A Numerical Approach for the Determination of Mode I Stress Intensity Factors in PMMA Materials

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***Abstract*—An evaluation technique of the K_I stress intensity factors (SIF) by a numerical investigation using line strain method is presented in this paper. The main purpose of this research is to re-analyze experimental results of fracture loads from polymethyl-metacrylate (PMMA) specimens (fully finite plates). Stress intensity factor equation calculation is derived from the Williams stress asymptotic expansion. Possible error caused by strain gradients across the gage length is minimized by integrating the equation in the K_I calculation. Theoretical and computed values using finite element analysis of stress intensity factors are compared with experimental results. A good agreement is observed between the present approach and experimental values. It is shown that, in the case of a through-plate crack, the stress intensity factor can be calculated with adequate accuracy using the proposed method.**

Keywords-Finite element; strain gage method; K_I; PMMA edge crack

I. INTRODUCTION

All standard paper components have been specified for three reasons: The measurement of the stress intensity factor with strain gages was first suggested by Irwin in 1957 [1]. Due to the local yielding effect in the crack vicinity, little progress has been made in implementing Irwin's suggestion. A valid strain gage technique for calculating the stress intensity factor K_I was first presented in [2-4]. In this method, a valid region was specified for locating the gages to get rid of the elastic-plastic crack-tip state caused by local yielding in the innermost region close to the crack tip. The error caused by strain gradient was minimized by placing the virtual strain gages sufficiently far from the crack tip. An extended over-deterministic approach was later proposed to significantly improve the accuracy [5].

A specified strain gage pattern has been proposed to measure the stress intensity factors of mixed mode problems [6]. Moreover, different strain gage approaches have been introduced to measure the variation of stress intensity factor of a propagating crack [7-12]. It has long been acknowledged that in plates of finite thickness the stress field near the crack tip is three-dimensional in nature. From the finite element results of [13, 14, 26-30] and the experimental results of [15] it has been shown that the local yielding effect is only significant within a radial distance about one-half thickness from the crack tip. For the rest of the area, the plane stress condition is expected to dominate.

The choice of the position and orientation of the point P for the calculation of K_I are studied by using virtual gages with two orientation angles (ϴ, α), as shown in Figure 1, when considering a line segment in a strain field near the crack tip. A different load and geometry of specimen are also employed to visualize the geometry effect on the evaluated stress intensity factors.

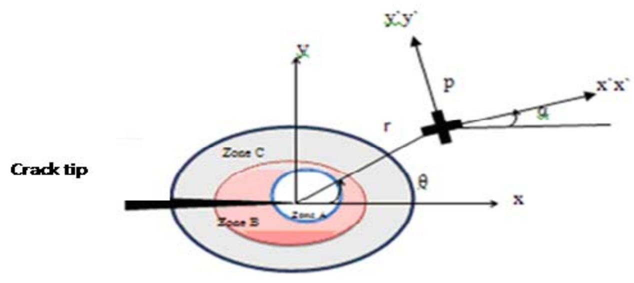


Fig. 1. Deferent area delimiting the vicinity of a crack tip.

In this work, we investigate the feasibility of the numerical determination of accurate opening mode of stress intensity factors using the Dally–Sanford method [2] for the cracked polymethyl-methacrylate (PMMA) specimens. In this procedure, gages can be placed at low strain gradient zones. In contrast with past research described in [2, 16-19] this work attempts to accurately calculate the SIF using a relative large strain points method (virtual strain gage) under monotonic increasing loads in fully finite edge-cracked plates subject. To validate the proposed method for the determination of SIFs, we compare experimental results [10] with our computer-calculated values obtained using ABQUS finite-element software.

II. BASIC THEORY AND MATHEMATICAL FORMULATION OF THE MODEL

The area adjacent to the crack tip was divided into three regions as shown in Figure 2. Region I very near the crack tip is invalid because of three-dimensional effects. Region III far from the crack tip is invalid because the truncated series solution does not adequately describe the strain field. Region II located between regions I and III is a valid area where the truncated-series solution represents the strain field to a specified accuracy. The size and shape of region II is presented for the compact-tension geometry. To identify a valid location for the virtual strain gages, Region II was subdivided into regions IIa and IIb. Region lIb was discarded because the strains in this area are too low for accurate measurement [2, 20].

