

Nonlinear Inelastic Analysis of 2D Steel Frames

An Improvement of the Plastic Hinge Method

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Abstract—In this study, a new method for nonlinear analysis of 2D steel frames, by improving the conventional plastic hinge method, is presented. The beam-column element is established and formulated in detail using a fiber plastic hinge approach. Residual stresses of I-shape sections are declared at the two ends through fibers. Gradual yielding by residual stresses along the member length due to axial force is accounted for by the tangent elastic modulus concept. The $P-\delta$ effect is captured by stability functions, whereas the $P-\Delta$ effect is estimated by the geometric stiffness matrix. A nonlinear algorithm is established for solving nonlinear problems. The present study predicts the strength and behavior of 2D steel frames as efficiently and accurately as the plastic zone method did.

Keywords—fiber plastic hinge; nonlinear algorithm; residual stress; stability functions; steel frames

I. INTRODUCTION

Nowadays, direct design for steel frames is permitted by the modern design codes. A direct design including the effects of geometric nonlinearity, inelasticity of materials, imperfections, residual stress, etc. is accounted directly and simultaneously in advanced analysis. There are usually two methods for advanced analysis: plastic hinge methods [1-15] and plastic zone methods [3, 16-25]. Authors in [14, 15, 22, 23, 26] invented a spring element for accounting stiffness of beam-to-column connections for nonlinear behavior analysis of steel frames with flexible connections. Recently, authors in [27-29] tried to investigate the behavior of steel frames with the effects of connections. Author in [30] investigated the effect of the Iranian standard no. 2800 on the elastic and inelastic behavior of dual steel systems by using the nonlinear pushover analysis of commercial software Sap2000. Up to now, in spite of the developments in computer science and technology, plastic zone methods are still expensive for the daily engineering design of steel frames. Plastic hinge methods are more simple, efficient in computational time, and with acceptable accuracy, so they are suitable for practical design. Plastic hinge methods have been studied widely from 1980 to 2000 [1-8]. Authors in [1, 3] used Hermite interpolation functions to predict the displacements of beam-column elements. Plastic hinges were assumed to concentrate at the two elemental ends. For considering geometric nonlinearity, the beam-column elements were divided into many short-elements. Residual stresses and imperfections were not accounted for in direct analysis.

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Authors in [6, 7] used a five-order interpolation function for considering the second-order effects of beam-column elements. The plasticity of cross-sections at the two ends of the element is modeled by two springs using the section assemblage concept. The tangent stiffness matrix of a structural system is established by integrating the stiffness of beam-column elements and the stiffness of springs. Authors in [10] derived a finite element formulation for a beam-column element that had an arbitrarily plastic hinge along the element length. Authors in [9] proposed a second-order inelastic large-deflection analysis method using only one element per member, including three plastic hinges in one member.

In 2014, Liu et al. [13] also proposed an arbitrarily-located plastic hinge element for direct analysis of planar steel frames. Their method was directly developed from the initial out-of-straight element. King et al. [5] proposed a second-order inelastic analysis method for steel frames. This method employed stability functions for predicting the second-order effects accurately. Gradual yielding at plastic hinges accounted for using LRFD's interactive equations. Gradual yielding by residual stresses along the member length due to axial force was calculated by the CRC tangent modulus concept. One element was used for modeling. In 2002, Ziemian and McGuire [8] improved the result of plastic hinge method using modified tangent modulus formulation. The result was nearly identical with the sophisticated plastic zone method in some examples, but the method should be verified even more with different problems. The plastic hinge method of [5] is effective and saves computational time because it uses only one element per member. However, residual stresses at two plastic hinges are not considered in the analysis, and in some problems, the result has a significant error when compared with the 'exact' solutions using plastic zone methods.

This study tries to develop a new plastic hinge method which can capture the nonlinear behavior of steel frames accurately. The proposed method employs the fiber discretization of a cross-section. Stability functions and the geometric stiffness matrix are utilized for accounting for the second-order effects. A nonlinear static algorithm implementing the generalized displacement method is established for solving nonlinear problems. By some examples, the present method is proved to be reliable and straightforward for tracing the nonlinear behavior of 2D steel frames.

