and A. F. Izzet, "Experimental and Numerical Comparison of Reinforced Concrete Gable Roof Beams with Openings of Different Configurations," *Engineering, Technology & Applied Science Research*, vol. 9, no. 6, pp. 5066–5073, Dec. 2019, https://doi.org/10.48084/etasr.3188.

[2] F. J. Alkhafaji and A. F. Izzet, "Prestress Losses in Concrete Rafters with Openings," *Engineering, Technology & Applied Science Research*, vol. 10, no. 2, pp. 5512–5519, Apr. 2020, https://doi.org/10.48084/etasr.3390.

[3] C. Hou, K. Matsumoto, and J. Niwa, "Shear failure mechanism of reinforced concrete haunched beams," *Journal of JSCE*, vol. 3, no. 1, pp. 230–245, May 2015, https://doi.org/10.2208/journalofjsce.3.1_230.

[4] M. A. J. Hassan and A. F. Izzet, "Serviceability of Reinforced Concrete Gable Roof Beams with Openings under Static Loads," *Engineering, Technology & Applied Science Research*, vol. 9, no. 5, pp. 4813–4817, Oct. 2019, https://doi.org/10.48084/etasr.3110.

[5] P. K. Naik and M. Manjunath, "Pushover Analysis of Multi-Storey Frame Structure with Haunched Beam," *International Journal of Trend in Research and Development*, vol. 4, no. 3, pp. 333–336, Jun. 2017.

[6] ACI Committee 318, *ACI CODE-318-14: Building Code Requirements for Structural Concrete and Commentary*. ACI, 2014.

[7] M. A. Mansur, "Effect of openings on the behaviour and strength of R/C beams in shear," *Cement and Concrete Composites*, vol. 20, no. 6, pp. 477–486, Jan. 1998, https://doi.org/10.1016/S0958-9465(98)00030-4.

[8] *Standard for the Structural Calculation of Reinforced Concrete Structures*. Tokyo, Japan: Architectural Institute of Japan, 1988.

[9] S. Y. Debaiky and E. I. Elniema, "Behavior and Strength of Reinforced Concrete Haunched Beams in Shear," *Journal Proceedings*, vol. 79, no. 3, pp. 184–194, May 1982, https://doi.org/10.14359/10896.

[10] *DIN 1045-1:2001-07. Concrete, reinforced and prestressed concrete structures - Part 1: Design and construction*. Berlin, Germany: Beuth Verlag, 2001.

[11] I. A. Macleod and A. Houmsi, "Shear Strength of Haunched Beams Without Shear Reinforcement," *ACI Structural Journal*, vol. 91, no. 1, pp. 79–89, Jan. 1994, https://doi.org/10.14359/4482.

[12] A. Tena-Colunga, H. I. Archundia-Aranda, and Ó. M. González-Cuevas, "Behavior of reinforced concrete haunched beams subjected to static shear loading," *Engineering Structures*, vol. 30, no. 2, pp. 478–492, Oct. 2008, https://doi.org/10.1016/j.engstruct.2007.04.017.

[13] M. E. Gulsan, H. M. Albegmprli, and A. Cevik, "Finite element and design code assessment of reinforced concrete haunched beams," *Structural Engineering and Mechanics*, vol. 66, no. 4, pp. 423–438, 2018, https://doi.org/10.12989/sem.2018.66.4.423.

[14] M. A. Hassan, "Strength and Serviceability of Reinforced Concrete Gable Roof Beam with Opening of Different Sizes," Ph.D. dissertation, University of Baghdad, Baghdad, Iraq, 2020.

[15] J. M. Savage, M. K. Tadros, P. Arumugasaamy, and L. G. Fischer, "Behavior and Design of Double Tees With Web Openings," *PCI Journal*, vol. 41, no. 1, pp. 46–62, Jan. 1996, https://doi.org/10.15554/pcij.01011996.46.62.

[16] F. J. A. Alkhafaji, "Performance of Perforated Prestressed Concrete Gable Rafters," Ph.D. dissertation, University of Baghdad, Baghdad, Iraq, 2020.