Backstepping Control for Induction Motors with Input and Output Constraints

Tung Lam Nguyen  
Department of Industrial Automation  
University of Science and Technology  
Hanoi, Vietnam  
lam.nguyentung@hust.edu.vn

Thanh Ha Vo  
Department of Electrical Engineering  
University of Transportation and Communications, Hanoi, Vietnam  
vothanhhaktd@utc.edu.vn

Nam Duong Le  
Department of Electrical Engineering  
Quy Nhon University  
Quy Nhon, Vietnam  
lenamduong@qnu.edu.vn

Abstract—In practice, the applied control voltage for an induction motor drive system fed by a voltage source inverter has a limit depending on the DC bus capacity. In certain operations such as accelerating, the motor might require an excessively high voltage value that the DC bus cannot supply. This paper presents a control solution for the bounded control input problem of the induction motor system by flexibly combining a hyperbolic tangent function in a backstepping control design procedure. In addition, the Barrier Lyapunov function is also employed to force speed tracking error in a defined value. The closed-loop system stability is proven, and the proposed control is verified through numerical simulations.

Keywords—backstepping; Barrier Lyapunov function; induction motor; FOC

I. INTRODUCTION

Induction motors have been serving as a major workforce in various industrial applications [1, 2] due to their robustness and ease of maintenance. Despite the fact that technologies used in induction motor drive systems are well-established, the drive system still draws control researchers’ attention due to its complicated dynamical properties [3, 5]. In the two most common induction motor drive control techniques, Flux Oriented Control (FOC) [6] based schemes are widely used when compared to Direct Torque Control (DTC) structure [7]. FOC renders the induction motor as a direct current motor in terms of decoupling torque producing and flux forming process. There have been many attempts to control the induction motor based on FOC, from classical PID control [2] to advance nonlinear strategies including model predictive control [8], sliding mode control [9], fuzzy-neural approach [10], and genetic algorithm [11]. Thanks to its systematic design and the ability to cope with system nonlinearities, backstepping method is intensively used in the induction motor drive. Authors in [12] successfully employed backstepping integrated with a high-gain observer for stabilizing the motor drive without information on the rotor speed, flux, and load torque. In order to compensate system uncertainties, backstepping was combined with a recurrent neural network to enhance tracking performance in induction servo systems [13]. A similar approach can be found in [14] where a radial basis function neural network was used. The proposed control guarantees system stability and bounded signals. Exploiting the robustness of sliding mode control, authors in [15] designed a backstepping sliding mode control for dealing with lumped uncertainties in linear induction motor drives. The effectiveness of the proposed control is verified numerically and experimentally. In [16], adaptive backstepping control supported by a fuzzy system for integral action was developed for a linear induction motor. Simulation study showed the control ability to cope with parameter variations and load disturbances. In the quest of compensating system mechanical parameters such as unknown viscous coefficient and load torque, an adaptive backstepping algorithm was designed in [17]. Other applications of backstepping control in induction motor drive can be seen in [18, 19].

The aforementioned researches mainly focus on ways to improve the dynamical responses of the induction motor drive closed-loop system regardless the limitations of the control inputs. It is clearly that in certain circumstances, the motor might require exceeded voltage value that the DC bus cannot supply. If the required voltage is not satisfied, it results in system performance degradation. In this paper, a method is introduced to tackle the problem by manipulating backstepping control with the assistance of a virtual control defined through a hyperbolic tangent function for limiting the control input in a specified value determined by the capacity of the DC bus. In addition, the paper also integrates Barrier Lyapunov function in the design steps in order to force the speed tracking error falling in a desired range.

II. MATHEMATICAL MODEL FORMULATION

The mathematical model of the induction motor is well defined, for more detailed derivation please refer to [20]. The induction motor model in the d-q reference frame is obtained as in (1) where: \( \sigma = 1 - \frac{T_{L}}{L_{d}} \) is the leakage factor, \( T_{r} = \frac{L_{d}}{R_{r}} \) the stator time constant, \( T_{r} = \frac{T_{r}}{R_{r}} \) the rotor time constant, \( k_{r} = \frac{T_{r}}{L_{r}} \), \( R_{o} = R_{r} + R_{s}k_{r}^{2} \), \( T_{r} = \frac{T_{r}}{T_{r}^{*}} \), \( \omega = \omega + \frac{T_{o}^{*}}{T_{r}^{*}} \) the slip estimation, \( \omega \) is the mechanical rotor speed, \( \omega^{*} \) is the number of pole pairs, \( J \) is the rotor and load lumped inertia, \( T_{L} \) the load torque, \( \psi_{sd} \) the
the rotor flux, and \( L_m, L_r, L_s \): mutual, rotor, and stator inductance.