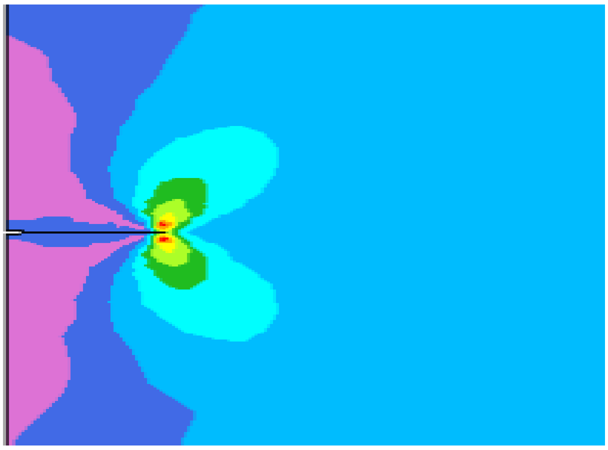


Fig. 2. Deferent area delimiting the vicinity of a crack tip.

Dally and Sanford [7] adopted an approach to three parameters and assumed that the deformation field in region II can be represented with sufficient accuracy by the three terms of the William's series [20]. The deformation field in this region for a state of plane stress is then written in the following form [9]:

$$E\varepsilon_{xx}=A_0 r^{-1/2}\cos(\theta/2)\,[(1-\nu)-(1+\nu)\sin(\theta/2)\sin(3\theta/2)] +2B_0+A_1 r^{1/2}\cos(\theta/2)[(1-\nu)+(1+\nu)\sin^2(\theta/2)] \quad (1)$$

$$E\varepsilon_{yy}=A_0 r^{-1/2}\cos(\theta/2)[(1-\nu)+(1+\nu)\sin(\theta/2)\sin(3\theta/2)] -2\nu B_0+A_1 r^{1/2}\cos(\theta/2)[(1-\nu)-(1+\nu)\sin^2(\theta/2)] \quad (2)$$

$$\mu\gamma_{xy}=A_0 r^{-1/2}\sin\theta\cos(3\theta/2)-A_1 r^{1/2}\sin(\theta/2)\cos(\theta/2)^2 \quad (3)$$

Where A_0, A_1 and B_0 are unknown coefficients which can be determined by using the specimen geometry and boundary conditions. Using the definition of K_I one can easily show that it is related to A_0 by: $K_I=\sqrt{2\pi}A_0$ [2-11].

A single strain gage is sufficient to evaluate the constant A_0 (hence K_I) by placing and orienting the strain gage as given below.

Using transformation equations of the deformation component ε_{xx} at the point P located at r and θ (Figure 1) [2, 4, 10] is given by:

$$E\varepsilon_{xx}=(1-\nu)[A_0 r^{-1/2}[\cos(\theta/2)]+A_1 r^{+1/2}[\cos(\theta/2)] -(1+\nu)[A_0 r^{-1/2}\sin(3\theta/2)\sin(\theta/2)\cos(\theta/2) -A_1 r^{+1/2}\sin^2(\theta/2)\cos(\theta/2)]+2B_0 \quad (4)$$

$$k=-\frac{1-\nu}{1+\nu} \quad (5)$$

the B_0 coefficient of (3) can be removed by selecting the angle α, so that

$$\cos 2\alpha=-k=-\frac{1-\nu}{1+\nu} \quad (6)$$

The coefficient A_1, can also be made zero by replacing α using:

$$\tan(\theta/2)=-\cot 2\alpha \quad (7)$$

Deformation $\varepsilon_{x'x'}$ is given by:

$$2G\varepsilon_{x'x'}=\frac{K_I}{\sqrt{2\pi r}}(k\cos\frac{\theta}{2}-\sin\theta/2\sin 3\frac{\theta}{2}\cos 2\alpha+ \frac{1}{2}\sin\theta\cos 3\frac{\theta}{2}\sin 2\alpha) \quad (8)$$

The angles α and θ depend only on the Poisson's ratio of the material. The location of the radius r_p strain gauge is chosen taking into account the effects of strain gradient [2-10].

