Global Sensitivity Analysis of In-plane Elastic Buckling of Steel Arches

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Abstract—**Steel arches are widely used in civil engineering and industrial structures. Their response depends on material properties, geometric dimensions, and boundary conditions. The objective of the current study is to perform global sensitivity analysis, and to assess the influence of random input parameters on the in-plane elastic buckling of steel arches. The in-plane elastic buckling load of steel arches under uniform compression proposed in previous studies is adopted. The influence of the random input variables of the structure is evaluated using Sobol's global sensitivity analysis. Monte Carlo simulation is also employed to rank the influence of input random variables.**

Keywords-in-plane elastic buckling; steel arches; sensitivity; global sensitivity; Sobol' indices; Monte Carlo simulation

I. INTRODUCTION

Global sensitivity consists of the individual sensitivity of each variable and its interaction with other variables. Sobol's sensitivity index is widely applied in the structural engineering field [1-5]. Steel structures often present high durability while including slender components. Sensitivity and reliability analyses for steel structures have been conducted by many researchers [6-12]. Steel arches are widely used in civil engineering and industrial structures. Arches resist external loadings by a combination of predominant axial compression and bending actions. The geometry of circular steel arches is shown in Figure 1, and the uniform compression on the steel arches is shown in Figure 2. If the steel arches are adequately designed with lateral bracings so that their out-of-plane failure is fully prevented, in-plane buckling failure may occur [13, 14]. The in-plane elastic buckling is important in the stability design of metal arch structures [15]. The in-plane elastic buckling of shallow arches with an arbitrary cross-section was investigated and the in-plane elastic buckling load of shallow arches was found to be much lower than that given by the classical theory [16, 17]. Moreover, the classical in-plane buckling load should not be used as the reference load in the development of design methods for the in-plane strength of shallow steel arches.

This paper studies the sensitivity assessment of the input random variables on the in-plane elastic buckling load of the steel arches under uniform compression. An algorithm using the global sensitivity method is proposed for utilization in analyses and assessments. The in-plane elastic buckling load of steel arches under uniform compression has been proposed in

[14]. In addition, the effects of input random variables on the in-plane elastic buckling load of the steel arches under uniform compression are also examined.
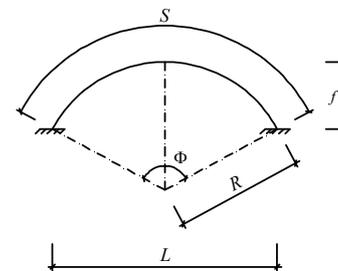

Fig. 1.   Geometry of the arches.
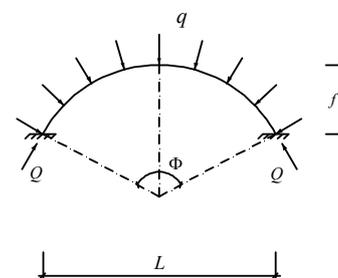

Fig. 2.   Uniform compression.

II. THEORETICAL FRAMEWORK

A. In-plane Elastic Buckling of Steel Arches

In this section, the buckling load of arches proposed in [14] is presented. Arches with an included angle $\Phi < 90^{\circ}$ are considered as shallow arches and the dimensionless parameter λ is defined as:

$$\lambda = \frac{S\Phi}{4r_x} \quad (1)$$

where S is the length of the arch, $r_x = \sqrt{I/A}$ is the radius of gyration of the cross-section, I is the second moment of the area of the cross-section about its major principal axis, and A is the area of the cross-section.

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The buckling load of the pin-ended shallow arch can be defined as in (2) and the buckling load of the fixed shallow arch is expressed as in (3):

$$N_{acr} = \begin{cases} \left(0.15 + 0.006\lambda^2\right)N_{cr}; & \text{for } 3.88 < \lambda < 9.38 \\ \left(0.26 + 0.74\sqrt{1 - 0.63\pi^4/\lambda^2}\right)N_{cr}; & \text{for } \lambda > 9.38 \end{cases} \quad (2)$$

$$N_{acr} = \begin{cases} \left(0.36 + 0.0011\lambda^2\right)N_{cr}; & \text{for } 9.87 < \lambda < 18.6 \\ \left(0.6 + 0.4\sqrt{1 - 3.109\pi^4/\lambda^2}\right)N_{cr}; & \text{for } \lambda > 18.6 \end{cases} \quad (3)$$