\[
\begin{align*}
\frac{di_{sd}}{dt} &= -\left( \frac{1}{\sigma T_i} + \frac{1-\sigma}{\sigma T_i} \right) i_{sd} + \omega_{r} i_{rd} + \frac{1-\sigma}{\sigma T_i} \psi_{rd} + \frac{1}{\sigma T_i} u_{sd} \\
\frac{di_{rd}}{dt} &= -\omega_{r} i_{sd} - \frac{1}{\sigma T_i} i_{rd} + \frac{1-\sigma}{\sigma T_i} \psi_{rd} + \frac{1}{\sigma T_i} u_{rd} \\
\frac{d\psi_{rd}}{dt} &= -\omega_{r} \psi_{rd} + \frac{1}{\sigma T_i} \psi_{rd} + \frac{1}{\sigma T_i} u_{rd} \\
m_{id} &= \frac{3}{2} z_p L_i \psi_{rd} i_{sd} = \frac{3}{2} z_p (1-\sigma)L_p \psi_{rd} i_{sd}
\end{align*}
\]

The first two equations characterize motor current dynamics. The last two equations denote motor flux forming process and equation of motion. It can be observed that the induction motor given in (1) is a coupled and nonlinear system. The block diagram representation of the induction motor is given in Figure 1.

**Fig. 1.** Structure of the induction motor in \(dq\)-coordinates.

### III. Backstepping Controller Design with Control Input and Output Constrains

#### A. Magnetic Flux Control Design

The control objective is to drive the motor magnetic flux to a desired value in such a way that the tracking error stays in a predefined range. In order to achieve this objective, the tracking error is defined as follows:

\[ z_1 = x_1 - \phi_{des} \]

and a Lyapunov candidate function is:

\[ V_1 = \frac{1}{2} \log \left( \frac{k^2}{k^2 - z^2_{1, id}} \right) \]

Taking the time derivative of \( V_1 \) gives:

\[ \dot{V}_1 = \frac{z_{1, id} (x_2 - \dot{q}_2)}{k_b - z^2_{1, id}} \]

At this point, \( x_2 \) is considered as a control input, the error between \( x_2 \) and the desired control value is defined as:

\[ z_2 = x_2 - \alpha_1 \]

where \( \alpha_1 \) is the virtual control. Substituting (5) into (4) gives:

\[ \dot{V}_1 = \frac{z_{1, id} (x_2 + \alpha_1 - \dot{q}_2)}{k_b - z^2_{1, id}} \]

Equation (6) suggests that the virtual control can be selected as:

\[ \alpha_1 = c_1 z_{1, id} + x_2 + \frac{\dot{q}_2 - \dot{q}_2}{k_b - z^2_{1, id}} \]

The virtual control renders \( \dot{V}_1 \) as:

\[ \dot{V}_1 = -c_1 z^2_{1, id} + z_{1, id} \dot{z}_2 \]

(8)

To deal with the coupling term \( z_1 z_2 \) and force \( z_1 \) to zero, we propose a Lyapunov candidate function as:

\[ V_2 = \frac{1}{2} z^2_{1, id} + V_1 \]

(9)

The derivative of \( V_2 \) will have the following form:

\[ \dot{V}_2 = -c_1 z^2_{1, id} + z_{1, id} \dot{z}_2 + z_{1, id} \dot{z}_2 \]

(10)

Taking the derivative of (5) we have:

\[ \dot{z}_2 = -c_1 z_{1, id} \left[ \frac{\dot{q}_2 - \dot{q}_2}{k_b - z^2_{1, id}} \right] \]

(11)

Substituting the first and second equations (10) to (11) results in:

\[ \dot{z}_2 = \frac{1}{T_e} \left( \frac{1}{k_b - z^2_{1, id}} \right) \left( \frac{1}{T_e} \right) + \frac{1}{T_e} \left( \frac{1}{k_b - z^2_{1, id}} \right) + \frac{1}{T_e} \left( \frac{1}{k_b - z^2_{1, id}} \right) \]