II. BEHAVIOR OF THE BEAM-COLUMN ELEMENT

A. $P-\delta$ Effect

Stability functions studied in [31] were utilized for predicting the $P-\delta$ effect. With one element for the member, it is efficient to economize sources and analysis time. The force-displacement relationship using the incremental form of a 2D beam-column element can be written as:

$$\begin{Bmatrix} \Delta P \\ \Delta M_I \\ \Delta M_J \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} A/I & 0 & 0 \\ 0 & S_1 & S_2 \\ 0 & S_2 & S_1 \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_I \\ \Delta \theta_J \end{Bmatrix} \quad (1)$$

where ΔP , ΔM_I , and ΔM_J are the axial force and moments, $\Delta \delta$, $\Delta \theta_I$, and $\Delta \theta_J$ are the axial movement and rotations, A is the sectional area, I is the moment of inertia around the z axis, L is the elemental length, E is the Young's modulus of the steel, and S_1 and S_2 are stability functions.

B. Fiber Plastic Hinge

Figure 1 illustrates the fiber plastic hinge method. In this method, two ends, I and J , of the element are monitored regarding the behavior of stress and strain of fibers. The force-displacement relation of a 2D element considering both the $P-\delta$ effect and plasticity can be formulated as in (2):

$$\begin{Bmatrix} \Delta P \\ \Delta M_I \\ \Delta M_J \end{Bmatrix} = \frac{E_t I}{L} \begin{bmatrix} A/I & 0 & 0 \\ 0 & \eta_I(S_1 - \frac{S_2^2}{S_1}(1-\eta_J)) & \eta_I \eta_J S_2 \\ 0 & \eta_I \eta_J S_2 & \eta_J(S_1 - \frac{S_2^2}{S_1}(1-\eta_I)) \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_I \\ \Delta \theta_J \end{Bmatrix} \quad (2)$$

where η_I and η_J are scalar parameters accounting for gradual yielding of fiber hinges.

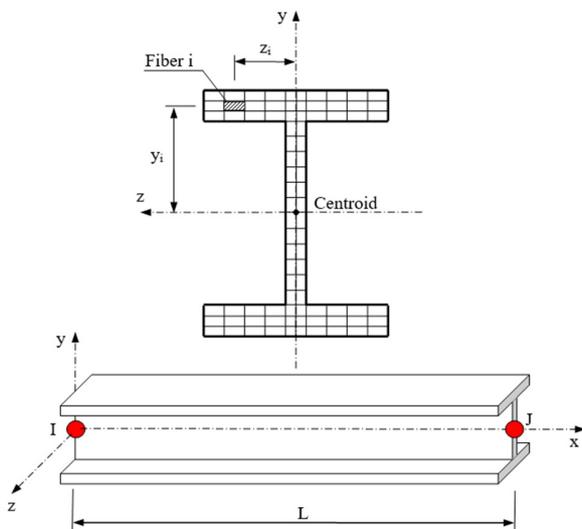


Fig. 1. Fiber plastic hinge method.

They are estimated as:

$$\eta_I = \frac{\sum_{i=1}^n E_{tIi} (A_i y_i^2 + I_i)}{E_t I} \quad (3)$$

$$\eta_J = \frac{\sum_{i=1}^n E_{tJi} (A_i y_i^2 + I_i)}{E_t I} \quad (4)$$

where n is the sum of fibers on the sections at I and J , E_{tIi} and E_{tJi} are the tangent modulus of the i^{th} fiber at I and J , A_i is the area of the i^{th} fiber, I_i is the moment of inertia of the i^{th} fiber, y_i is the center coordinate of the i^{th} fiber, shown in Figure 1, and E_t is the tangent modulus of an element.