When an arch has an included angle $\Phi > 90^{o}$, it is considered to be a deep arch, and its elastic buckling load is given by [14]:

$$N_{acr} = \left[1 - \left(\Phi/\pi\right)^2\right]N_{cr}; \quad \text{for pin-ended arches} \quad (4)$$

$$N_{acr} = \left[1 - \left(\Phi/1.4304\pi\right)^2\right]N_{cr}; \quad \text{for fixed arches} \quad (5)$$

where N_{cr} is the second mode elastic flexural buckling load of a corresponding fixed or pin-ended column with the same length. N_{cr} is determined as:

$$N_{cr} = \frac{\pi^2 EI}{(kS)^2}, \text{ with } k = \begin{cases} 0.5; & \text{for pin-ended arches} \\ 0.35 & \text{for fixed arches} \end{cases} \quad (6)$$

where E is the Young's modulus and I is the second moment of the area of the cross-section about its major principal axis.

B. Global Sensitivity Analysis

Sensitivity presents the influence of input design parameters (i.e. variables) $\boldsymbol{X} = \left(X_1, X_2, ..., X_m\right)$ on the output model $\boldsymbol{Y} = f\left(\boldsymbol{X}\right)$. Here, $\boldsymbol{X} = \left(X_1, X_2, ..., X_m\right)$ is the vector of design parameters in $\mathbb{R}^m$ space and $\boldsymbol{Y} = \left(Y_1, Y_2, ..., Y_n\right)$ is the vector of output value in $\mathbb{R}^n$ space. Let us consider an integral function $\boldsymbol{Y} = f\left(\boldsymbol{X}\right)$ which can be decomposed into elementary functions, expressed as [5]:

$$f\left(\boldsymbol{X}\right) = f_0 + \sum_i f_i\left(X_i\right) + \sum_{i<j} f_{ij}\left(X_i, X_j\right) + \cdots$$
$$\cdots + f_{1\cdots m}\left(X_1, ..., X_m\right) \quad (7)$$

where X_i represents the input random variables with independent probability while the variance of the output variable can be expressed as [4, 5, 18]:

$$Var[Y] = \sum_{i=1}^{m} V_i + \sum_{1 \leq i < j \leq m}^{m} V_{ij} + ... + V_{1...m} \quad (8)$$

where

$$V_i = Var\left[E\left[Y|X_i\right]\right]; \ V_{ij} = Var\left[E|Y|X_iX_j\right] - V_i - V_j$$

$$V_{1...n} = Var[Y] - \sum_{i=1}^{m} V_i - \sum_{1 \leq i < j \leq m}^{m} V_{ij} - \sum_{1 \leq i_1 < ... < i_{n-1} \leq m}^{m} V_{i_1...i_{m-1}}$$

The first-order Sobol's indices are determined by:

$$S_i = \frac{V_i}{Var[Y]} \quad (9)$$

The second-order Sobol's indices are determined by:

$$S_i = \frac{V_{ij}}{Var[Y]} \quad (10)$$

The total-order Sobol's indices are then determined by:

$$S_{Ti} = S_i + S_{ij} + S_{ikl} + ... + S_{i...n} \quad (11)$$

The Sobol's indices are commonly estimated with the Monte Carlo method [19, 20]. In this research, the Sobol's indices of global sensitivity with Monte Carlo simulation are established with the use of Matlab, the flowchart of the procedure is shown in Figure 3.
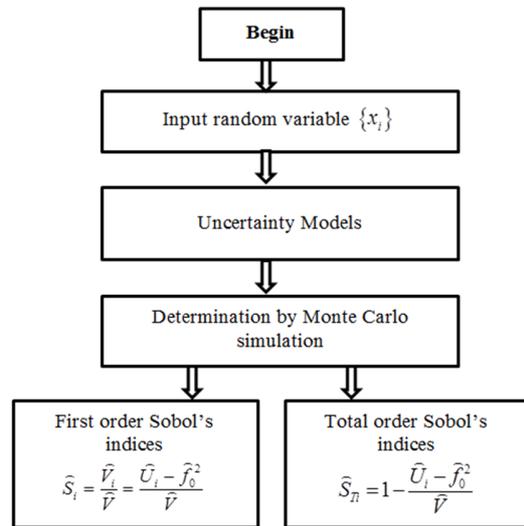

Fig. 3.   Flowchart of the Sobol's indices sensitivity analysis by Monte Carlo simulation.