(12)

From (10) and (12) the control signal can be achieved guaranteeing \( x_1 \) converges to \( dq \) with the dynamical error confined in a range defined by \( k_b \). Another control objective is to specify the control input subject to a limitation. It is necessary to consider input constraints in the control design since the applied voltage is supplied from an inverter whose output voltage is practically limited. Toward this end, the limitation of the output voltage is described as a tangent function.

\[ u_{sd} = g(\theta) = u_{sd} \tan \left( \frac{\theta}{u_{sd}} \right) \]

(13)

where \( u_{sd} \) is the control limit. The design of \( u_{sd} \) is passed to the construction of \( \theta \). Equation (13) results in:
\[
g'(u) = \frac{\partial g(v)}{\partial u} U (14)\]

The control design of \(u_{\text{ref}}\) is subject to \(u_{\text{ref}} \leq u_{\text{ref}}\) is a shift to a task of finding a virtual signal \(U\) defined by:

\[
\dot{u} = \frac{\partial g(v)}{\partial u} U (15)\]

To proceed we define:

\[
z_1 = u_{\text{ref}} - \alpha_z = g(v) - \alpha_z (16)\]

From (12) and (16) the virtual control \(\alpha_z\) can be selected as:

\[
\begin{align*}
\alpha_z &= -\sigma L_t \sigma (k_1^2 - z_1^2) z_t + \frac{1}{\sigma T_x} \left( \frac{x_2}{T_x} + \omega \sigma z_1 \right) \\
+ &\frac{1}{\sigma T_x} \left( \frac{x_2}{T_x} + \omega \sigma z_1 \right) \\
+ &\frac{1}{\sigma T_x} \left( \frac{x_2}{T_x} + \omega \sigma z_1 \right) \\
&+ \frac{2z_1^2}{(k_1^2 - z_1^2)^2} \left( \frac{1}{T_x} \sigma z_1 \right) \\
&- \dot{z}_2 + c_z \dot{z}_2
\end{align*}
\]

Virtual control \(\alpha_z\) renders \(\dot{V}_2\) as:

\[
\dot{V}_2 = -c_z z_1^2 - c_z z_1^2 + \frac{1}{(k_1^2 - z_1^2)^2} \left( \frac{x_2}{T_x} + \omega \sigma z_1 \right) \sigma z_1 + c_z \dot{z}_2
\]

In order to remove the coupling term related to \(z_2 z_3\) and drive \(g(v)\) to track \(\alpha_z\), a Lyapunov candidate function is proposed as follows:

\[
V_3 = \frac{1}{2} z_1^2 + \dot{V}_2 (19)
\]

The derivative of (19) yields:

\[
\dot{V}_3 = -c_z z_1^2 - c_z z_1^2 + \frac{1}{(k_1^2 - z_1^2)^2} \left( \frac{x_2}{T_x} + \omega \sigma z_1 \right) \sigma z_1 + c_z \dot{z}_2 (20)
\]

Taking the derivative of (16) and substituting the result into (20), we can finally get the virtual control that guarantees bounded control input as:

\[
U_1 = -\frac{1}{(k_1^2 - z_1^2)^2} \left( \frac{x_2}{T_x} + \omega \sigma z_1 \right) \sigma z_1 + \dot{z}_2 - c_z \dot{z}_2 (21)
\]

Substituting the proposed control into (20) gives:

\[
\dot{V}_3 = -c_z z_1^2 - c_z z_1^2 - c_z z_1^2 \leq 0 (22)
\]

and the design process for magnetic flux is completed.

B. Speed Control Design

The speed control design with input and output constrains is carried out in the same manner as in the previous section. For the sake of abstract presentation, control design steps are summarized in this section as follows:

\[
t_1 = x_3 - \omega_{\text{ref}} (23)
\]

\[
\beta_1 = -d_1 t_1 + x_3 - \frac{x_3 - \omega_{\text{ref}}}{k_1} (24)
\]

\[
t_2 = x_4 - \beta_2 (25)
\]