C. Residual Stresses

From [5], the CRC tangent modulus E_t is applied with the aim to consider the effects of residual stresses along the length. This effect is similar to the spread of plasticity on the length due to axial force, formulated as:

$$E_t = E \text{ for } P \leq 0.5P_y \quad (5)$$

$$E_t = 4 \frac{P}{P_y} \left(1 - \frac{P}{P_y} \right) E \text{ for } P > 0.5P_y \quad (6)$$

where P_y is squash load. ECCS residual stress [2] is admitted as initial condition to fibers, as shown in Figure 2.

D. Fiber State

The cross-section ends of the element are divided into several fibers n for considering gradual yielding of two fiber plastic hinges at I and J , as shown in Figure 1. Fibers are monitored and their behavior (stress, strain) is updated. If the fiber is yielded, its elastic modulus is equal to zero. Axial strain $\Delta \epsilon$ and curvature $\Delta \chi$ of cross-section, and section forces are written as:

Sectional force vector:

$$\{\Delta N \quad \Delta M\}^T \quad (7)$$

Sectional deformation vector:

$$\{\Delta \epsilon \quad \Delta \chi\}^T \quad (8)$$

The sectional deformation vector is estimated as:

$$\begin{Bmatrix} \Delta \epsilon \\ \Delta \chi \end{Bmatrix} = \begin{bmatrix} \sum_{i=1}^n E_i A_i & -\sum_{i=1}^n E_i A_i y_i \\ -\sum_{i=1}^n E_i A_i y_i & \sum_{i=1}^n E_i (A_i y_i^2 + I_i) \end{bmatrix}^{-1} \begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix} \quad (9)$$

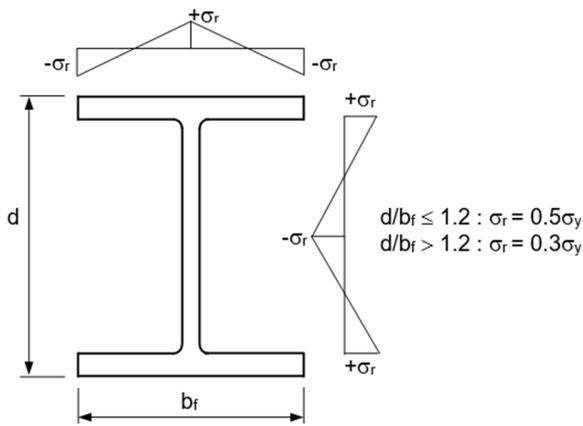


Fig. 2. ECCS residual stress pattern.

E. $P-\Delta$ Effect

The elemental tangent stiffness matrix is written as:

$$[K]_T = [T]_{3 \times 5}^T [K_e]_{3 \times 3} [T]_{3 \times 5} + [K_g]_{5 \times 5} \quad (10)$$

where the transformation matrix $[T]_{3 \times 5}$ of the element is formulated as:

$$[T]_{3 \times 5} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/L & -1/L \\ 0 & 0 & 1 & 1/L & -1/L \end{bmatrix} \quad (11)$$

and $[K_g]_{5 \times 5}$ is the geometric stiffness matrix:

$$[K_g]_{5 \times 5} = \begin{bmatrix} 0 & 0 & 0 & \frac{M_I + M_J}{L^2} & -\frac{M_I + M_J}{L^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{M_I + M_J}{L^2} & 0 & 0 & \frac{P}{L} & -\frac{P}{L} \\ -\frac{M_I + M_J}{L^2} & 0 & 0 & -\frac{P}{L} & \frac{P}{L} \end{bmatrix} \quad (12)$$

III. NONLINEAR SOLUTION

The nonlinear solution algorithm invented by Yang and Shieh [32] is developed to find a solution to the structural system. Yang and Shieh's method is one of the most efficient and stable numerical methods. It can easily capture the problems with several critical points. The equilibrium equation of steel frames is:

$$[K_{j-1}^i] \{\Delta D_j^i\} = \lambda_j^i \{\hat{P}\} + \{R_{j-1}^i\} \quad (10)$$

where $[K_{j-1}^i]$ is the tangent stiffness matrix, $\{\Delta D_j^i\}$ is the displacement vector, $\{\hat{P}\}$ is the consultation load vector, $\{R_{j-1}^i\}$ is the residual force vector, λ_j^i is the load coefficient.