III.   METHODOLOGY

A. Safe Condition

According to [14], the safe condition of the in-plane elastic buckling load of steel arches, determined by (3)-(5), can be rewritten as follows:

$$N_{saf} \leq N_{acr} \quad (12)$$

where N_{saf} is the external load, which can fluctuate randomly.

B. Deterministic Model and Uncertainty Model

Deterministic model is the above buckling analysis problem, in which the input parameters are those of geometry $\left(S, B, D, t_w, t_f, L, \Phi\right)$ and the Young's modulus of elasticity of

steel (E). This model can be written in the form of $\boldsymbol{X} = \left[S, B, D, t_w, t_f, E, L, \Phi \right]$:

$$N_{acr} = \Im(\boldsymbol{X}) \quad (13)$$

The uncertainty model is constructed based on the deterministic model by taking into account the randomness of input parameters. The parameters assumed to be random are $\boldsymbol{X} = \left[S(\omega), B(\omega), D(\omega), t_w(\omega), t_f(\omega), E(\omega), L(\omega), \Phi(\omega) \right]$, where ω represents the randomness of the parameters. This model can be written as:

$$N_{acr}(\omega) = \Im\big(\boldsymbol{X}(\omega)\big) \quad (14)$$

C. Global Analysis by Monte Carlo Simulation

Sensitivity analysis for in-plane elastic buckling load of the steel arches has been performed using the global sensitivity, which is based on Sobol's sensitivity indices. Monte Carlo simulation is established with the use of Matlab. The flowchart is shown in Figure 4.
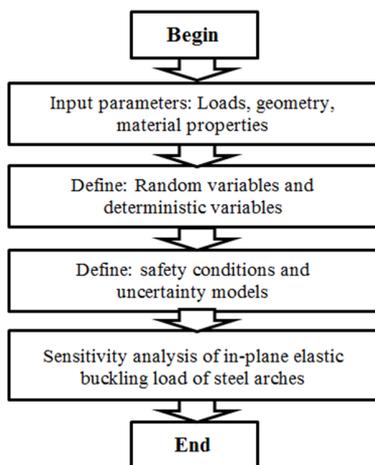

Fig. 4. Flowchart of the sensitivity analysis for in-plane elastic buckling load of steel arches based on the Sobol's indices of global sensitivity by Monte Carlo simulation.

IV. NUMERICAL RESULTS

A. Validation of the Matlab Code of Sobol's Indices Sensitivity Analysis

In order to validate the Matlab code of the Sobol's indices of global sensitivity by Monte Carlo simulation, a test function, namely Ishigami's function was employed, defined as [21]:

$$Y(X) = \sin X_1 + 7 \sin^2 X_2 + \frac{1}{10} X_3^4 \sin X_1 \quad (15)$$

where X_1, X_2, X_3 are uniform distributions between $-\pi$ and π. The numerical results are shown in Table I. A comparison with the results from [5] implies the reliability of the proposed program.

TABLE I.   MEAN ESTIMATION WITH 10,000 MONTE CARLO SAMPLES OF FIRST AND TOTAL SOBOL'S INDICES

X_i		Proposed code		Result in [5]		Error (%)	
		S_i	S_{Ti}	S_i	S_{Ti}	S_i	S_{Ti}
1	X_1	0.315	0.577	0.313	0.576	0.63	0.17
2	X_2	0.432	0.440	0.434	0.438	0.46	0.45
3	X_3	0.001	0.258	0.001	0.254	0.00	1.55

B. Sensitivity Analysis of In-plane Elastic Buckling Load of Steel Arches

Global sensitivity based on Sobol's sensitivity indices was applied to evaluate the influence of input random variables on the in-plane elastic buckling load of steel arches under uniform compression. The input parameters are divided in two cases:

Case study 1: The in-plane elastic buckling load of the fixed steel arches under uniform compression with input parameters $S, B, D, t_w, t_f, E, L, \Phi$.