III. NUMERICAL PROCEDURES

A. *Material constitutive model*

Specimens are made of polymer PMMA (Plexiglas sheet) [10]. Material properties are E=2300 MPa, υ=0.37, K_{IC} = 1.9 $MPam^{0.5}$ and G_{Ic}= 0.4 kJm^{-2}.

B. *Finite element model*

Natural elements iso-parametric quadratic triangular (T6) and the singular point due to symmetry is used and only half of the plate was taken into account in the analysis. Figure 4 shows a typical mesh used for determining the standard SIF. These meshes were used for all values of a/W and h/W. When the mesh was refined significant improvements were noted in the values of SIF [25].

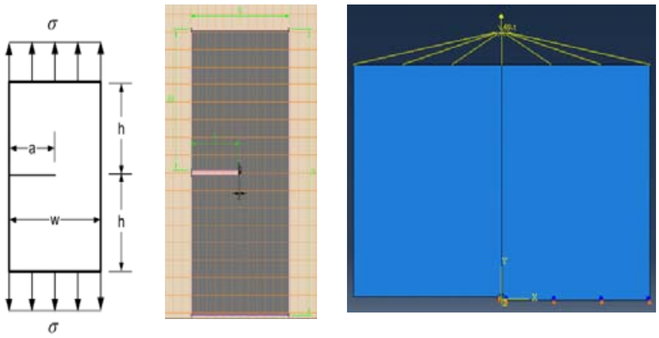
The SIF of a plate of finite width cracked edge is given by:

$$K_I=\sigma\sqrt{2\pi}F_1\left(\frac{a}{w},\frac{h}{w}\right) \quad (9)$$

Where $F_1\left(\frac{a}{w},\frac{h}{w}\right)$ is the normalized function which shows the effect of the geometry of the specimen [10, 32].

C. *Loading and boundary conditions*

Fully finite edge-cracked plates specimen with key dimensions are shown in Figure 3b. The thickness was 6 mm in a plane strain state. The width of all samples was maintained at W = 150 mm, while the length of the crack and the height h of the sample were changed. For the simulation, the values of a/W were 0.3 and 0.5, and the value of h/W was equal to 0.2 to 1.1 in step of 0.1. The load is pointed and semi distributed uniformly as shown in Figure 3c.



a) Specimen studie b) Geometry of the specimen. c) Load and boundary conditions of half specimen.

Figure.3 centrally notched rectangular specimen under uniaxial tension.

D. Computational code

A finite element analysis using ABAQUS 6.11.1 [25] was conducted. The extrapolation method of integral J was used to calculate standardized SIF mode I opening [31]. We know that this method is consistent and profitable for a very specific SIF. Corresponding to the element the value of 0.25 was used [24].

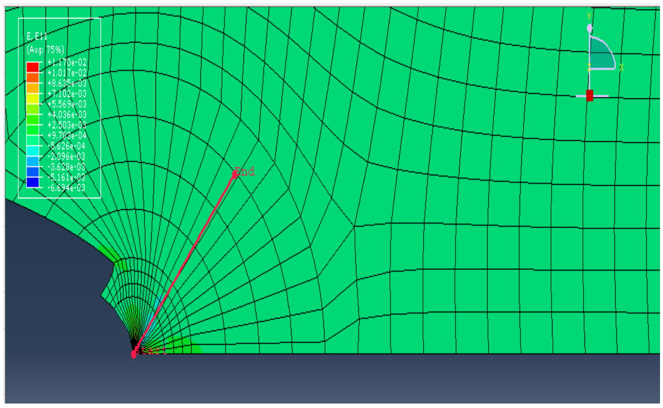


Fig. 4. A typical finite element mesh used for modeling the specimen, the Integration contours for evaluation of the M-integral and the line strain.

θ and α in (4) and (5) are equal to 58.69° and 54.76° respectively. The radial position of the strain gage r=10 mm was chosen in all tests based on the stress gradient analysis presented in [12]. Some results of SIF calculated in digital studied samples are shown in Figures 5 and 6.