IV. EXAMPLES AND DISCUSSION

A. Column Flexural Buckling

Figure 3 illustrates a simply supported steel column under axial compression. Young's modulus is $E=200000\text{Mpa}$ and Poisson's ratio of steel is $\nu=0.3$. A horizontal load is put in the middle of the column for considering geometry imperfection.

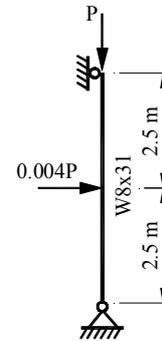


Fig. 3. Simply supported column.

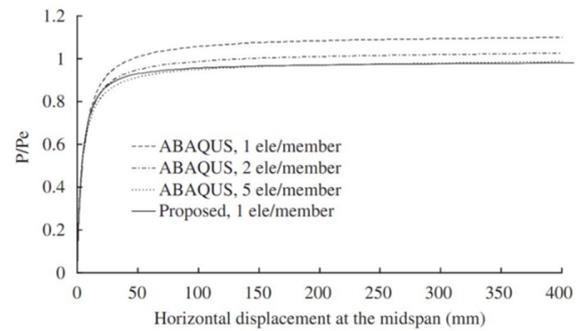


Fig. 4. Load-deflection curves of the column.

The load-deflection curves of the column captured by the present study and ABAQUS are plotted in Figure 4. By one element per column, the presented method can predict precisely the behavior and strength of the column, but ABAQUS overestimates the strength by 10.6%. ABAQUS needs more than five elements for obtained accuracy. This example illustrates the accuracy of the presented study in predicting the second-order effect.

B. Portal Steel Frame

Vogel [3] invented the portal steel frame for benchmark second-order inelastic methods. Nguyen and Kim [25] proposed a plastic zone method for analyzing this frame. The frame configuration is described in Figure 5. The elastic modulus is $E=205000\text{Mpa}$, the yield stress of the steel is $\sigma_y=235\text{MPa}$. The cross-sections are HEA340 and HEB300. Authors in [3, 25] used 50 elements for columns and 40 elements for the beam for analyzing the frame, while this present program uses one element for beam-column members, and I-shape cross-sections have been meshed into 24 fibers for flanges and 18 fibers for the web. The load-deflection curve of the present study closely matches with Vogel's result, as plotted in Figure 6. Nguyen and Kim's result is lower than Vogel's result: -2.05% error is obtained when compared with

Vogel's result. The collapse load coefficient of the different applied methods is listed in Table I. Less than 0.8% error of the present study is achieved when comparing with Vogel's result. Analyzing this problem on a computer, with a configuration of Intel Core i7-7500 4CPUs 2.70GHz and 16GB RAM, the analyzing time is only 15s. This shows the accuracy and computation speed of the proposed method.

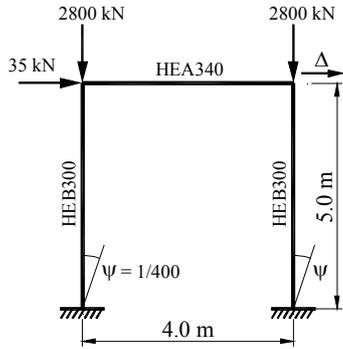


Fig. 5. Portal steel frame.

TABLE I. COLLAPSE LOAD COEFFICIENT FOR PORTAL FRAME

Method	Collapse load coefficient	Error (%)
[3]	1.022	-
[25]	1.001	-2.05
Present study	1.014	-0.78

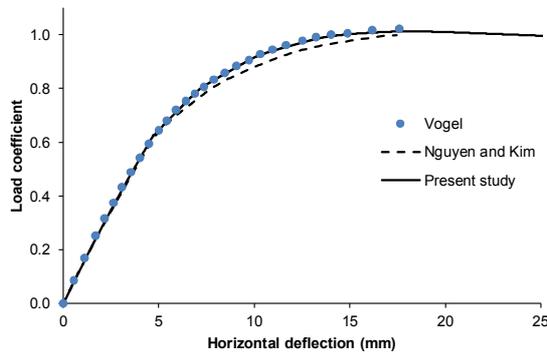


Fig. 6. Load-deflection curves of portal frame.