Case study 2: The in-plane elastic buckling load of the pin-ended steel arches under uniform compression with input parameters $S, B, D, t_w, t_f, E, L, \Phi$.

1) Case Study 1

The fixed steel arch under uniform compression is shown in Figure 5. The input variables are shown in Table II. Global sensitivity based on Sobol's sensitivity indices is applied to evaluate the influence of input random variables on the in-plane elastic buckling load of the fixed steel arches under uniform compression. After 100,000 simulations in 67.0 minutes, the results of Sobol's sensitivity indices are shown in Table III and Figure 6.
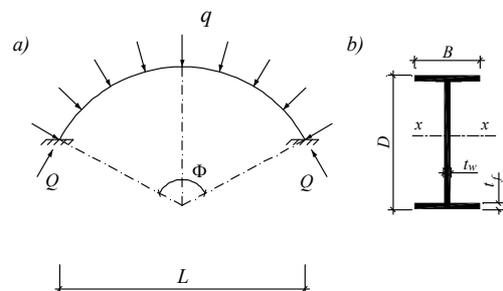

Fig. 5. The fixed steel arches under uniform compression and the cross-section of the arch.

TABLE II.   STATISTICAL PROPERTIES OF RANDOM VARIABLES FOR SOBOL'S INDICES SENSITIVITY ANALYSIS

Properties	Variables	Distribution	Range	Units
Geometry	S	Uniform	/6.283 – 7.679/	m
	L	Uniform	/9.00 – 10.00/	m
	Φ	Uniform	/0.628 – 0.768/	Rad
Material	E	Uniform	/18E5 – 22E5/	Mpa
Cross-section	D	Uniform	/0.235 – 0.287/	m
	B	Uniform	/0.136 – 0.166/	m
	t_w	Uniform	/0.007 – 0.008/	m
	t_f	Uniform	/0.011 – 0.014/	m

TABLE III.   MEAN ESTIMATION WITH 100,000 MONTE CARLO SAMPLES OF FIRST AND TOTAL SOBOL'S INDICES

	1st order effects	Total effects
X_1 (S)	0.2942	0.2819
X_2 (E)	0.0795	0.0690
X_3 (D)	0.6577	0.6510
X_4 (B)	0.0425	-
X_5 (t_w)	0.0425	-
X_6 (t_f)	0.0425	-
X_7 (Φ)	0.0425	-
X_8 (L)	0.0425	-

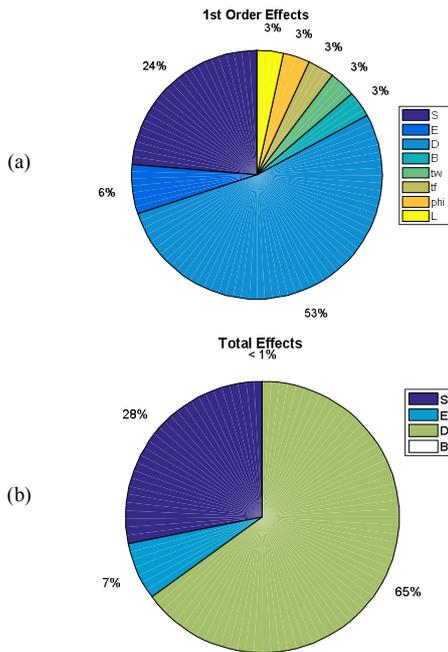Fig. 7.   The pin-ended steel arches under uniform compression and the cross-section of the arch.

Fig. 6.   (a) First order and (b) total Sobol's sensitivity indices of the fixed steel arches under uniform compression.

Fig. 8.   (a) First order and (b) total Sobol's sensitivity indices of the pin-ended steel arches under uniform compression.