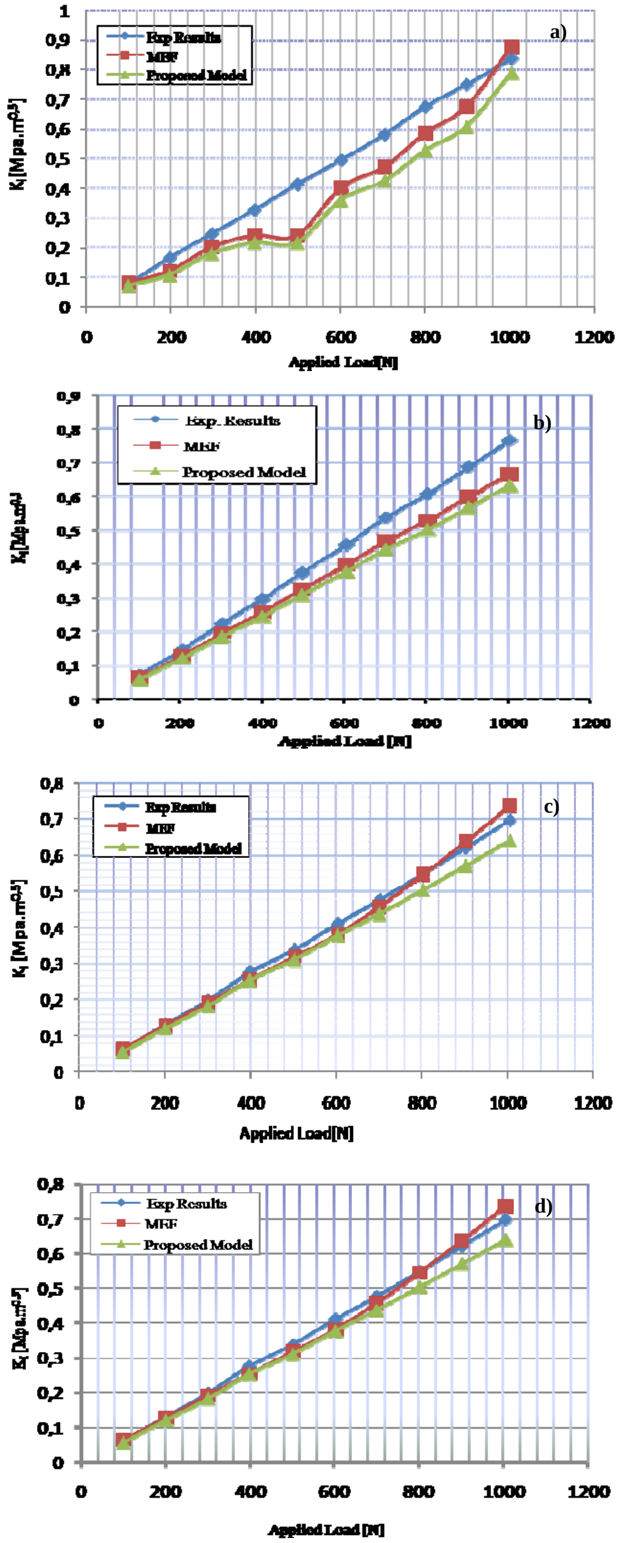


Figure .5: A comparison between the Variations values of normalized and experimental SIF [10] and with different values of h/w; Parameters adopted: a / w = 0, 3