C. Six-story Steel Frame

The six-story steel frame plotted in Figure 7 was firstly analyzed by Vogel in [3]. Vogel used both plastic zone and plastic hinge methods, Chan and Chui [7] used the refined plastic hinge method, Nguyen and Kim in [22] used a fiber beam-column method and in [25] the plastic zone method. All the columns are inclined with an angle of $\psi = 1/450$. Properties of steel are $E=205000\text{MPa}$ and $\sigma_y=235\text{MPa}$. In the present study, five elements for beams and one for columns were used for modeling. Figure 8 and Table II show the results predicted by various methods. In this frame, the predicted load-deflection curve of various methods is not much different. The load-deflection curve of the present study is almost identical to Vogel's plastic zone method. The critical strength predicted by the proposed method (1.116) has less than 0.45% error when compared with Vogel's result (1.111) using

the plastic zone method. Analyzing this problem on the same Intel computer, it takes only 53s, showing that the proposed program is accurate and efficient in predicting the nonlinear behavior and the strength of 2D steel frames.

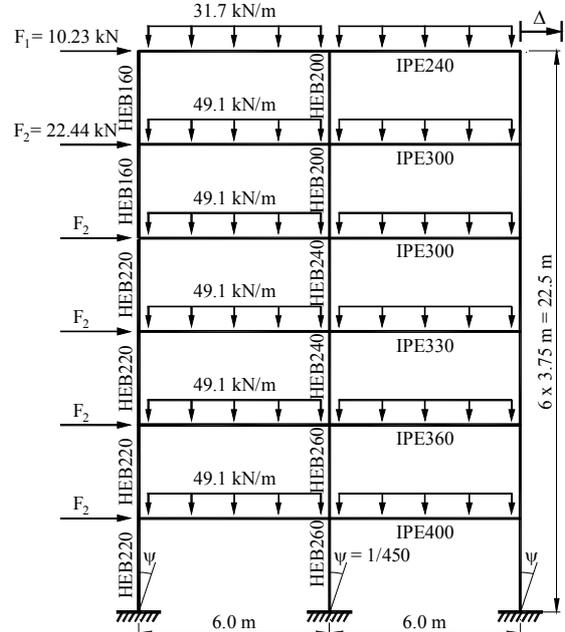


Fig. 7. Six-story steel frame.

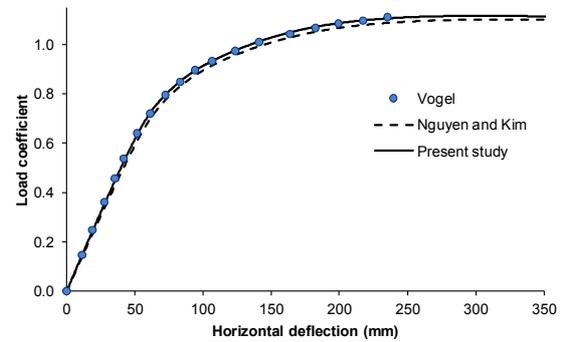


Fig. 8. Load-deflection curves of a six-story frame.

TABLE II. COLLAPSE LOAD COEFFICIENT FOR A SIX-STORY FRAME

Method	Collapse load coefficient	Error (%)
[3]	1.111	-
[25]	1.100	-0.99
Present study	1.116	+0.45

V. CONCLUSION

A second-order inelastic analysis program based on the finite element method for 2D steel frames has been developed successfully. The effects of P-δ, P-Δ, inelasticity of materials, residual stresses, and imperfections have been accounted for the nonlinear analysis by the generalized displacement method. The proposed method is simple, accurate, and efficient in predicting the strength and behavior of steel frames. The

proposed method can be integrated into commercial software for daily engineering design using advanced analysis.

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