\[
\begin{align*}
\beta_2 &= -\frac{J \sigma L_t (k_1^2 - t_1^2)}{k_1 x_1} \left( \frac{x_2}{T_x} \sigma x_1 - \omega_{\text{ref}} - \frac{1}{\sigma} \right) \\
&+ \frac{k_1 x_1}{J (k_1^2 - t_1^2)} \left( \frac{1}{T_x} \sigma x_1 - \omega_{\text{ref}} \right) t_1 \\
&+ \frac{2 \omega^2}{(k_1^2 - t_1^2)^2} \left( \frac{k_1 x_1 x_2}{J} - \omega_{\text{ref}} \right) + d_1 \left( \frac{k_1 x_1 x_2}{J} - \omega_{\text{ref}} \right) + d_2 t_1 \\
t_3 &= u_{\text{ref}} - \beta_2 = g(v) - \beta_2 (27)
\end{align*}
\]

where \(\beta_1, \beta_2\) are virtual controls and \(t_1, t_2, t_3\) are the backstepping design errors in each step. The control signal guaranteeing bounded input and output is \(U_2\).

IV. SIMULATION RESULTS

A. Settings

Simulations were conducted on an IM machine with the structure control for induction motor shown in Figure 2. The space vector modulation is used for the inverter part. \(R_s, R_r,\) and \(R_t\) are flux, speed, and \(dq\)-current controllers, respectively. Simulation parameters are given in Table I. In the simulation, two control schemes were implemented: conventional and input-output embedded backstepping controls.

<table>
<thead>
<tr>
<th>Table I. Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rated power</strong></td>
</tr>
<tr>
<td><strong>Speed motor</strong></td>
</tr>
<tr>
<td><strong>Rated phase current</strong></td>
</tr>
<tr>
<td><strong>Number of pole pairs</strong></td>
</tr>
<tr>
<td><strong>Rotor resistance</strong></td>
</tr>
<tr>
<td><strong>Stator resistance</strong></td>
</tr>
<tr>
<td><strong>Rotor inductance</strong></td>
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<tr>
<td><strong>Stator inductance</strong></td>
</tr>
<tr>
<td><strong>Mutual inductance</strong></td>
</tr>
<tr>
<td><strong>Total inertia</strong></td>
</tr>
</tbody>
</table>

B. Simulation Procedure

At \(t=0\)s, magnetic current is created and at \(t=2\)s the system speeds up to 1200rpm. In the simulation scenario, a sudden torque with rated value of 1.5Nm is applied on the motor shaft. The results of the simulation of stator current and speed controllers using conventional backstepping control method and backstepping with control input and output constrains are shown in Figures 3-8 where (a) represents the response of conventional backstepping and (b) represents the backstepping with input and output constraints. The proposed backstepping controller is applied to the induction motor. The coefficients are chosen as: \(c_p=1000, c_r=1000, c_p=5150, c_r=1000, d_p=500, d_r=500, d_p=1000, d_r=500.\)
In the simulation result, the control signal constraint is selected as:

$$U_c = \frac{u_{sd}^2 + u_{sq}^2}{2} \leq 160V \quad (29)$$

Constraints on $dq$ are determined according to rectangular approximation:

$$u_{dq}^{\text{max}} = \alpha \frac{V_{dc}}{\sqrt{3}} \quad \text{and} \quad u_{ad}^{\text{max}} = \sqrt{1-\alpha^2} \frac{V_{dc}}{\sqrt{3}} \quad (30)$$

where $V_{dc}=300V$ and $\alpha=0.8$. This condition implies that control inputs must satisfy:

$$|u_{dq}| \leq u_{dq}^{\text{max}} \quad \text{and} \quad |u_{sd}| \leq u_{ad}^{\text{max}} \quad (31)$$

The designed controller can also limit the system’s output through the value $k_b$. This helps us to control the overshoot of the system. In the simulation, we set the value $k_b=0.1$.

For the system controlled by the conventional backstepping method, we set the dc bus voltage to $V_{dc} = 300V$. This implies that there is no hard limit applied in this case. On the contrary, the dc bus voltage is restricted to 160V when the induction motor drive system is regulated by backstepping control with input and output constraints. In both cases, simulation results show that the two approaches provide good tracking performance with fast response time as can be seen in Figure 3.
the control design to deal with the voltage saturation problem. This approach differs from other works where either torque producing or flux forming voltage is sacrificed when the input voltage enters the saturation zone. The simulation results show that the control can suppress the control input voltage in the prescribed domain while maintaining the tracking error due to the help of the barrier Lyapunov function. In future research, we aim at deploying the proposed control in an experimental rig.

REFERENCES


Nguyen et al.: Backstepping Control for Induction Motors with Input and Output Constrains