(a) h/w =0,3 et (b) h/w =0,5

(c) h/w = 0,7 et (d) h/w =1,0

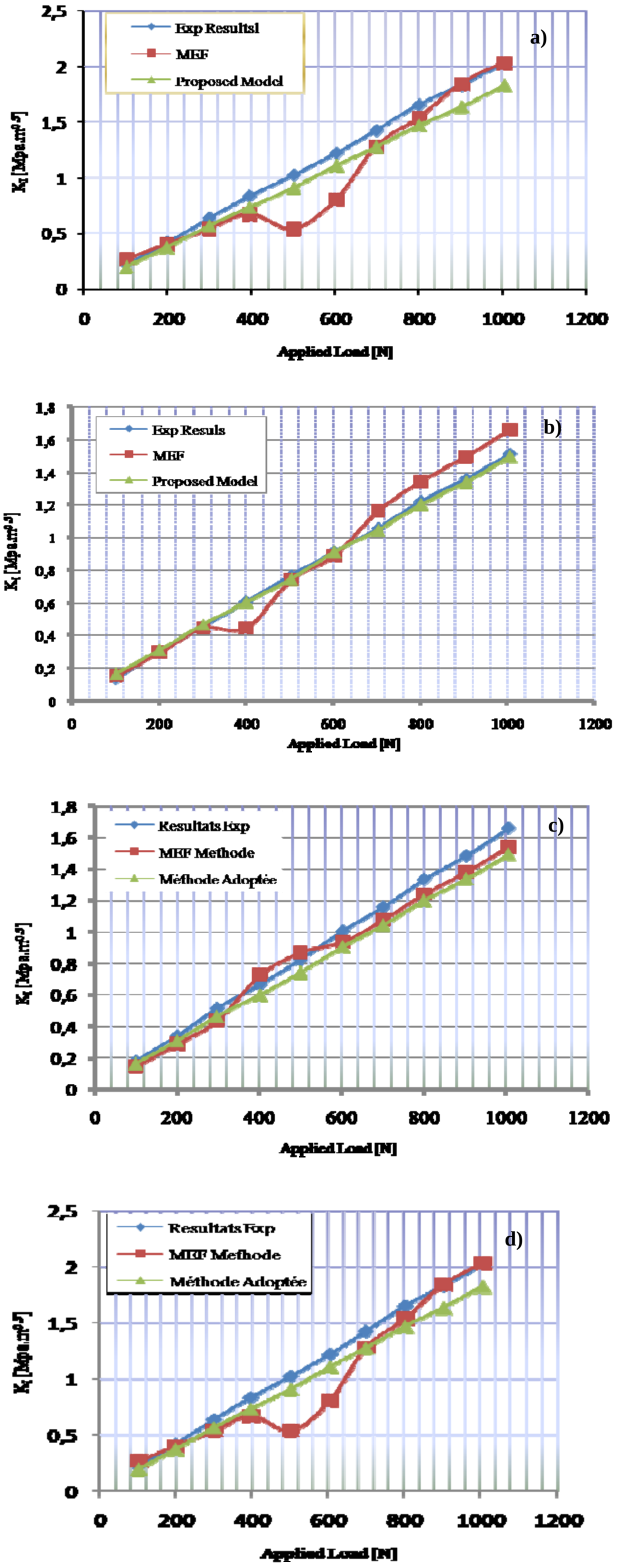


Figure.6: A comparison between the Variations values of normalized and experimental SIF [10] and with different values of h/w; Parameters adopted: a / w = 0, 5
(a) h/w =0,3 et (b) h/w =0,5
(c) h/w = 0,7 et (d) h/w =1,0

IV. RESULTS AND DISCUSSION

Figures 5 and 6 show the deformation data measured experimentally [10] and the results obtained by the proposed model for a/w = 0.3 and 0, 5 with h/w = 0.3, 0.5, 0.7, 1.0.

A very good linear relationship can be noted between the load and the deformation in all figures for the eight samples. From the linear equation presented in the literature, the deformations (measured) were calculated using the proposed model for different loads. Thereafter, SIF values were determined by finite elements analysis (Abaqus). Figures 5 and 6 shows the dependence between SIF (K_I) and the different loads. The figures show that the measured strains are linearly proportional with the applied load.

V. CONCLUSIONS

The results of this work allow us to draw the following conclusions:

- The digital program that was used for the study of a PMMA's sample applying a process of the single virtual strain gage, had showed that this latter is the most powerful tool for the measurement of the SIF (K_I) in the PMMA's specimen as was demonstrated in the present study.
- The latter conclusion is confirmed by the relative consistency between the FIC values calculated based on this approach, and those obtained experimentally.
- That said, the proposed approach represents an alternative tool in the sense of quantitative and qualitative effectiveness for the accurate measurement of FIC in a polymer plate with finite dimensions.
- The results obtained in this study show that the use of a single strain gage is very effective in the case of PMMA.

The major interest of this technique is its ability to approach the problem in more complex configurations.

A continuation of this work suggests a coupling between the T-Stress and K_I, with more complicated loading. An extension will be proposed for different geometries, and of course, a validation on polymeric materials other than PMMA will be considered in order to make a conclusion on the generalization of the proposed approach.

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