TABLE IV.   MEAN ESTIMATION WITH 100,000 MONTE CARLO SAMPLES OF FIRST AND TOTAL SOBOL'S INDICES

	1st order effects	Total effects
X_1 (S)	0.2337	0.2788
X_2 (E)	0.0472	0.0738
X_3 (D)	0.6611	0.6844
X_4 (B)	0.2337	-
X_5 (t_w)	-	-
X_6 (t_f)	-	-
X_7 (Φ)	-	-
X_8 (L)	-	-

Table III and Figure 6 show the influence of input random variables on the in-plane elastic buckling load of the fixed steel arches under uniform compression based on Sobol's sensitivity indices. It can be seen that *S, E* and *D* are the most influential input random variables with proportions of 24.0%, 6.0%, and 53.0% of the first-order Sobol's sensitivity indices, respectively and the total order sensitivity indices are 28.0%, 7.0%, and 65.0%, respectively. Meanwhile, the least influential input random variables are *B, t_w, t_f, L* and Φ with a proportion of 3.0%. This is consistent with the qualitative concept and that means the importance of sensitivity analysis of the in-plane elastic buckling load of the fixed steel arches under uniform compression.

2) Case Study 2

The input variables are shown in Figure 7 and Table II. Global sensitivity based on Sobol's sensitivity indices was applied to evaluate the influence of the input variables on the in-plane elastic buckling load of the pin-ended steel arches under uniform compression. The results after 100,000 simulations in 69.0 minutes are shown in Table IV and Figure 8.
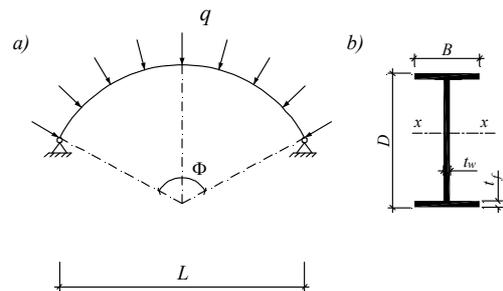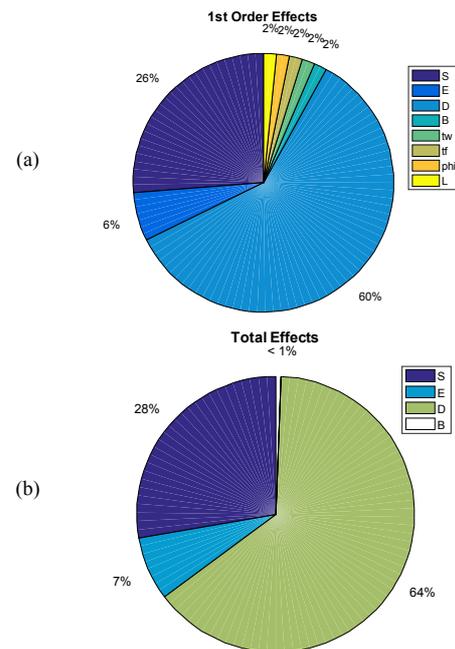
Table IV and Figure 8 show the influence of the input variables on the in-plane elastic buckling load of the pin-ended steel arches under uniform compression based on Sobol's sensitivity indices. It can be seen from Table IV and Figure 9 that *S, E* and *D* are the most influential input random variables with proportions of 26.0%, 6.0% and 60.0% of the first order Sobol's sensitivity indices, respectively and the total order sensitivity indices are 28.0%, 7.0% and 64.0% respectively.

Meanwhile, the least influential input random variables are B, t_w, t_f, L and Φ with a proportion of 2.0%.

From the above case studies, it can be seen that Sobol's sensitivity indices of random input variables on the in-plane elastic buckling load of steel arches condition of the fixed and pin-ended steel arches under uniform compression are not significant. The input random variables have a high sensitivity including S, E, and D. The obtained results show the significance of this research.

V. CONCLUSIONS

This paper proposed an algorithm to assess the structural sensitivity of the in-plane elastic buckling load of steel arches. The numerical process is developed based on Sobol's sensitivity indices and Monte Carlo simulation. The in-plane elastic buckling load of steel arches condition of the fixed and pin-ended steel arches under uniform compression is considered in the structural sensitivity assessment. From the numerical analysis, the flowing conclusions can be drawn:

- The algorithm is based on Sobol's sensitivity indices and Monte Carlo simulation and is developed for the sensitivity analysis of steel arches. A verification of the proposed algorithm is conducted using the Ishigami's test function.

- The developed procedure can be applied for the in-plane elastic buckling load of steel arches as an important recommendation for the selection of the input random variables based on numerical analysis.

- An extended application for other types of structures is highly feasible, however additional numerical tests and verifications are required.

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